6.622 Power Electronics *Prof. David Perreault*

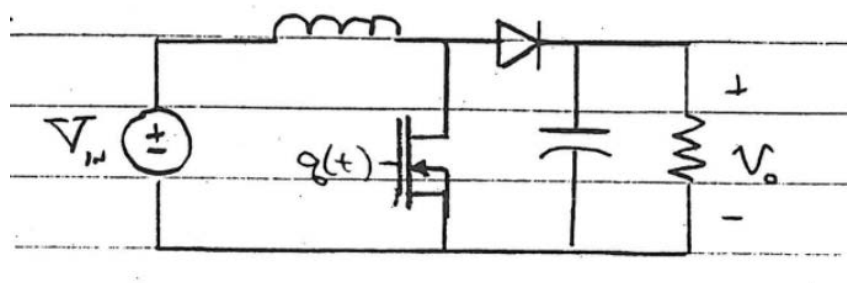
Lecture 24 - Control 1

1 Modeling and Control

1.1 Direct circuit averaging

Consider a boost converter

$$V_0 = \frac{V_{in}}{1-D}$$



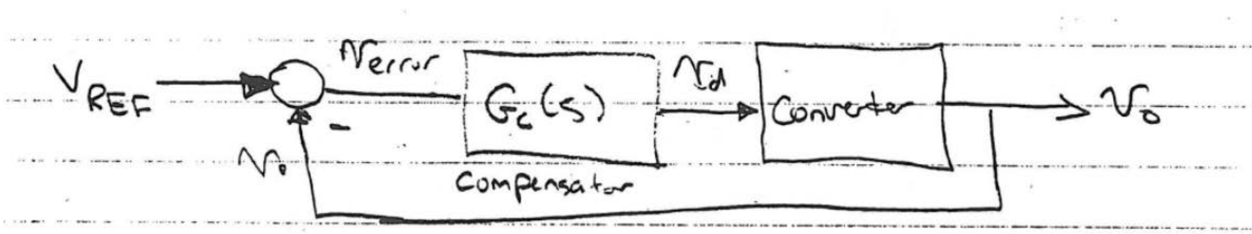
Desire to regulate the output voltage v_0 in the face of

1. Load disturbances $\Rightarrow R_{min} \leq R \leq R_{max}$
2. Input voltage variations $\Rightarrow V_{min} \leq V \leq V_{max}$

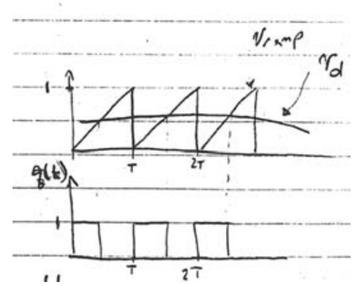
$\Rightarrow$ Feedforward has problems

1. Control depends on idealized modeling assumptions
2. Doesn't let us control response to load variations

$\Rightarrow$ Use <u>feedback!!</u> Change d depending on output voltage. (Check % that had 6.302...)



V_d is a voltage of $0 < V_d < 1$ representing duty ratio $\Rightarrow$ generate q(t) switching fn
Over 1 cycle $< q(t) > = < v_d > \rightarrow$ PWM generation



We need a dynamic model for the converter
Switched models are not easy to use

- They carry too much information about the waveforms
- We want to know about low frequency variations, not switching

(See example simultation of boost to illustrate this: Ex 1 → can show demo boost $d < t$ (@ fixed D))
To study low-frequency "averaged" behavior, we can look at the local average value of the waveforms.

Define local average operator:

$$\overline{x(t)} = \frac{1}{T}\int_{t-T}^{t} X(\tau)d\tau$$

(Moving average over 1 cycle)

→ local average tracks low freq variations, suppresses switching ripple info
Properties of this operator

differentiation

$$\overline{(\frac{dx}{dt})} = \frac{d}{dt}(\overline{x})$$

(Proof trivial)

Note: in general $\overline{x(t)y(t)} \neq \overline{x}(t)\overline{y}(t)$
But if x(t) or y(t) has both

1. small ripple
2. slow variation wrt

Then $\overline{x(t)y(t)} \approx \overline{x}(t)\overline{y}(t)$

Linearity $\overline{(ax+by)} = a\overline{x} + b\overline{y}$ (proof trivial)

Time invariance $\overline{x(t-t_0))} = \overline{x}(t-t_0))$ (proof trivial)

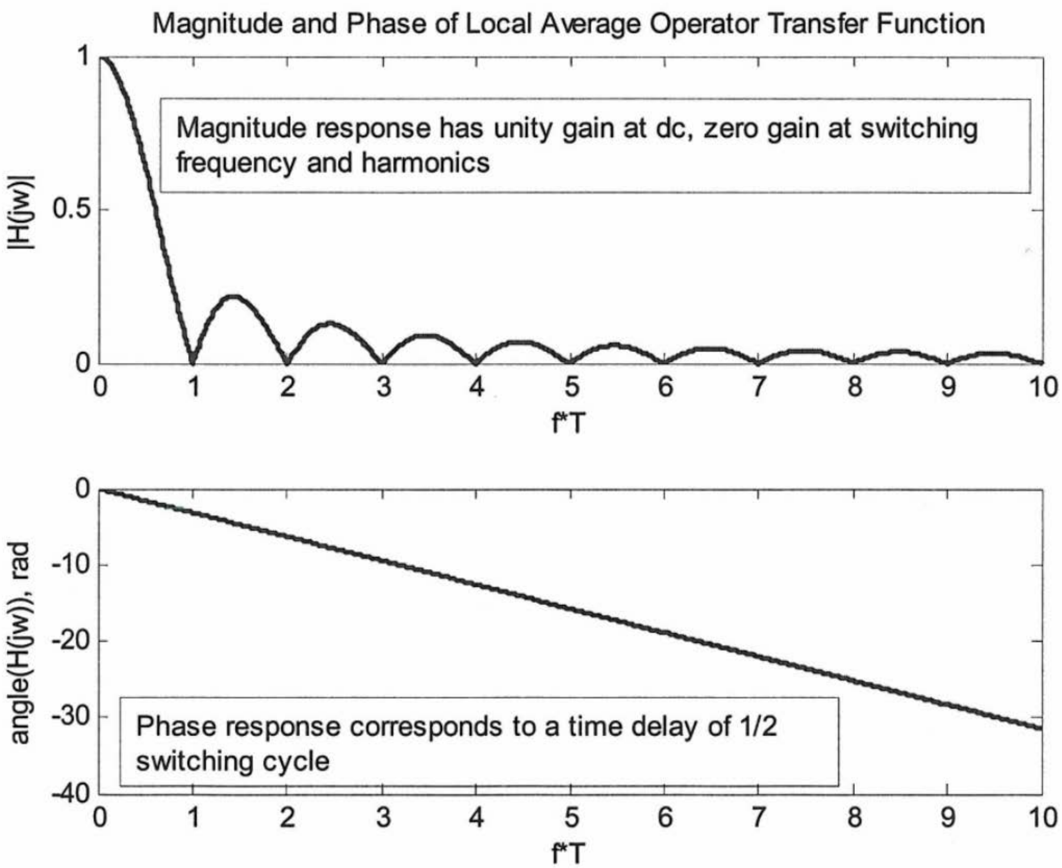
2 Local average operator

Operator definition

$$\overline{X(t)} = \frac{1}{T}\int_(t-T)^t x(\tau)d\tau$$

Transfer function

$$H(j\omega) = \frac{\overline{X}(j\omega)}{X(j\omega)} = \frac{1-e^{-j\omega T}}{j\omega T} = sinc(\frac{\omega T}{2\pi})\cdot e^{-j\omega T/2}$$


Magnitude and Phase of Local Average Operator Transfer Function
Magnitude response has unity gain at dc, zero gain at switching frequency and harmonics
|H(jw)|
f*T
angle(H(jw)), rad
Phase response corresponds to a time delay of 1/2 switching cycle
f*T

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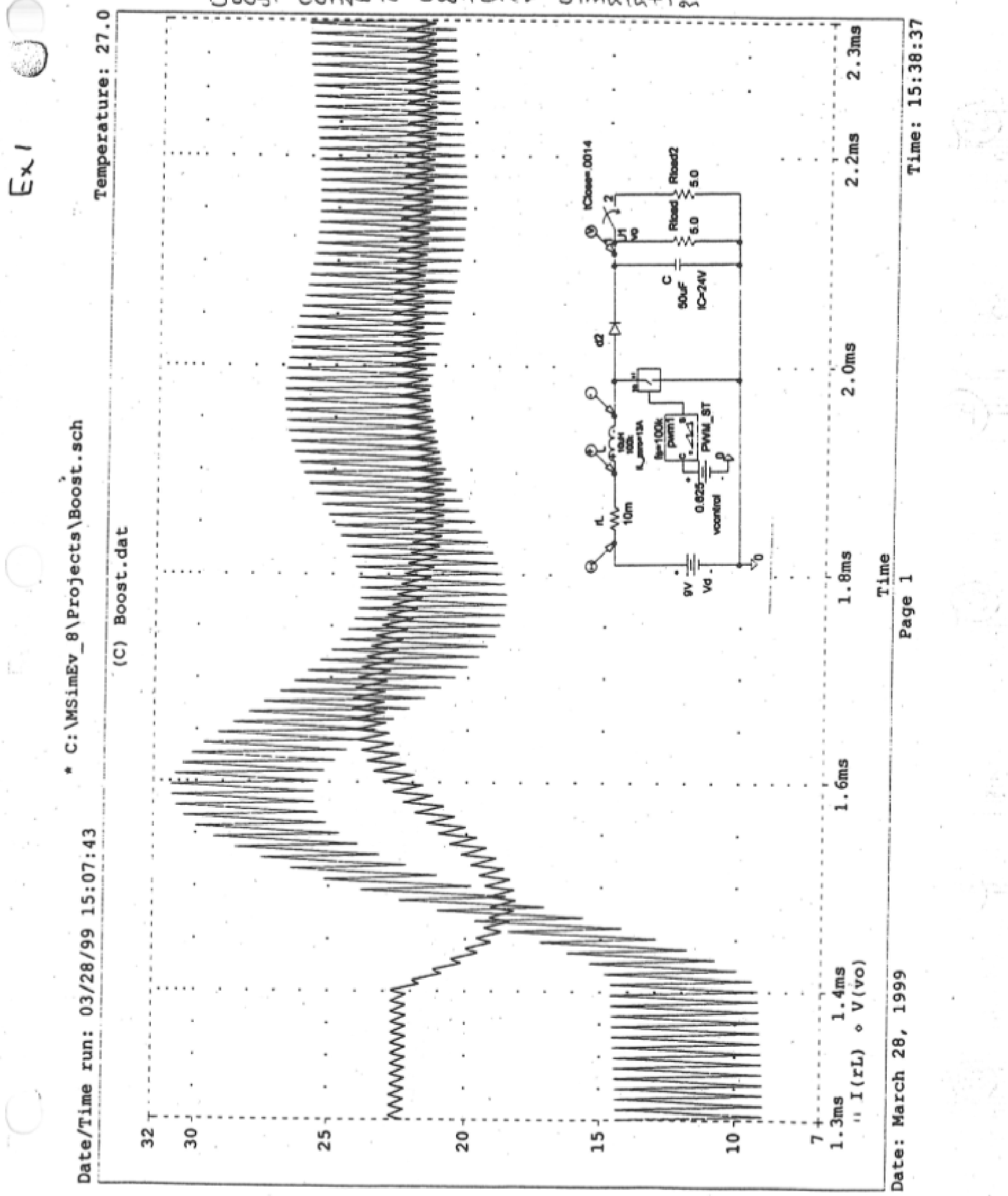
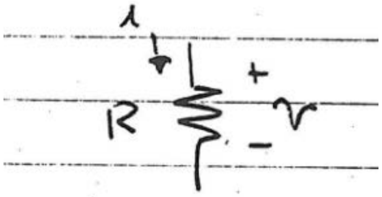
Power Electronics Notes - D. Perreault (EX1)

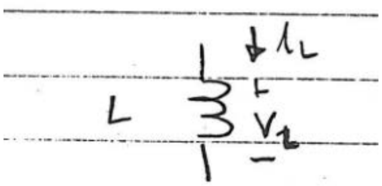
Boost Converter Switched Simulation



Consider the use of this operator on a circuit: because LTI, KVL + KCL are satisfied for average vars
KVL: $\Sigma v_{ij} = 0 \rightarrow \Sigma \overline{v_{ij}} = 0$
KCL: $\Sigma i_j = 0 \rightarrow \Sigma \overline{i_j} = 0$
We can apply averaging in circuits
Consider constitutive laws for averaged vars:

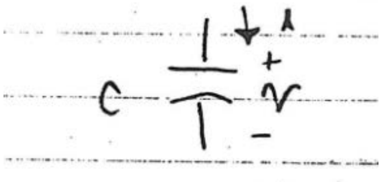


$$v(t) = i(t)R$$
$$\overline{v(t)} = \overline{i(t)}R$$



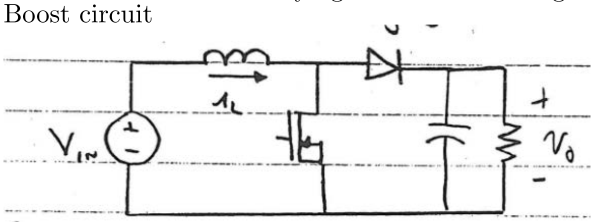
$$v(t) = L\frac{di}{dt}$$
$$\overline{v(t)} = L\frac{d\bar{i}}{dt}$$

LTI circuit elements constitutive relationships do not change!

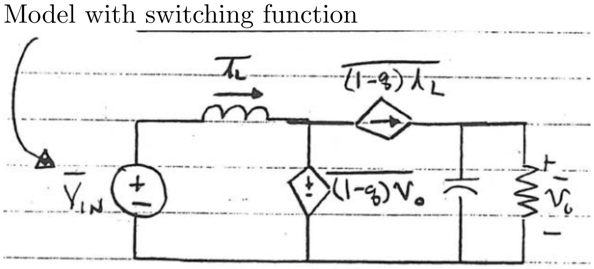


$$i(t) = C\frac{dv}{dt}$$
$$\overline{i(t)} = C\frac{d\overline{v}}{dt}$$

Nonlinear or time varying elements do change!
Boost circuit



Model with switching function



$\overline{x(t)y(t)} \neq \overline{x}(t)\overline{y}(t)$
but if x(t) or y(t) has

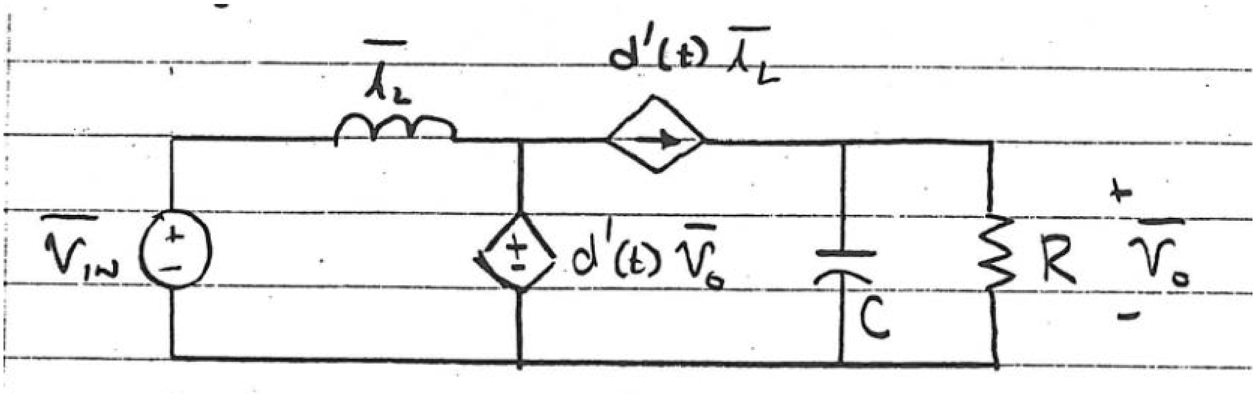
1. small ripple
2. and slow variation wrt T

$\Rightarrow \overline{x(t)y(t)} \approx \overline{x}(t)\overline{y}(t)$

To average, place line over all variables (LTI elements do not change)

If i_L has small ripple, slow variation $\therefore \overline{(1-q)i_L} \approx \overline{1-q} \cdot \overline{i_L} = d'\overline{i_L}$
v_c has small ripple, slow variation (ie $\approx$ const over a cycle) $\therefore \overline{(1-q)v_0} \approx \overline{1-q} \cdot \overline{v_0} = d'\overline{v_0}$
Where $\Sigma d(t) = \overline{q(t)}$

3 Averaged circuit model



- In this circuit model, we have no more switching (only depends on averaged duty cycle d(t) = $\overline{q(t)}$)
- model is not linear in our control variable d(t) (because of d'v_0, d'i_L terms)
- model is very simple, + should be accurage for averaged variables if our assumptions are valid (ie $\overline{qi_L} = \overline{q}\overline{i_L}$, etc)

Ex2. (Show simulation w/ both switched averaged models.)

1. good results (small offset due to evice mods)
2. will be very useful for control!!

Note: not useful for some things. Ex switch power dissipation
$\underline{P(t)} = \underline{V(t)i(t)}$
$\overline{P(t)} = \overline{V(t)i(t)} \neq \overline{v(t)}\,\overline{i(t)}$ (small ripple assumtion not met)

Boost Converter Switched + Averaged simulation

Ex 2