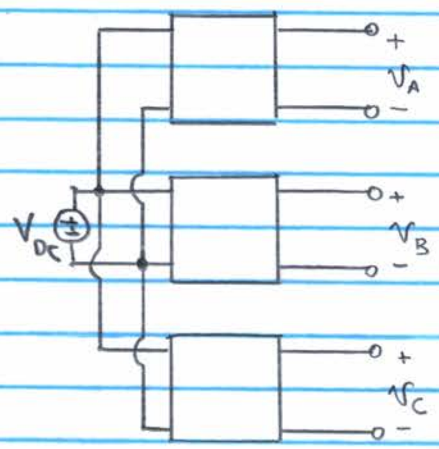
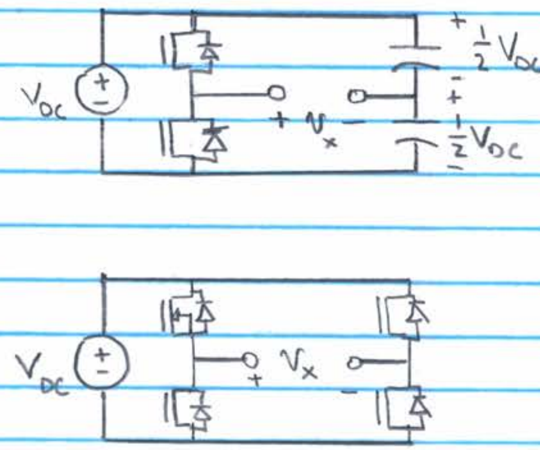
6.334 3-phase inverters

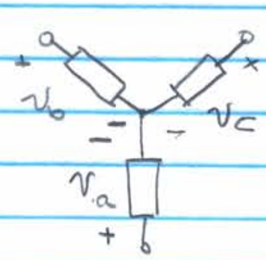
Consider implementation of an inverter for 3-phase using three single-phase inverters (e.g. full-bridge or half-bridge), one for each phase:

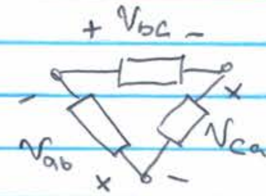




A half-bridge inverter requires only two devices and can synthesize a positive and a negative output $\{+\frac{1}{2}V_{DC}, -\frac{1}{2}V_{DC}\}$ but no zero state, while a full-bridge inverter can generate any of positive, negative and zero $\{+V_{DC}, -V_{DC}, 0\}$.

One might think that to realize a balanced 3-phase inverter would require as many as twelve devices to synthesize the desired output patterns. However, most 3-phase loads are connected in wye or delta, placing constraints on the instantaneous voltages that can be applied to each branch of the load



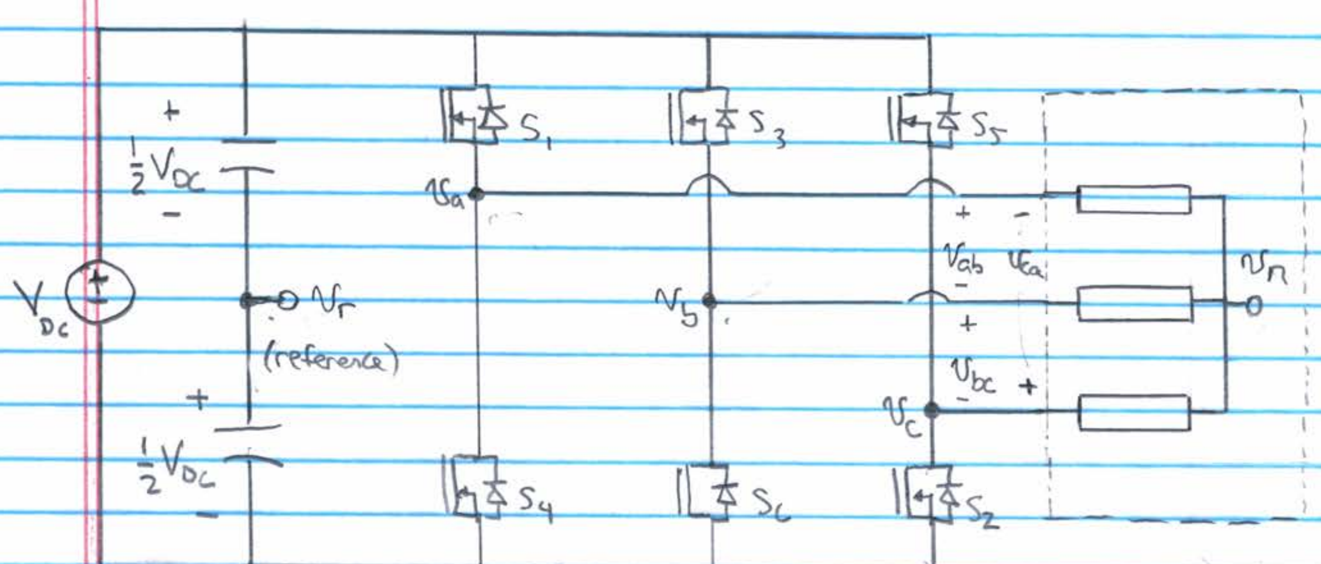


For the wye connection all the "negative" terminals of the inverter outputs are tied together, and for the delta connection the inverter output terminals are cascaded in a ring.

6.334 3-phase inverters

The load connections both limit the instantaneous voltages that may be synthesized with inverters comprising bridge legs fed from a single dc bus (without shorting the dc bus) and reduce the number of half-bridges needed to synthesize the allowed patterns.

In particular considering "full-bridge" structures, half of the devices become redundant, and we can realize a 3-phase bridge inverter using only six switches (three half-bridge legs):



The 3-phase bridge comprises 3 half-bridge legs (one for each phase: a, b, c). The devices are often traditionally numbered as illustrated (conveying conduction order in "square wave" or "six step" operation, as is done for rectifiers. For symmetry and convenience, we utilize the midpoint of the dc bus as a voltage reference node. The connected load could be wye or delta, but we illustrate it as a wye connection with internal (unconnected) neutral point.

6.334 3-phase inverters

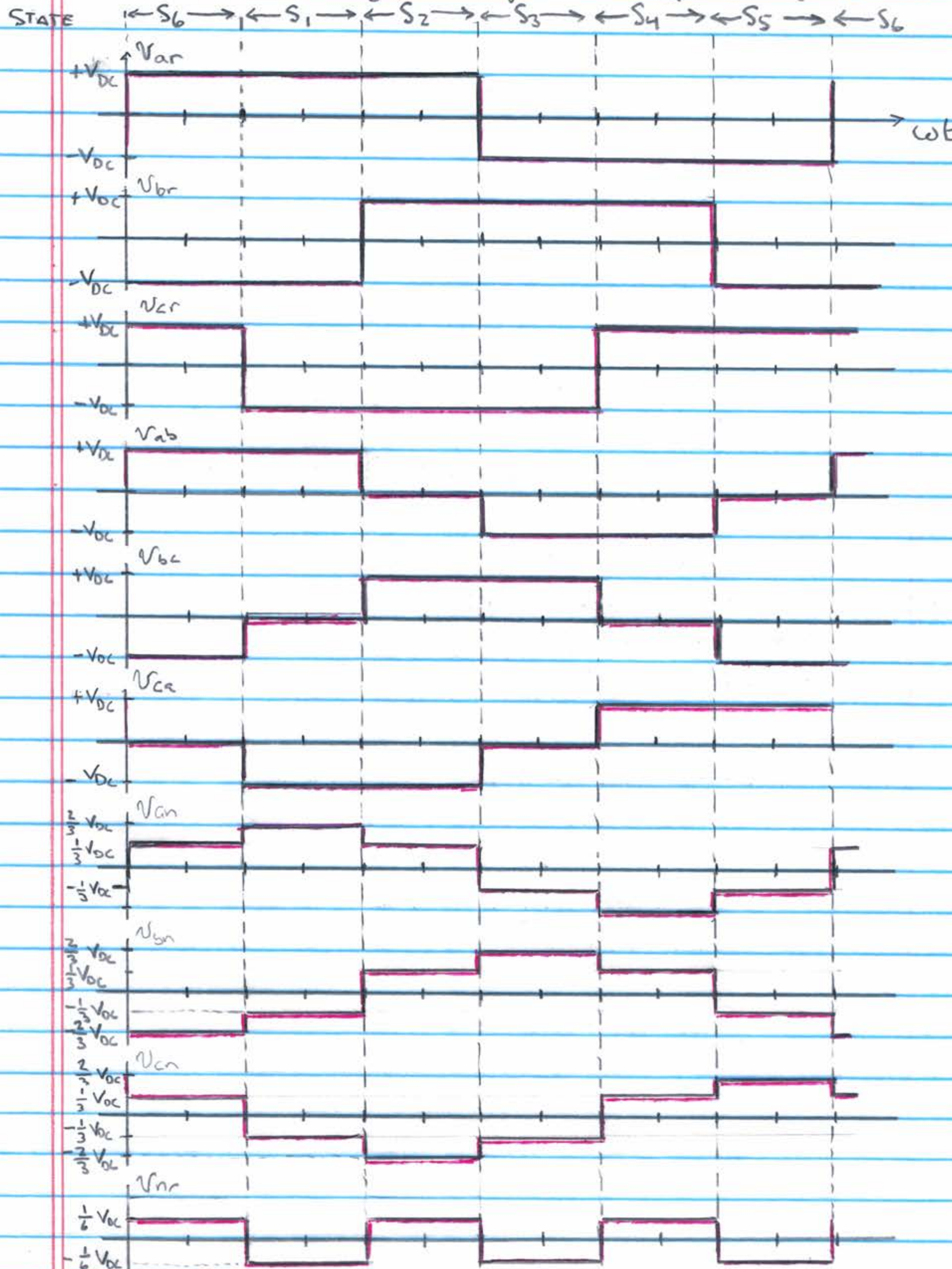
Considering inverter states in which one switch in each half-bridge is always on (for current continuity at the load) there are $2^3 = 8$ switch state possibilities for the 3-phase inverter. We give each state a vector designation and associated number corresponding to whether the top or bottom switch in each half-bridge is on.

- We can directly calculate the bridge output to reference voltages (V_{ar}, V_{br}, V_{cr}) and the output line-to-line voltages (V_{ab}, V_{bc}, V_{ca}) for each switch state.
- For a balanced 3-phase load we can calculate what the line to (unconnected) neutral point voltages would be (and the neutral point to reference voltage), if the load had such a neutral point. By balanced load in this instance, we refer to a load which acts to divide the applied voltages just as would a set of 3 equal impedances connected to a (floating) neutral point.

Vector name	Switches	V/V_{DC}									
{abc}	on	V_{ar}	V_{br}	V_{cr}	V_{ab}	V_{bc}	V_{ca}	V_{an}	V_{bn}	V_{cn}	V_{nr}
$S_0=\{000\}$	4,6,2	$-1/2$	$-1/2$	$-1/2$	0	0	0	0	0	0	$-1/2$
$S_1=\{100\}$	1,6,2	$+1/2$	$-1/2$	$-1/2$	1	0	-1	$2/3$	$-1/3$	$-1/3$	$-1/6$
$S_2=\{110\}$	1,3,2	$+1/2$	$+1/2$	$-1/2$	0	1	-1	$1/3$	$1/3$	$-2/3$	$+1/6$
$S_3=\{010\}$	4,3,2	$-1/2$	$+1/2$	$-1/2$	-1	1	0	$-1/3$	$2/3$	$-1/3$	$-1/6$
$S_4=\{011\}$	4,3,5	$-1/2$	$+1/2$	$+1/2$	-1	0	1	$-2/3$	$1/3$	$1/3$	$+1/6$
$S_5=\{001\}$	4,6,5	$-1/2$	$-1/2$	$+1/2$	0	-1	1	$-1/3$	$-1/3$	$2/3$	$-1/6$
$S_6=\{101\}$	1,6,5	$+1/2$	$-1/2$	$+1/2$	1	-1	0	$1/3$	$-2/3$	$1/3$	$+1/6$
$S_7=\{111\}$	1,3,5	$+1/2$	$+1/2$	$+1/2$	0	0	0	0	0	0	$+1/2$

6.334 3-phase inverters

Lets consider inverter waveforms for the simple case where the 3 bridge legs each generate square waves spaced by $1/3\ T$:



6.334 3-phase inverters

This inverter operating mode – sometimes aptly called "six-step" mode – cycles sequentially through six of the 8 states defined above. The other two states are "zero" states which effectively short circuit the load terminals together. These provide means to apply zero-state voltages to the load when desired (e.g. for PWM control of the fundamental output voltage amplitudes)

Note that as the line-to-line voltages are formed as differences of identical waveforms shifted by $T/3$, they cannot contain any triple-n harmonics (in this mode or any other balanced 3-phase operating pattern).

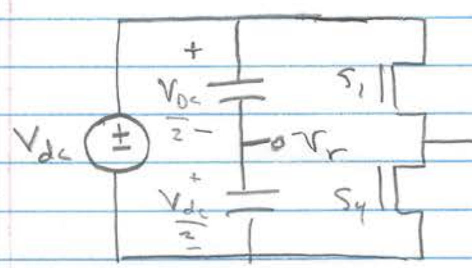
Likewise, for a 3-phase load network acting like 3 identical impedances connected to a (floating) neutral point, the neutral-point voltage becomes the average of the three phase voltages. If v_{ar}, v_{br}, v_{cr} are identical but shifted by $T/3$, they all have the exact same triple-n harmonic frequency content, and thus the neutral point "average" voltage v_{nr} will also have the exact same triple-n harmonic content. Consequently, the line-to-neutral voltages ($v_{xn} = v_{xr} - v_{nr}, x \in a,b,c$) will have no triple-n harmonic content. For the special case of six-step operation (v_{ar}, v_{br}, v_{cr} square waves), the neutral voltage v_{nr} becomes a square wave at a frequency 3X the fundamental.

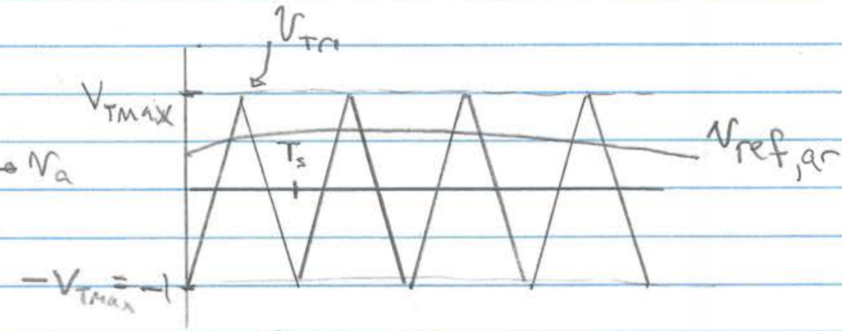
The harmonic cancellation effect does limit the amplitude of the l-l and l-n fundamental voltages that can be synthesized to < that achievable with square waves (e.g. as seen in the waveforms of v_{ab} or v_{an}).

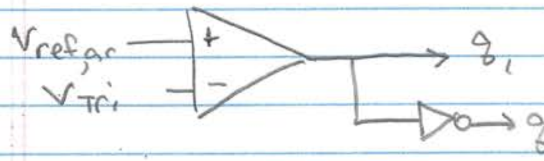
If the neutral point (when available) is connected to the reference point, one gets very different operation. In that case we have three separate half bridges driving three independent phase windings, with no limitations on harmonic content, a slightly larger achievable fundamental, but all the synthesis limitations of half bridges.

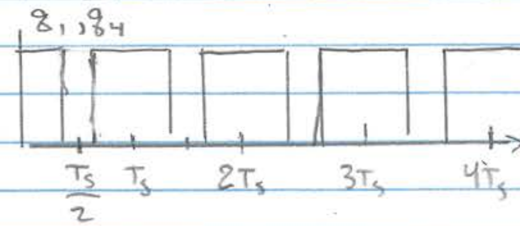
6.334 3-phase inverters

Pulse-width modulation. There are multiple ways PWM might be realized. A simple one is to realize "sine Δ" pwm on each half-bridge









over a switching period $\overline{v_{ar}} = \frac{V_{dc}}{2} \cdot \left(\frac{v_{ref,ar}}{V_{TMAX}}\right)$

So long as $|v_{ref}| < V_{Tmax}$

Suppose we wanted to synthesize a sine wave at a "line" frequency $\omega_\ell \ll \omega_s$ ($\omega_s = 2\pi/T_s$)

$\overline{v_{ar}} = V_m \sin(\omega_\ell t)$ then $v_{ref,ar} = \boxed{\frac{V_m}{V_{dc}/2\,(V_{Tmax})}} \sin(\omega_\ell t)$

for $V_{T,max} = 1$ $v_{ref,ar} = \frac{V_m}{V_{dc}/2} \cdot \sin(\omega_\ell t)$

modulation index m

This synthesizes a low-frequency sinusoidal component with a modulation index $m = \frac{V_m}{V_{dc}/2}$.

- for $\omega_s \gg \omega_\ell$ we get no low-frequency distortion in $\overline{v_{ar}}$ (or other output quantities) so long as modulation index $m \leq 1$
- For a balanced load (Δ or y connected with floating neutral) we cannot synthesize any triple-n components of ω_ℓ

6.334 3-phase inverters

It is possible to synthesize outputs having a slightly larger amplitude than modulation index $m=1$ without low-frequency distortion.

For synthesizing a balanced 3-phase output set with modulation m, we get (local average over a cycle) switch duty ratios

$$d_1 = 1-d_4 = \frac{1}{2} + \frac{m}{2}\sin(\omega_e t) \Rightarrow \bar{v}_{an} = m\frac{V_{dc}}{2}\sin(\omega_e t)$$

$$d_3 = 1-d_6 = \frac{1}{2} + \frac{m}{2}\sin(\omega_e t - \tfrac{2\pi}{3}) \Rightarrow \bar{v}_{bn} = m\frac{V_{dc}}{2}\sin(\omega_e t - \tfrac{2\pi}{3})$$

$$d_5 = 1-d_2 = \frac{1}{2} + \frac{m}{2}\sin(\omega_e t + \tfrac{2\pi}{3}) \Rightarrow \bar{v}_{cn} = m\frac{V_{dc}}{2}\sin(\omega_e + \tfrac{2\pi}{3})$$

we need to keep all duty ratios between 0-1 to avoid low-frequency distortion.

If we add a signal at the 3rd harmonic of ω_e to each reference waveform, we can get to slightly larger values of m without saturating duty ratios at 0 or 1. The third harmonic that is synthesized is cancelled in the l-l and l-n waveforms (for a balanced load), so doesn't practically affect the output.

This technique is called "third harmonic injection" and lets us reach a modulation index m of up to $2/\sqrt{3} \simeq 1.15$

To do this, set the references such that:

$$d_1 = 1-d_4 = \frac{1}{2} + \frac{m}{2}\sin(\omega_e t) + \frac{m}{12}\sin(3\omega_e t)$$

$$d_3 = 1-d_6 = \frac{1}{2} + \frac{m}{2}\sin(\omega_e t - \tfrac{2\pi}{3}) + \frac{m}{12}\sin(3\omega_e t)$$

$$d_5 = 1-d_2 = \frac{1}{2} + \frac{m}{2}\sin(\omega_e t + \tfrac{2\pi}{3}) + \frac{m}{12}\sin(3\omega_e t)$$

add $\frac{V_{T,max}}{12} m\sin(3\omega_e t)$ to each reference waveform