6.622 Power Electronics *Prof. David Perreault*

Lecture 25 - Control 2

1 State-space averaging, Linearization

Reading: KS+V 12.1-12.4, 13.1-13.2

1.1 Intro

State space averaging: different (more methodical) approach to the same type of model we built last time

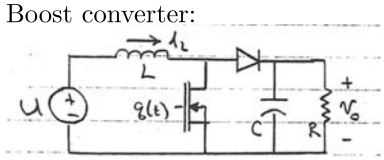
Review:

Local average

$$\overline{x}(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)d\tau$$

1. Linear
2. Time invariant
3. $\overline{(\frac{dx}{dt})} = \frac{d\overline{x}}{dt}$
4. $\overline{x(t)y(t)} \approx \overline{x}(t)\overline{y}(t)$ if only x or y has only slow variation and small ripple

Boost converter:



q(t) = 1 switch on; 0 switch off

$d(t) = \overline{q}$
$d'(t) = \overline{q'}$

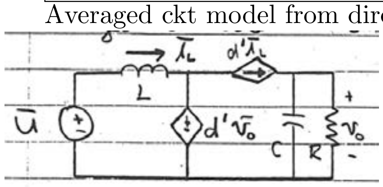
State equations: i_l, v_o are state variables

$$\begin{cases} \frac{di_L}{dt} = \frac{u}{L}q(t) + \frac{(u-v_0)}{L}(1-q(t)) \\ \frac{dv_0}{dt} = -\frac{1}{RC}vq(t) + (\frac{1}{C}i_L - \frac{1}{RC}v)(1-q(t)) \end{cases}$$

Regroup and average:

$$\begin{cases} \frac{d\overline{i_L}}{dt} = \frac{\overline{u}}{L} + \overline{\frac{(v_0)}{L}q'(t)} \approx \frac{1}{L}\overline{u} - \frac{1}{L}\overline{v}d' \\ \frac{d\overline{v_0}}{dt} = -\frac{1}{RC}\overline{v_0} + \frac{1}{C}\overline{i_L q'(t)} \approx -\frac{1}{RC}\overline{v_0} + \frac{1}{C}\overline{i_L}d' \end{cases}$$

Averaged ckt model from direct ckt averaging



$$\begin{cases} \frac{di_L}{dt} = \frac{\overline{u}}{L} + \frac{\overline{(v_0)}}{L}d'(t) \\ \frac{d\overline{v_0}}{dt} = -\frac{1}{RC}\overline{v_0} + \frac{1}{C}\overline{i_L}d' \end{cases}$$

Same state equations: circuit average and state avg are the same! (circuit view vs equation view)

2 Linearization

To do linear control design, linearize system about operationg point → explain what linearized dynamics mean

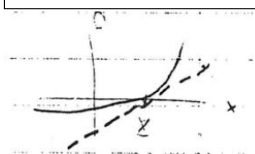
$x = \overline{X} + \tilde{x} \Rightarrow \tilde{x} = x - \overline{X}$

Formal definition $\overline{u} = U + \tilde{u}, \overline{i_L} = I_L + \tilde{i_L}, \overline{v_0} = V_0 + \tilde{v_0}, d = D + \tilde{d}$

given

$$\frac{dx}{dt} = f(x, r, t), f(\underline{X}, R, t) = 0$$

$$\Rightarrow \cancel{\frac{\underline{X}}{dt}} + \frac{\tilde{d}}{dt} = \frac{\partial f}{\partial x}|_{\underline{X},R}\tilde{x} + \frac{\partial f}{\partial r}|_{\underline{X},R}\tilde{r} + f(\underline{X}, R)$$



Approx f with a line near $x = \overline{X}$

Intuitive approach: substitute expanded variables in + simplify (all purely S.S. terms <u>must</u> go away by definintion of S.S.)

$$\begin{cases} \cancel{\frac{dI_L}{dt}} + \frac{d\tilde{i_L}}{dt} = \frac{1}{L}U + \frac{1}{L}\tilde{u} - \frac{1}{L}(V_0 + \tilde{v_0})(1 - D - \tilde{d}) \\ \cancel{\frac{dV_0}{dt}} + \frac{d\tilde{v_0}}{dt} = -\frac{1}{RC}V_0 - \frac{1}{RC}\tilde{v_0} + \frac{1}{C}(I_L + \tilde{i_L})(1 - D - \tilde{d}) \end{cases}$$

$$\frac{d\tilde{i_L}}{dt} = \frac{1}{L}U - \frac{1}{L}V_0 D' + \frac{1}{L}\tilde{u} - \frac{D'}{L}\tilde{v_0} + \frac{V_0}{L}\tilde{d} + \frac{1}{L}\tilde{v_0}\tilde{d}$$

$$\frac{d\tilde{v_0}}{dt} = -\frac{1}{RC}V_0 + \frac{D' I_L}{C} - \frac{1}{RC}\tilde{v_0} + \frac{D'}{C}\tilde{i_L} - \frac{I_L}{C}\tilde{d} - \frac{1}{C}\tilde{i_L}\tilde{d}$$

$$\frac{d\tilde{i_L}}{dt} = \frac{1}{L}\tilde{u} - \frac{D'}{L}\tilde{v_0} + \frac{V_0}{L}\tilde{d}$$

$$\frac{dv_0}{dt} = -\frac{1}{RC}\tilde{v_0} + \frac{D'}{C}\tilde{i_L} - \frac{I_L}{C}\tilde{d}$$

Linearized model at op pt. U, D, V_0, I_L

Assume u = U (no perturbation in U); Laplace transform:

$$\begin{cases} s\tilde{i_L} = -\frac{D'}{L}\tilde{v_0} + \frac{V_0}{L}\tilde{d} \\ s\tilde{v_0} = -\frac{1}{RC}\tilde{v_0} + \frac{D'}{C}\tilde{i_L} - \frac{I_L}{C}\tilde{d} \end{cases}$$

$$s\tilde{v_0} = -\frac{1}{RC}\tilde{v_0} + \frac{D'}{sC}(-\frac{D}{L}v_0 + \frac{V_0}{L}\tilde{d}) - \frac{I_L}{C}\tilde{d}$$

$$(S + \frac{D'^2}{sCL} + \frac{1}{RC})v_0 = (\frac{v_0 D'}{sLC} - \frac{I_L}{C})\tilde{d}$$

$$\frac{\tilde{v_o}}{\tilde{d}} = \frac{-s\frac{I_L}{C} + \frac{V_0 D'}{LC}}{s^2 + \frac{1}{RC}s + \frac{D'^2}{LC}}$$

2nd order system

- 2 LHP poles (underdamped)
- 1 RHP zero (yuck!)

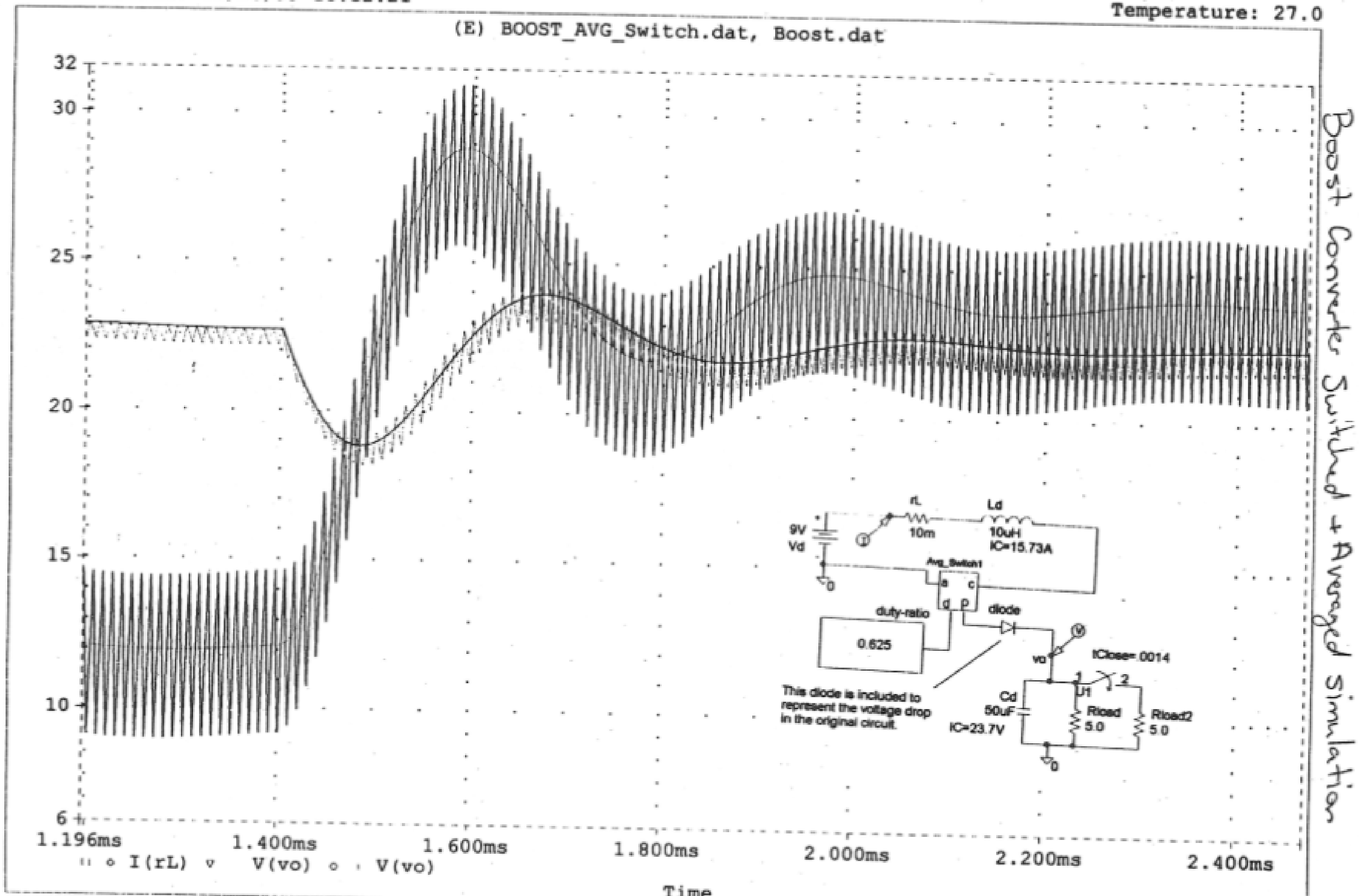
Poles move w/ operating point!!
Ex/ U = 9V, V=24V, D=0.625, I_L ≈25.6A, L = 10μH, C = 50μF, R = 2.4Ω

$$\frac{\tilde{v_0}}{\tilde{d}} = \frac{-512000s + 1.8 \times 10^{10}}{s^2 + 8000s + 2.81 \times 10^8}$$

$$\begin{cases} \text{zero @ s = 35,156} \frac{rad}{sec} \\ \text{poles @ s = -4000} \pm \text{j 16,279} \frac{rad}{sec} \end{cases}$$

Ex 2

* C:\MSimEv_8\Projects\BOOST_AVG_Switch.sch, * C:\MSimEv_8\Projects\Boost.sch
Date/Time run: 03/28/99 15:12:21 Temperature: 27.0



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