Power Electronics Notes - D. Perreault

★ State-Space Averaging, Linearization

Reading: KS+V 12.1-12.4, 13.1-13.2

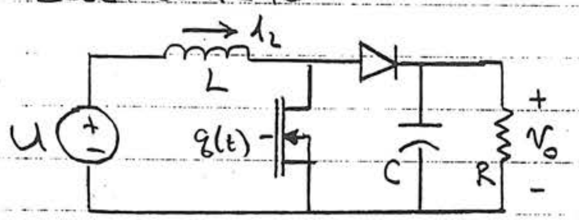
Intro: State-space averaging: different (more methodical) approach to the same type of model we built last time.

→ Review: Local Average

$$\bar{x}(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)\,d\tau$$

1. Linear
2. Time Invariant
3. $\overline{\left(\frac{dx}{dt}\right)} = \frac{d\bar{x}}{dt}$
4. $\overline{x(t)y(t)} \approx \bar{x}(t)\bar{y}(t)$ if x or y has slow variation & small ripple

Boost Converter



$$q(t) = \begin{cases} 1 & \text{switch on} \\ 0 & \text{switch off} \end{cases} \qquad q' = 1-q \qquad \begin{aligned} d(t) &= \bar{q} \\ d'(t) &= \bar{q}' \end{aligned}$$

State equations: i_L, v_o are state variables

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{U}{L}q(t) + \dfrac{(U-v_o)}{L}(1-q(t)) \\[2ex] \dfrac{dv_o}{dt} = -\dfrac{1}{RC}v\,q(t) + \left[\dfrac{1}{C}i_L - \dfrac{1}{RC}v\right](1-q(t)) \end{cases}$$

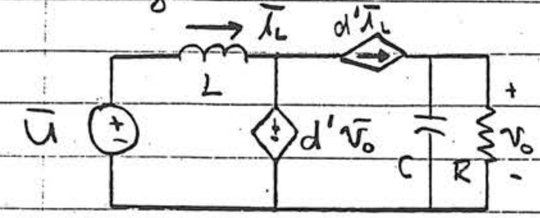
regroup & average

$$\boxed{\begin{cases} \dfrac{d\bar{i}_L}{dt} = \dfrac{\bar{U}}{L} - \dfrac{\overline{v_o q'(t)}}{L} \approx \dfrac{1}{L}\bar{U} - \dfrac{1}{L}\bar{v}d' \\[2ex] \dfrac{d\bar{v}_o}{dt} = -\dfrac{1}{RC}\bar{v}_o + \dfrac{1}{C}\overline{i_L q'(t)} \approx -\dfrac{1}{RC}\bar{v}_o + \dfrac{1}{C}\bar{i}_L d' \end{cases}}$$ ★

because $\overline{xy} \approx \bar{x}\bar{y}$

Power Electronics Notes – D. Perreault

Averaged ckt model from direct ckt. averaging



$$\Rightarrow \begin{cases} \dfrac{d\bar{i}_L}{dt} = \dfrac{1}{L}\bar{u} - \dfrac{1}{L}\bar{v}_o d' \\[2ex] \dfrac{d\bar{v}_o}{dt} = -\dfrac{1}{RC}\bar{v}_o + \dfrac{1}{C}\bar{i}_L d' \end{cases}$$

Same state eqns: Circuit avg + state space avg are the same!
(circuit view vs. eqn. view.)

★ Linearization

To do linear control design, Linearize system about operating point.
→ explain what linearized dynamics mean. $x = \underset{\text{s.s.}}{X} + \underset{\text{pert}}{\tilde{x}} \Rightarrow \tilde{x} = x - X$ deviation from s.s

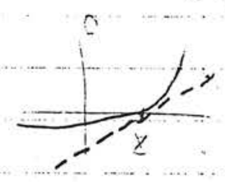
$$\boxed{\bar{u} = U + \tilde{u} \quad \bar{i}_L = I_L + \tilde{i}_L \quad \bar{v}_o = V_o + \tilde{v}_o \quad d = D + \tilde{d}}$$

formal def: given $\dfrac{dx}{dt} = f(x(t), r(t), t)$, $f(X, R, t) = 0$ op. point

$$\Rightarrow \cancel{\frac{dX}{dt}}^{\,0} + \frac{d\tilde{x}}{dt} = \left.\frac{\partial f}{\partial x}\right|_{X,R} \tilde{x} + \left.\frac{\partial f}{\partial r}\right|_{X,R} \tilde{r} + \underbrace{f(X,R)}_{0}$$

$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial r}$ must be continuous!!

1st order Taylor Expansion of f about X, R



approx f with a line near $x = X$

intuitive approach: substitute expanded variables in + simplify
(all purely s.s. terms must go away by definition of s.s.)

$$\begin{cases} \cancel{\dfrac{dI_L}{dt}}^{\,0} + \dfrac{d\tilde{i}_L}{dt} = \dfrac{1}{L}U + \dfrac{1}{L}\tilde{u} - \dfrac{1}{L}(V_o + \tilde{v}_o)(1 - D - \tilde{d}) \\[2ex] \cancel{\dfrac{dV_o}{dt}}^{\,0} + \dfrac{d\tilde{v}_o}{dt} = -\dfrac{1}{RC}V_o - \dfrac{1}{RC}\tilde{v}_o + \dfrac{1}{C}(I_L + \tilde{i}_L)(1 - D - \tilde{d}) \end{cases}$$

$$\frac{d\tilde{i}_L}{dt} = \overbrace{\cancel{\frac{1}{L}U - \frac{1}{L}V_o D'}}^{\to 0} + \frac{1}{L}\tilde{u} - \frac{D'}{L}\tilde{v}_o + \frac{V_o}{L}\tilde{d} + \cancel{\frac{1}{L}\tilde{v}_o\tilde{d}}^{\,\to 0}$$

$$\frac{d\tilde{v}_o}{dt} = \cancel{-\frac{1}{RC}V_o + \frac{D' I_L}{C}} - \frac{1}{RC}\tilde{v}_o + \frac{D'}{C}\tilde{i}_L - \frac{I_L}{C}\tilde{d} - \cancel{\frac{1}{C}\tilde{i}_L\tilde{d}}^{\,\to 0}$$

$$\boxed{\begin{aligned} \frac{d\tilde{i}_L}{dt} &= \frac{1}{L}\tilde{u} - \frac{D'}{L}\tilde{v}_o + \frac{V_o}{L}\tilde{d} \\ \frac{d\tilde{v}_o}{dt} &= -\frac{1}{RC}\tilde{v}_o + \frac{D'}{C}\tilde{i}_L - \frac{I_L}{C}\tilde{d} \end{aligned}}$$

⟵ Linearized model at op pt. U, D, V_o, I_L

Power Electronics Notes – D. Perreault

Assume $u = \bar{U}$ (no perturbation in U); Laplace transform:

$$\begin{cases} s\tilde{i}_L = -\frac{D'}{L}\tilde{v}_o + \frac{V_o}{L}\hat{d} \\ s\tilde{v}_o = -\frac{1}{RC}\tilde{v}_o + \frac{D'}{C}\tilde{i}_L - \frac{I_L}{C}\hat{d} \end{cases}$$

$$s\tilde{v}_o = -\frac{1}{RC}\tilde{v}_o + \frac{D'}{sC}\left(-\frac{D'}{L}v_o + \frac{V_o}{L}\hat{d}\right) - \frac{I_L}{C}\hat{d}$$

$$\left(s + \frac{D'^2}{sCL} + \frac{1}{RC}\right)v_o = \left(\frac{V_o D'}{sLC} - \frac{I_L}{C}\right)\hat{d}$$

$$\boxed{\frac{\tilde{v}_o}{\hat{d}} = \frac{-s\frac{I_L}{C} + \frac{V_o D'}{LC}}{s^2 + \frac{1}{RC}s + \frac{D'^2}{LC}}}$$

→ 2nd order system
- → 2 LHP poles (underdamped)
- → 1 RHP zero (yuck!)

★ → poles move w/ operating point !!

one op pt only! → Ex/ $\bar{U} = 9V$, $\bar{V} = 24\ V$, $D = 0.625$, $I_L \approx 25.6\ A$
$L = 10\,\mu H$, $C = 50\,\mu F$, $R = 2.5\,\Omega$

$$\frac{\tilde{v}_o}{\hat{d}} = \frac{-512000\,s + 1.8\times10^{10}}{s^2 + 8000\,s + 2.81\times10^{8}}$$

$$\begin{cases} \text{zero @ } s = 35{,}156 \text{ rad/sec} \\ \text{poles @ } s = -4{,}000 \pm j\,16{,}279 \text{ rad/sec} \end{cases}$$

≈ .4 msec osc. period

example from simulation (Ex 1/2)
∴ compare to it!!

6.334 Lecture Control #1

(6)

Power Electronics Notes - D. Perreault (EX2)

Boost Converter Switched + Averaged simulation

Ex 2