Power Electronics Notes - D. Perreault

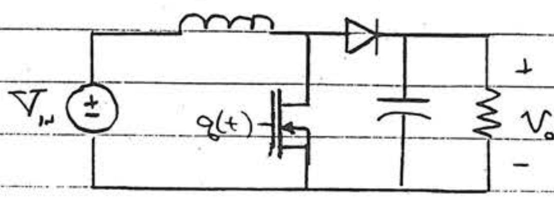
Modeling And Control

Direct Circuit Averaging

READ KSV 11.1-11.3.4

Consider a Boost Converter

$$V_o = \frac{V_{in}}{1-D}$$



Desire to regulate the output voltage V_o in the face of:

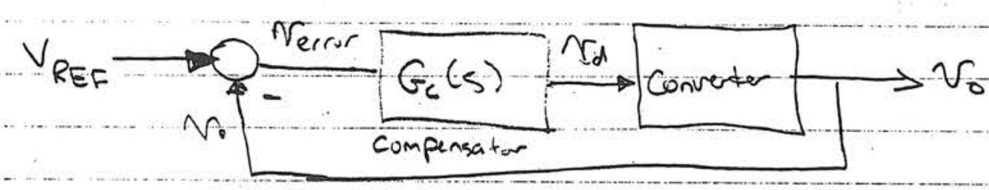
1. Load disturbances $\rightarrow R_{min} \leq R \leq R_{max}$
2. Input voltage variations. $\rightarrow V_{min} \leq V \leq V_{max}$

→ Feedforward has problems

1. control depends on idealized modeling assumptions
2. doesn't let us control response to load variations

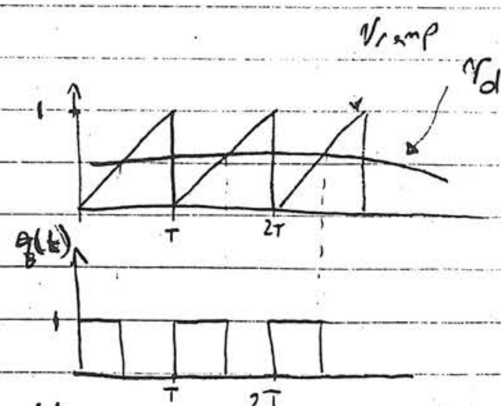
⇒ Use Feedback!! change d depending on output voltage.

(check % that has had 6.302...)



v_d is a voltage $0 < V_d < 1$ representing duty ratio

⇒ generate $q(t)$ switching fn.



over 1 cycle $\langle q(t) \rangle = \langle v_d \rangle$

→ PWM generation

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We need a dynamic model for the converter

Switched models are not easy to use

→ they carry too much information about the waveforms

→ we want to know about low frequency variations, not switching

Ex 1

(see example simulation of boost to illustrate this : Ex 1)

★ → Can show demo boost ckt (@ fixed D)

To study low-frequency "averaged" behavior, we can look at the local average value of the waveforms.

Define Local Average operator: (moving avg over 1 cycle)

$$\boxed{\bar{x}(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)\,d\tau}$$

→ local average tracks low freq. variations, suppresses switching ripple info.

Properties of this operator

differentiation $\overline{\left(\frac{dx}{dt}\right)} = \frac{d}{dt}(\bar{x})$ (proof trivial)

Note: In general $\overline{x(t)\,y(t)} \neq \overline{x(t)}\;\overline{y(t)}$

★ But if $x(t)$ or $y(t)$ has both 1.) Small ripple 2.) slow variation wrt

then $\overline{x(t)y(t)} \approx \overline{x(t)}\;\overline{y(t)}$

linearity $\overline{(ax+by)} = a\bar{x} + b\bar{y}$

(proof trivial)

time invariance $\overline{(x(t-t_0))} = \bar{x}(t-t_0)$ (proof trivial)

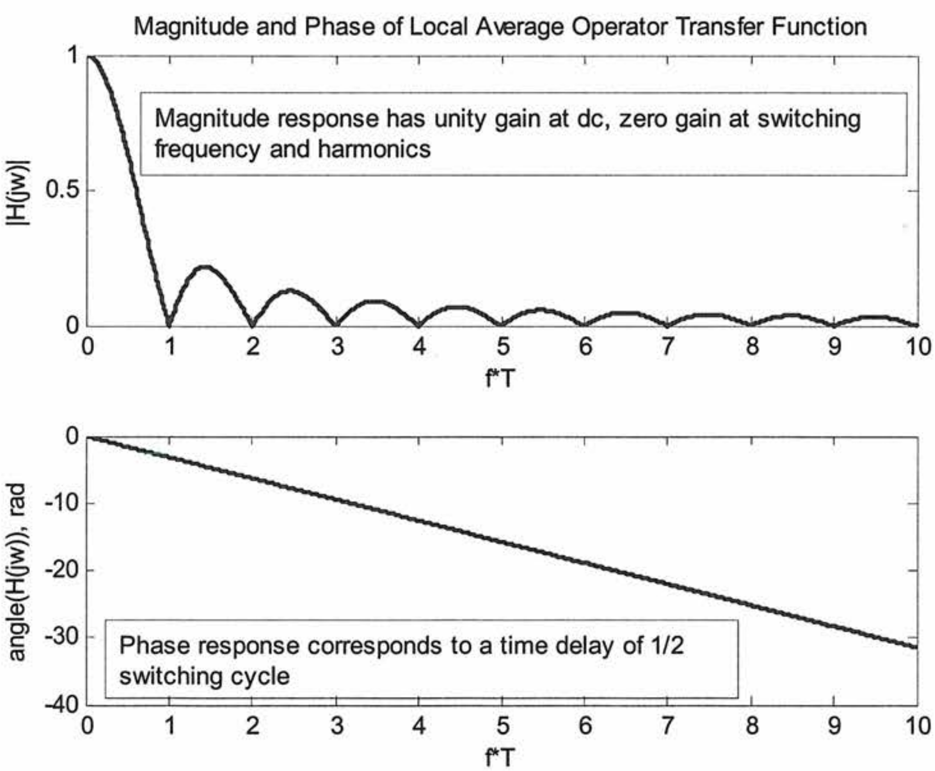
Local Average Operator

Operator Definition

$$\overline{X}(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)\,d\tau$$

Transfer function

$$H(j\omega) = \frac{\overline{X}(j\omega)}{X(j\omega)} = \frac{1-e^{-j\omega T}}{j\omega T} = \text{sinc}\left(\frac{\omega T}{2\pi}\right)\cdot e^{-j\omega T/2}$$



6.334 Lecture Control #1

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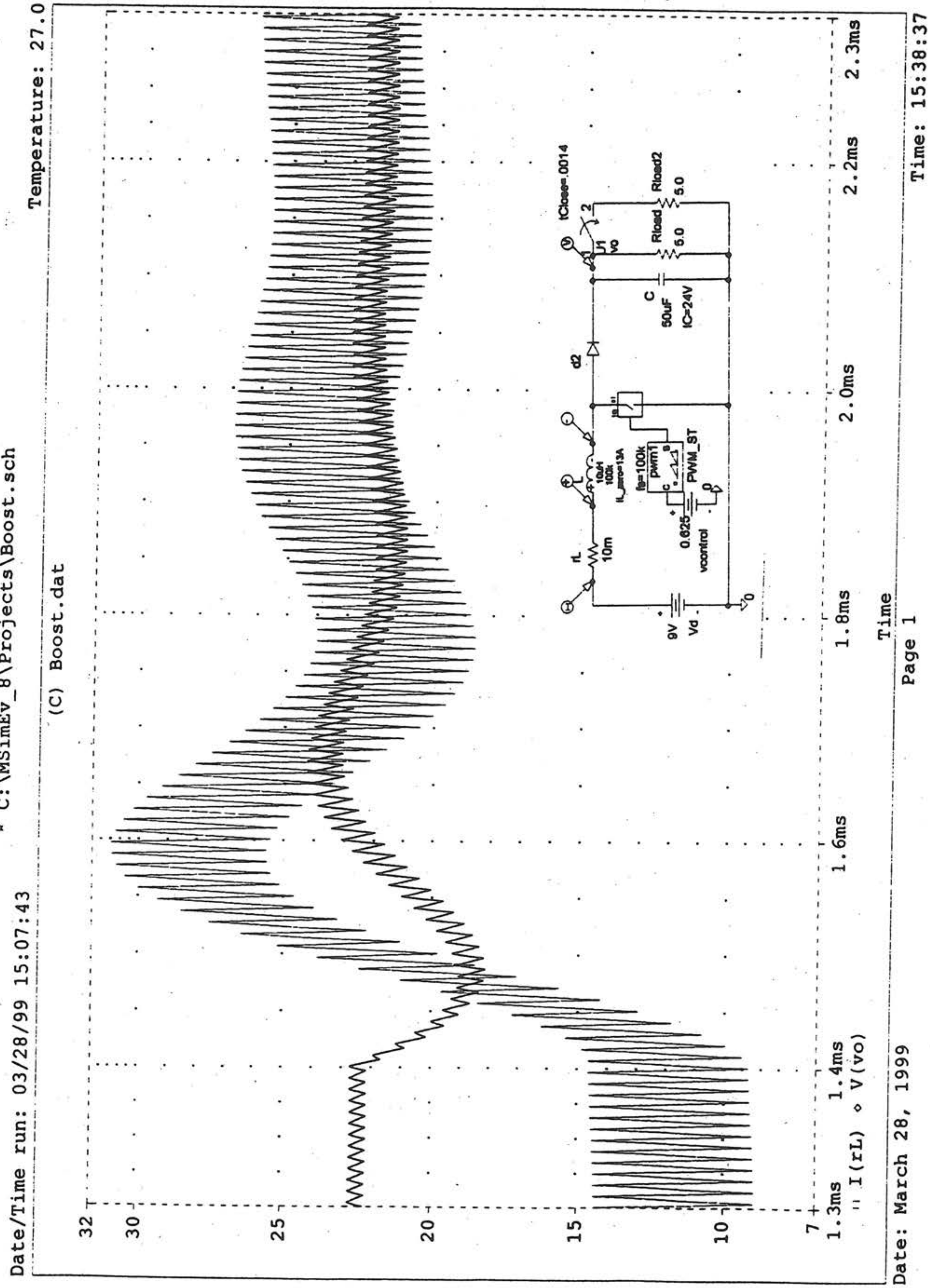
Boost Converter Switched Simulation

Ex1



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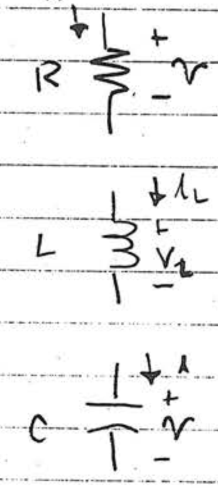
Consider the use of this operator on a circuit:

⇒ ☆ **Because LTI, KVL + KCL are satisfied for averaged vars**

$$\text{KVL}: \sum v_{ij} = 0 \rightarrow \sum \bar{v}_{ij} = 0$$
$$\text{KCL}: \sum i_j = 0 \rightarrow \sum \bar{i}_j = 0$$

We can apply avg'ing in circuits

Consider constitutive laws for averaged vars:



$$v(t) = i(t) R$$
$$\bar{v}(t) = \bar{i}(t) R$$

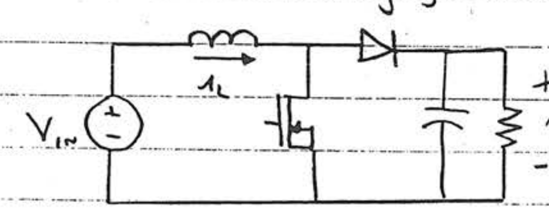
$$v = L \frac{di}{dt}$$
$$\bar{v} = L \frac{d\bar{i}}{dt}$$

$$i = C \frac{dv}{dt}$$
$$\bar{i} = C \frac{d\bar{v}}{dt}$$

LTI circuit elements
Constitutive relationships do not change!

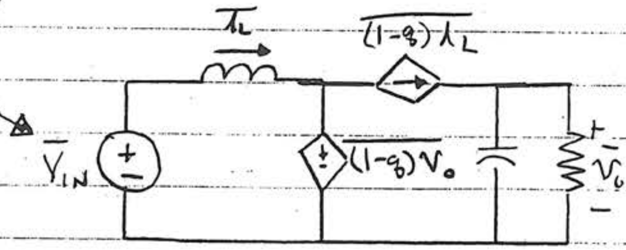
Nonlinear or time varying elements do change!

Boost ckt



$$\boxed{\begin{array}{l} \overline{x(t)\,y(t)} \neq \bar{x}(t)\,\bar{y}(t) \\ \text{but IF } x(t) \text{ or } y(t) \text{ has} \\ \quad 1.\ \text{Small ripple} \\ \text{and } 2.\ \text{slow variation wrt } T \\ \Rightarrow \overline{x(t)\,y(t)} \approx \overline{x(t)}\;\overline{y(t)} \end{array}}$$

model w/switching function



to average, place ‾ over all variables.
(LTI elements do not change)

If i_L has small ripple, slow variation $\therefore \overline{(1-q)i_L} \approx \overline{1-q} \cdot \bar{i}_L = d'\bar{i}_L$

v_C has small ripple, slow variation $\therefore \overline{(1-q)v_o} \approx \overline{1-q} \cdot \bar{v}_o = d'\bar{v}_o$

↳ i.e. ≈ const over a cycle

where $\{ d(t) = \overline{q(t)}$

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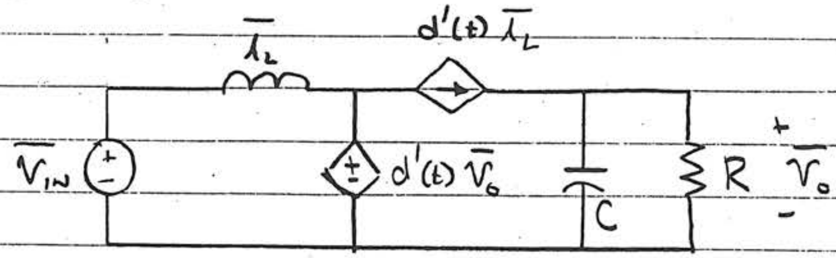
Averaged Circuit model



★ → in this circuit model we have no more switching (only depends on averaged duty cycle $d(t) = \overline{q(t)}$)

★ → model is not linear in our control variable $d(t)$ (because of $d'\hat{v}_o$, $d'i_L$ terms)

★ → model is very simple, + should be accurate for averaged variables if our assumptions are valid (i.e. $\overline{q i_L} = \bar{q}\,\bar{i}_L$, etc.)

Ex 2 (show simulation w/ both switched + avg'd models.)

→ good results (small offset due to device mods.)

★ → will be very useful for control!!

Note: not useful for some things. Ex switch power dissipation

$$P(t) = v(t)\,i(t)$$

$$\overline{p(t)} = \overline{v(t)\,i(t)} \neq \bar{v}(t)\,\bar{i}(t)$$

↳ small ripple assumption not met ...

→ Review how we got here + address questions!

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(6)

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Boost Converter Switched + Averaged simulation

Ex 2