Power Electronics Notes – D. Perreault

★ Control Design Example

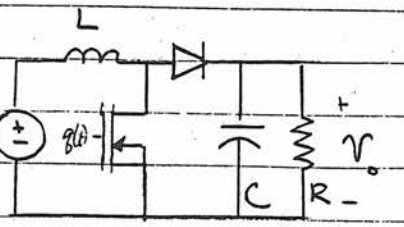
Boost Converter

$L = 10\mu H$ $C = 50\mu F$

$f_{sw} = 100KHz$ $V_{o,REF} = 24\,V$

$U_{Nom} = 9V$, $8V < U < 10V$

$2\Omega < R < 10\Omega$



Start with switched equations of state:

$$\begin{cases} \dfrac{d\lambda_L}{dt} = \dfrac{U}{L} q(t) + \dfrac{(U - v_o)}{L}(1 - q(t)) \\ \dfrac{dv_o}{dt} = -\dfrac{1}{RC} v_o q(t) + \left[\dfrac{1}{C}\lambda_L - \dfrac{1}{RC} v_o\right](1 - q(t)) \end{cases}$$

AVG. State-space averaging with $\bar{x} = \frac{1}{T}\int_{t-T}^{t} x(\tau)\,d\tau$ results in nonlinear averaged model

$$\begin{cases} \dfrac{d\bar{\lambda}_L}{dt} \approx \dfrac{1}{L}\bar{u} - \dfrac{1}{L}\bar{v}\,d' \\ \dfrac{d\bar{v}_o}{dt} \approx -\dfrac{1}{RC}\bar{v}_o + \dfrac{1}{C}\bar{\lambda}_L d' \end{cases}$$

Linearize: Linearization about op. point U, I_L, V_o, D yields LTI linearized model of incremental dynamics

$$\begin{cases} \dfrac{d\hat{\lambda}_L}{dt} = \dfrac{1}{L}\hat{u} - \dfrac{D'}{L}\hat{v}_o + \dfrac{V_o}{L}\hat{d} \\ \dfrac{d\hat{v}_o}{dt} = -\dfrac{1}{RC}\hat{v}_o + \dfrac{D'}{C}\hat{\lambda}_L - \dfrac{I_L}{C}\hat{d} \end{cases}$$

LTI analysis: Using Laplace transform & Identity $\frac{V_o}{R} = D' I_L$

$$H(s) = \frac{\hat{v}_o(s)}{\hat{d}(s)} = \frac{-s\frac{V_o}{RCD'} + \frac{V_o D'}{LC}}{s^2 + \frac{1}{RC}s + \frac{D'^2}{LC}}$$

← Transfer function depends on op. pt.

@ $U = 9$, $V = 24$, $D = 0.625$

$I_L \approx 25.6A$, $R = 2.5\Omega$, $C = 50\mu F$, $L = 10\mu H$

$$\frac{\hat{v}_o}{\hat{d}}: \frac{-512{,}000\,s + 1.8\times10^{10}}{s^2 + 8000s + 2.71\times10^8}$$

Zero @ $s = 35{,}156$ → EXPLAIN!

poles @ $s = -4000 \pm j16{,}279$

period ≈ 0.4 msec

SEE TRANSIENT IN Ex/2

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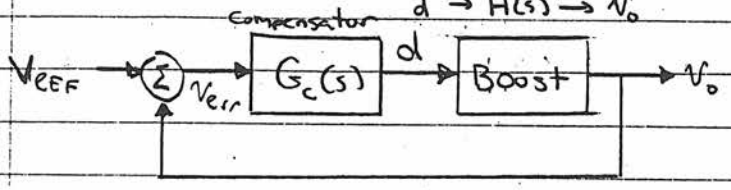
Back to original goal: **Control!**

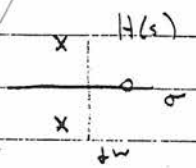
$\tilde{d} \rightarrow H(s) \rightarrow \tilde{v}_o$



Explain source of RHP zero

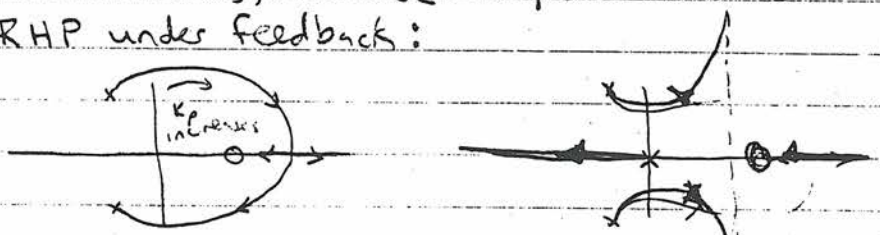
Looking at our transfer function, we have

1. 2 lightly-damped poles in LHP
2. 1 RHP zero



Note sign in numerator of H(s)

This is tricky, because the poles tend to move to the RHP under feedback:



Proportional $G_c(s) = K_p$ Integral $G_c(s) = \frac{K_I}{s}$

→ So we must pick control gains that are not too high for stability...

Also, the poles move with operating point! (variations in U, R for example)

→ look at slide for variations of poles with R.
(problem becomes more difficult at light load...)

★ → could use a damping leg  !! (neat trick)

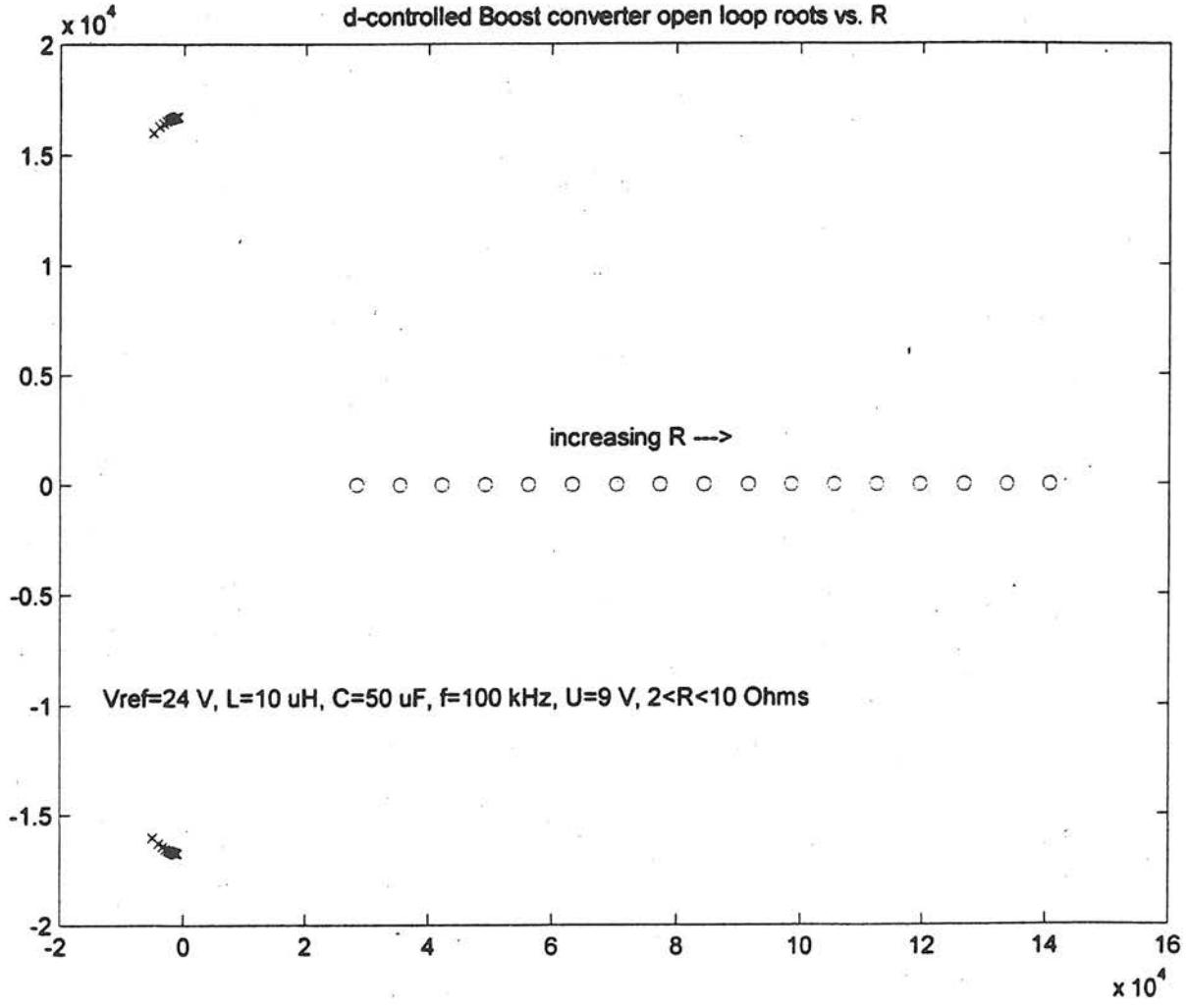
must design controller valid over all operating points

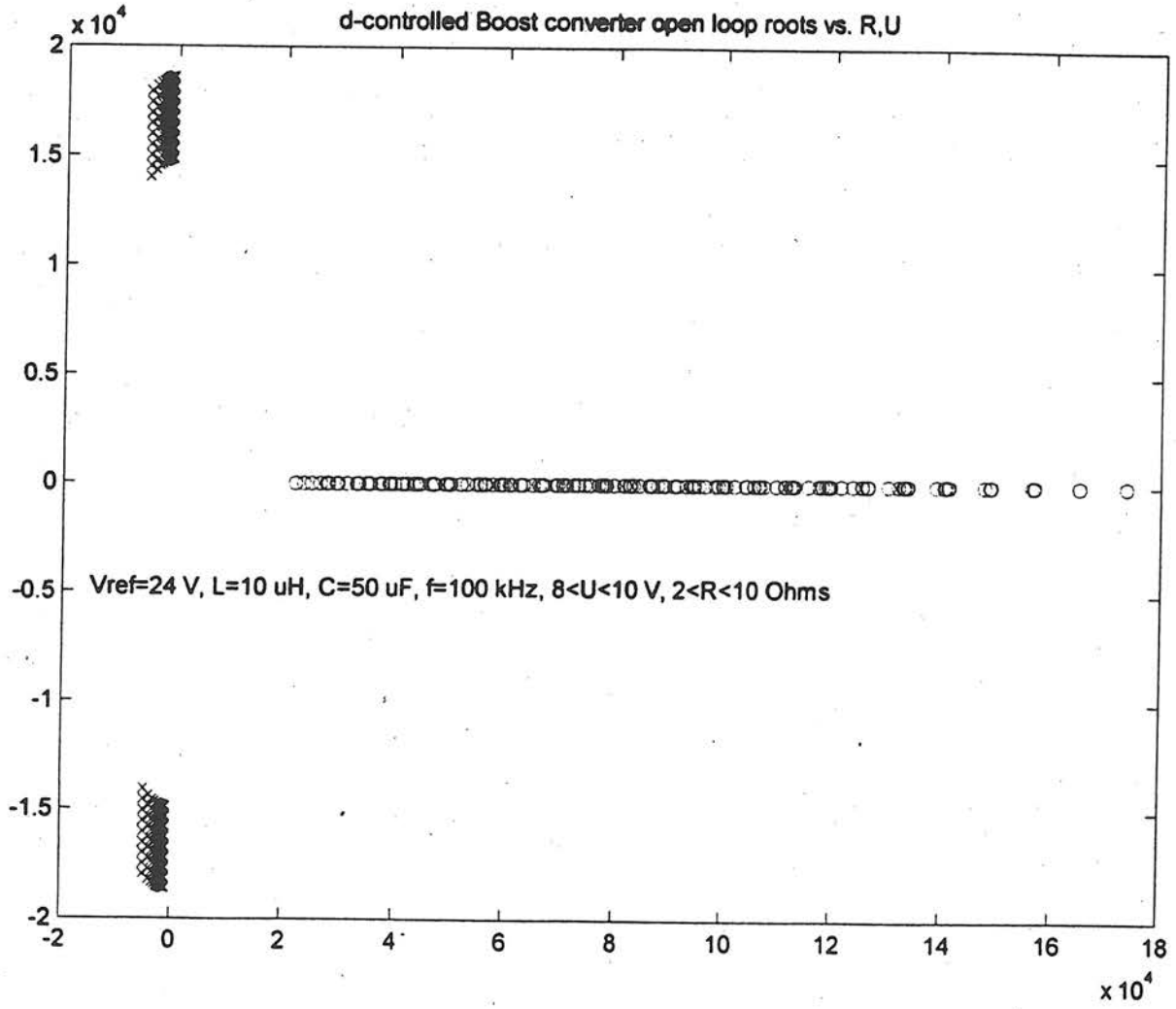
→ look at simple integral controller design

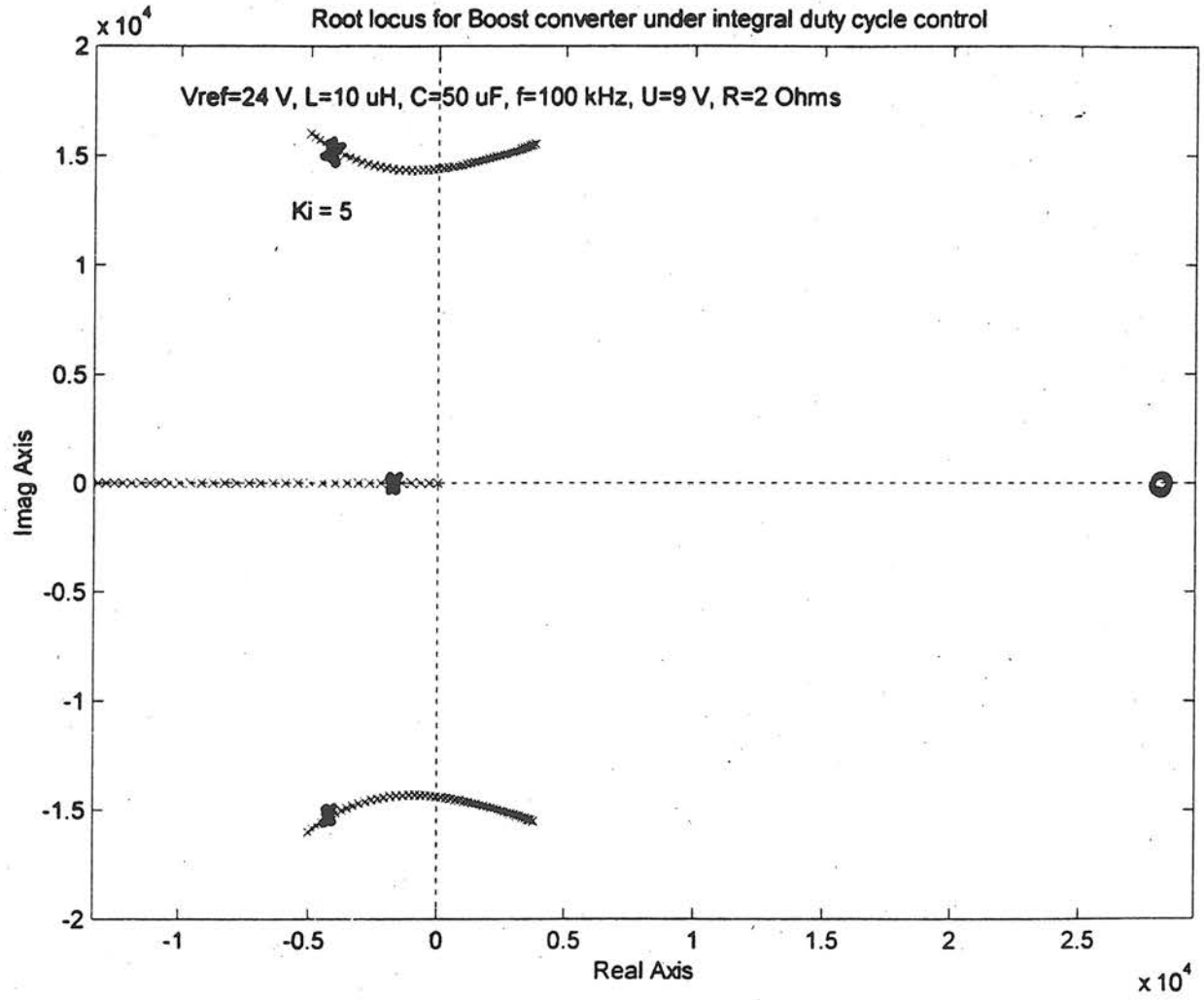
→ note: technically only small signal dynamics are determined, but power converters are forgiving in this respect.

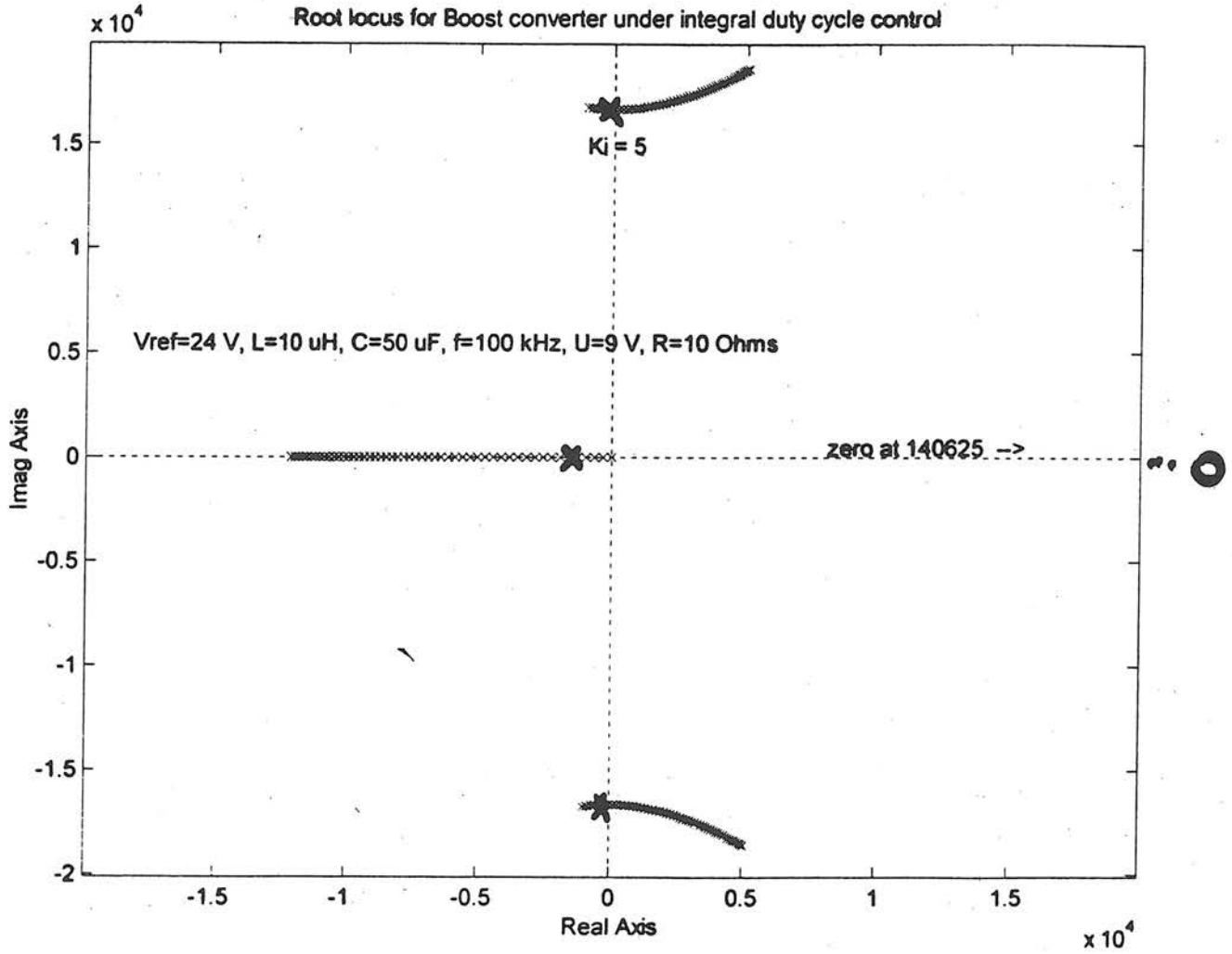
Note: This controller may not be ok in practice due to lightly-damped poles (noise sensitive, transient perf. etc.)

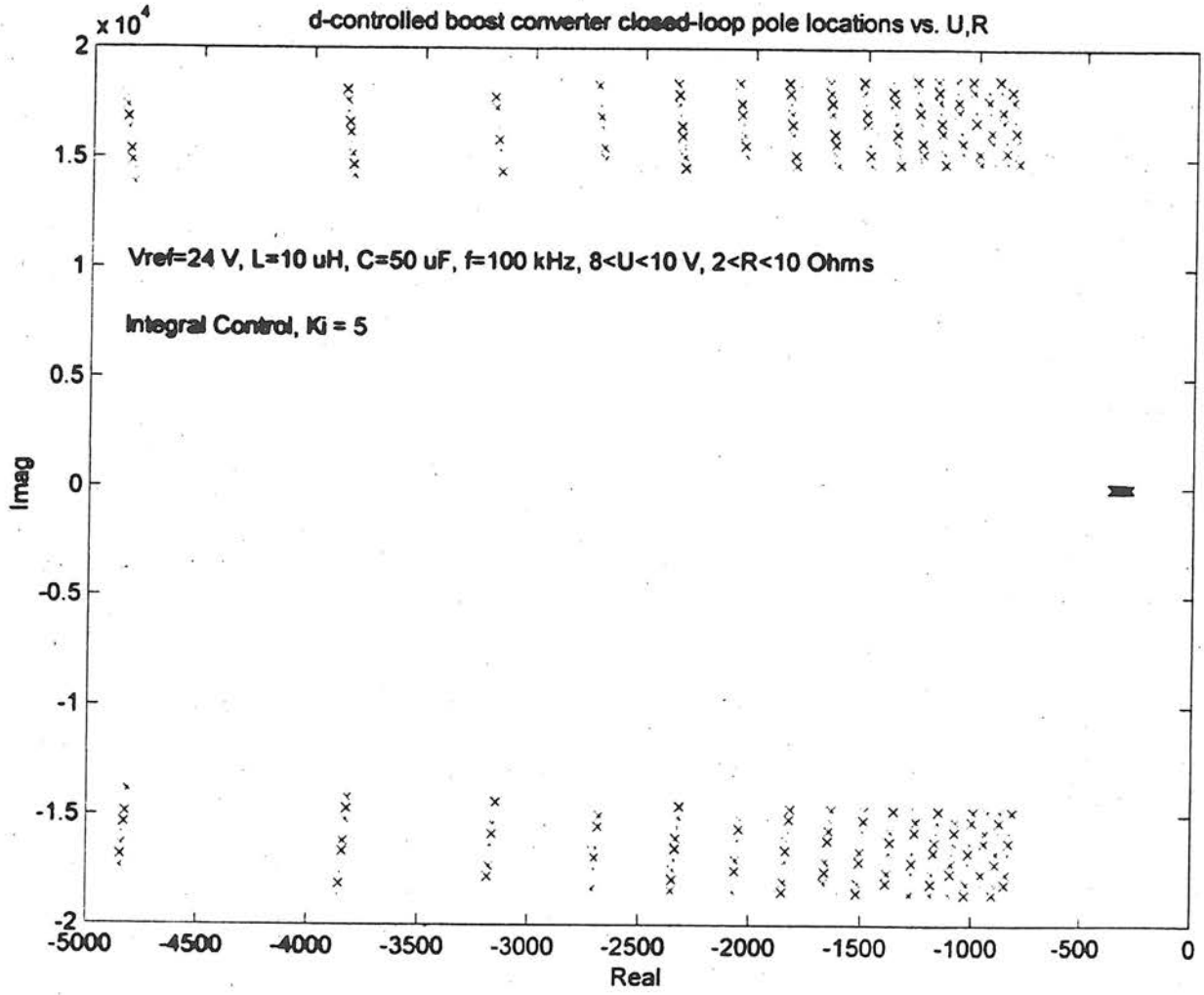
Main drawback is RHP zero!! → go to current mode control!!

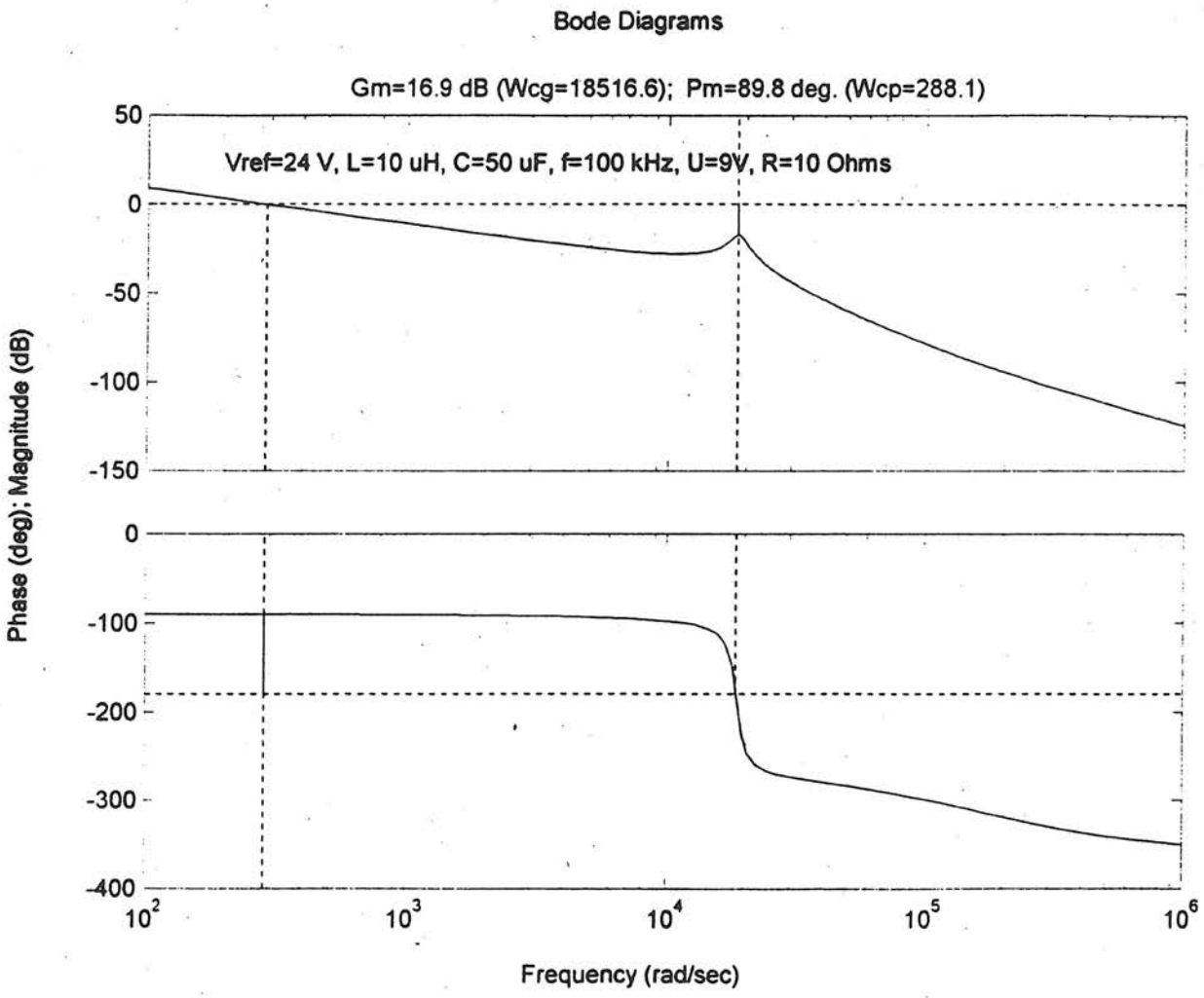

d-controlled Boost converter open loop roots vs. R
x 10^4
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
increasing R --->
Vref=24 V, L=10 uH, C=50 uF, f=100 kHz, U=9 V, 2<R<10 Ohms
-2
0
2
4
6
8
10
12
14
16
x 10^4

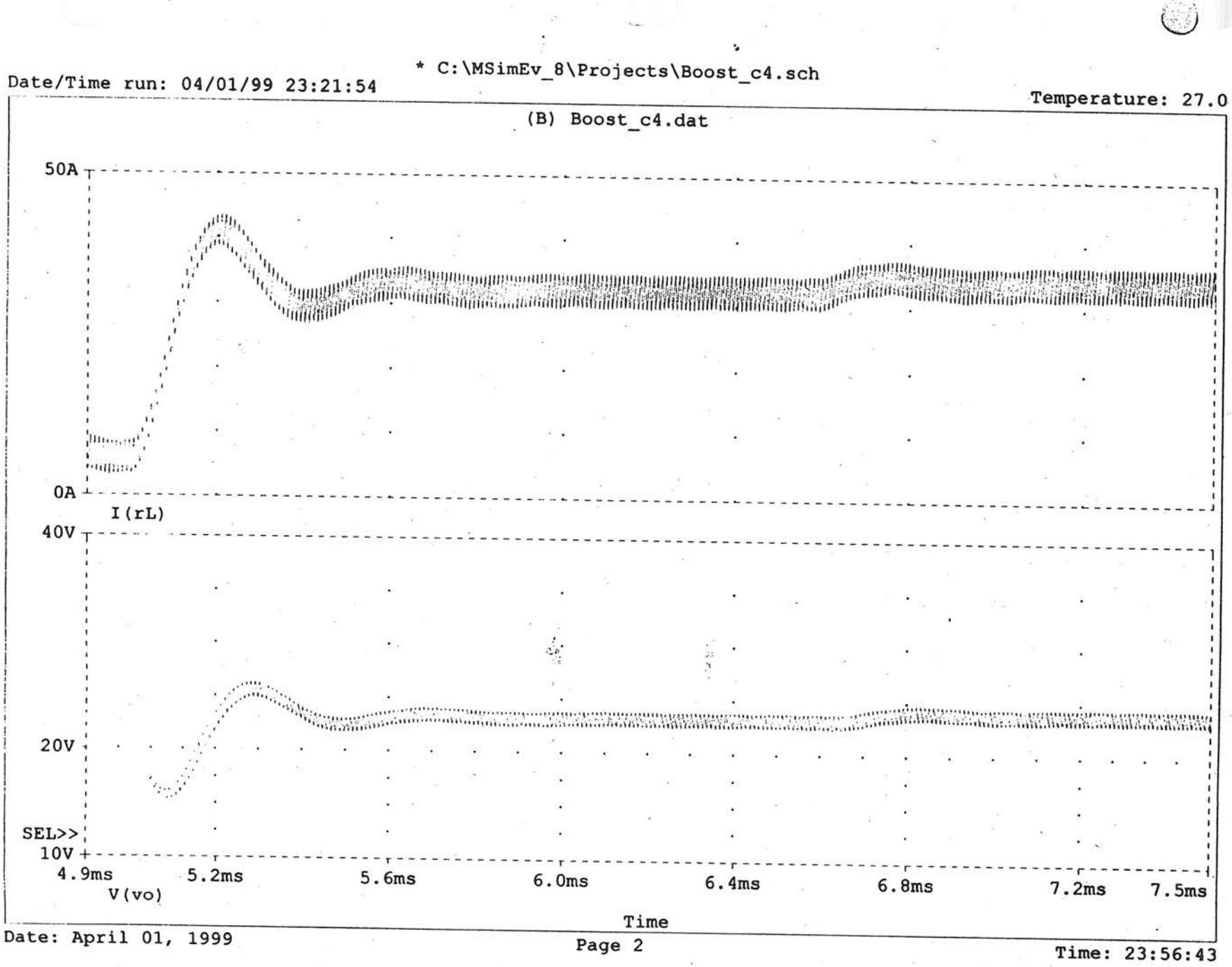

d-controlled Boost converter open loop roots vs. R,U
x 10^4
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
0
2
4
6
8
10
12
14
16
18
x 10^4
Vref=24 V, L=10 uH, C=50 uF, f=100 kHz, 8<U<10 V, 2<R<10 Ohms

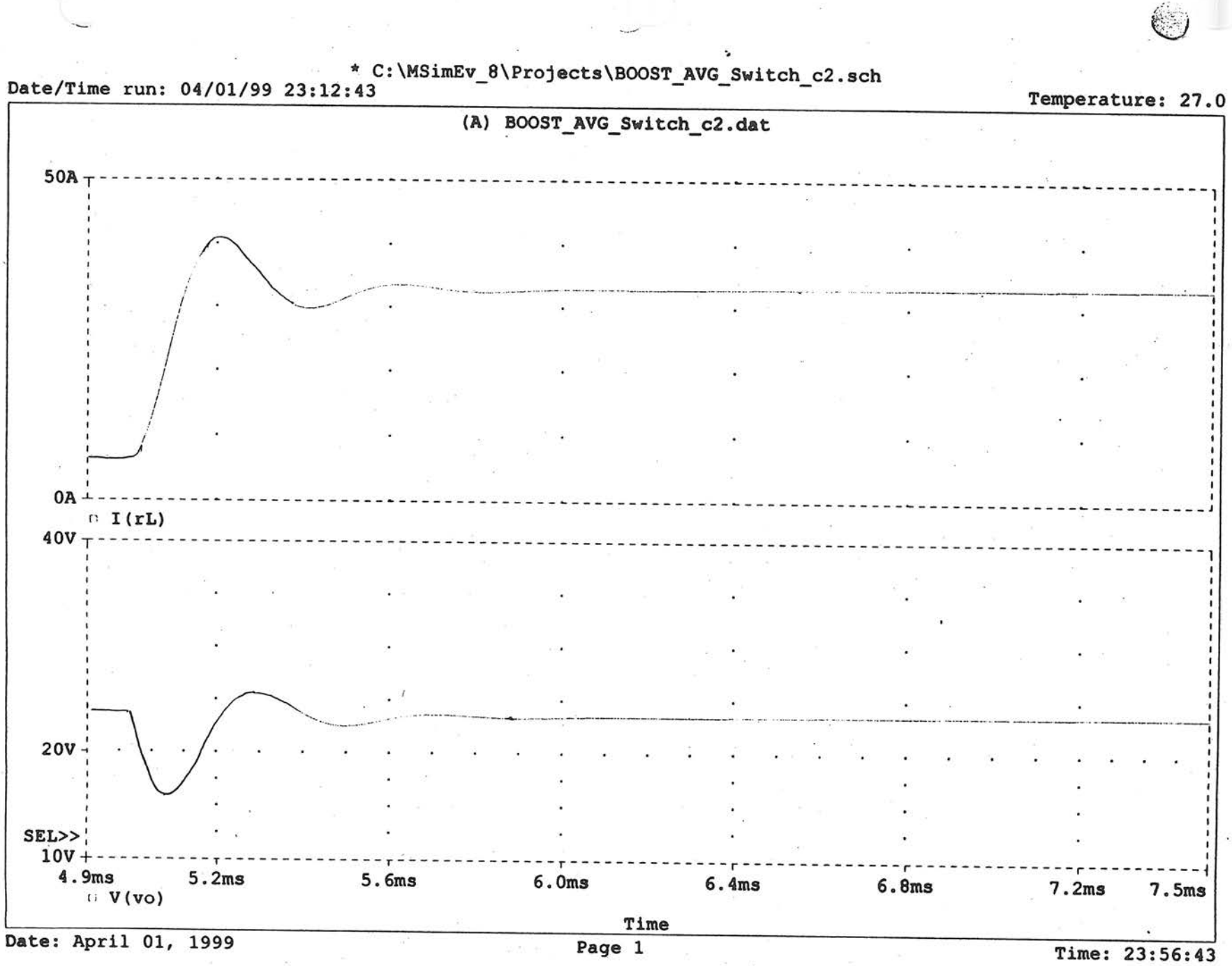

Root locus for Boost converter under integral duty cycle control
Vref=24 V, L=10 uH, C=50 uF, f=100 kHz, U=9 V, R=2 Ohms
Ki = 5
x 10^4
Imag Axis
Real Axis
x 10^4
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1
-0.5
0
0.5
1
1.5
2
2.5


Root locus for Boost converter under integral duty cycle control
x 10^4
Ki = 5
Vref=24 V, L=10 uH, C=50 uF, f=100 kHz, U=9 V, R=10 Ohms
zero at 140625 -->
Imag Axis
Real Axis
x 10^4
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5


d-controlled boost converter closed-loop pole locations vs. U,R
x 10^4
Vref=24 V, L=10 uH, C=50 uF, f=100 kHz, 8<U<10 V, 2<R<10 Ohms
Integral Control, Ki = 5
Imag
Real
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-5000
-4500
-4000
-3500
-3000
-2500
-2000
-1500
-1000
-500
0


Bode Diagrams
Gm=16.9 dB (Wcg=18516.6); Pm=89.8 deg. (Wcp=288.1)
Vref=24 V, L=10 uH, C=50 uF, f=100 kHz, U=9V, R=10 Ohms
Phase (deg); Magnitude (dB)
50
0
-50
-100
-150
0
-100
-200
-300
-400
10^2
10^3
10^4
10^5
10^6
Frequency (rad/sec)

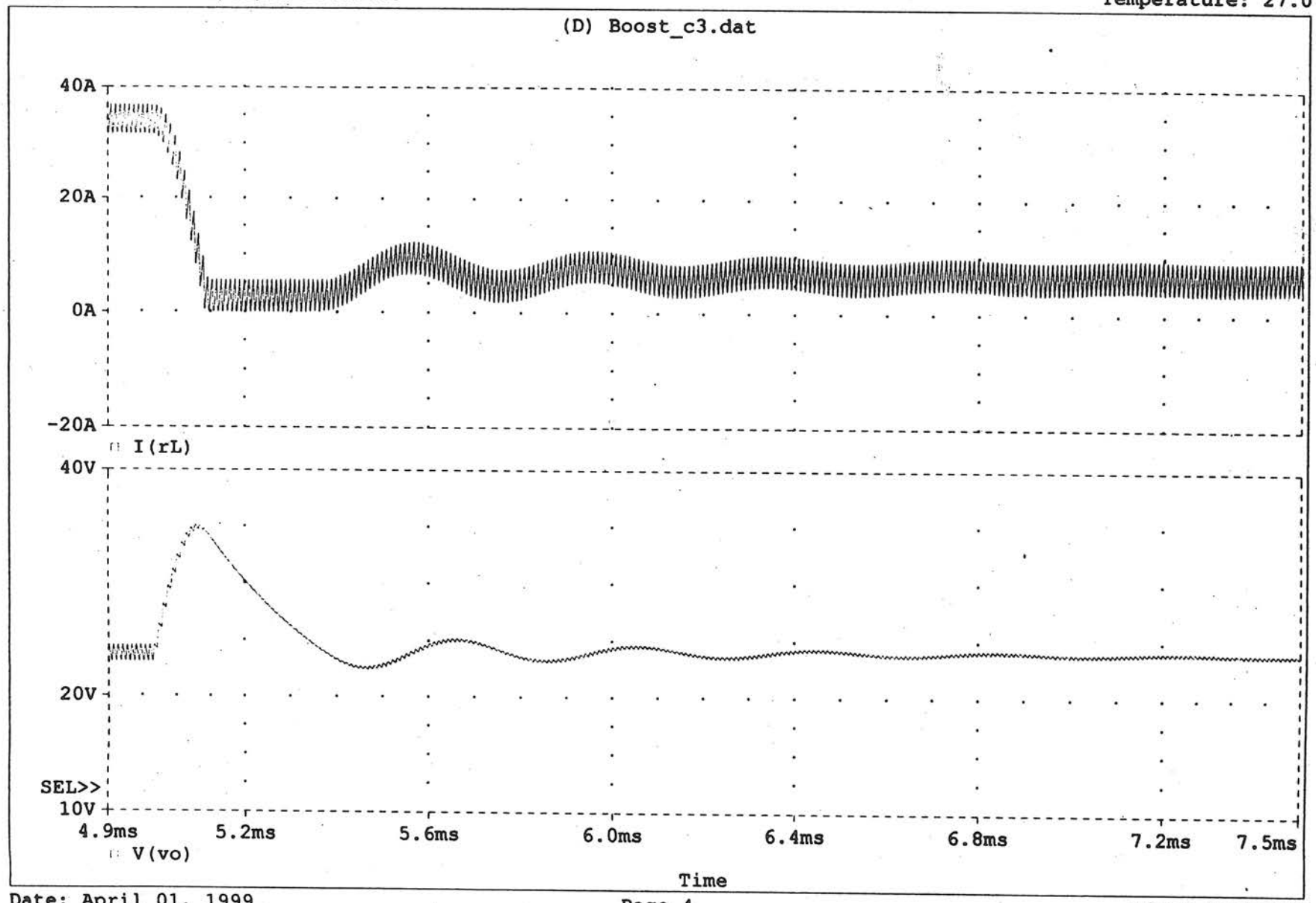

* C:\MSimEv_8\Projects\Boost_c4.sch
Date/Time run: 04/01/99 23:21:54
Temperature: 27.0
(B) Boost_c4.dat
50A
0A
I(rL)
40V
20V
SEL>>
10V
4.9ms
5.2ms
5.6ms
6.0ms
6.4ms
6.8ms
7.2ms
7.5ms
V(vo)
Time
Date: April 01, 1999
Page 2
Time: 23:56:43


* C:\MSimEv_8\Projects\BOOST_AVG_Switch_c2.sch
Date/Time run: 04/01/99 23:12:43
Temperature: 27.0
(A) BOOST_AVG_Switch_c2.dat
50A
0A
I(rL)
40V
20V
SEL>>
10V
4.9ms
5.2ms
5.6ms
6.0ms
6.4ms
6.8ms
7.2ms
7.5ms
V(vo)
Time
Date: April 01, 1999
Page 1
Time: 23:56:43

* C:\MSimEv_8\Projects\Boost_c3.sch

Date/Time run: 04/01/99 23:18:45

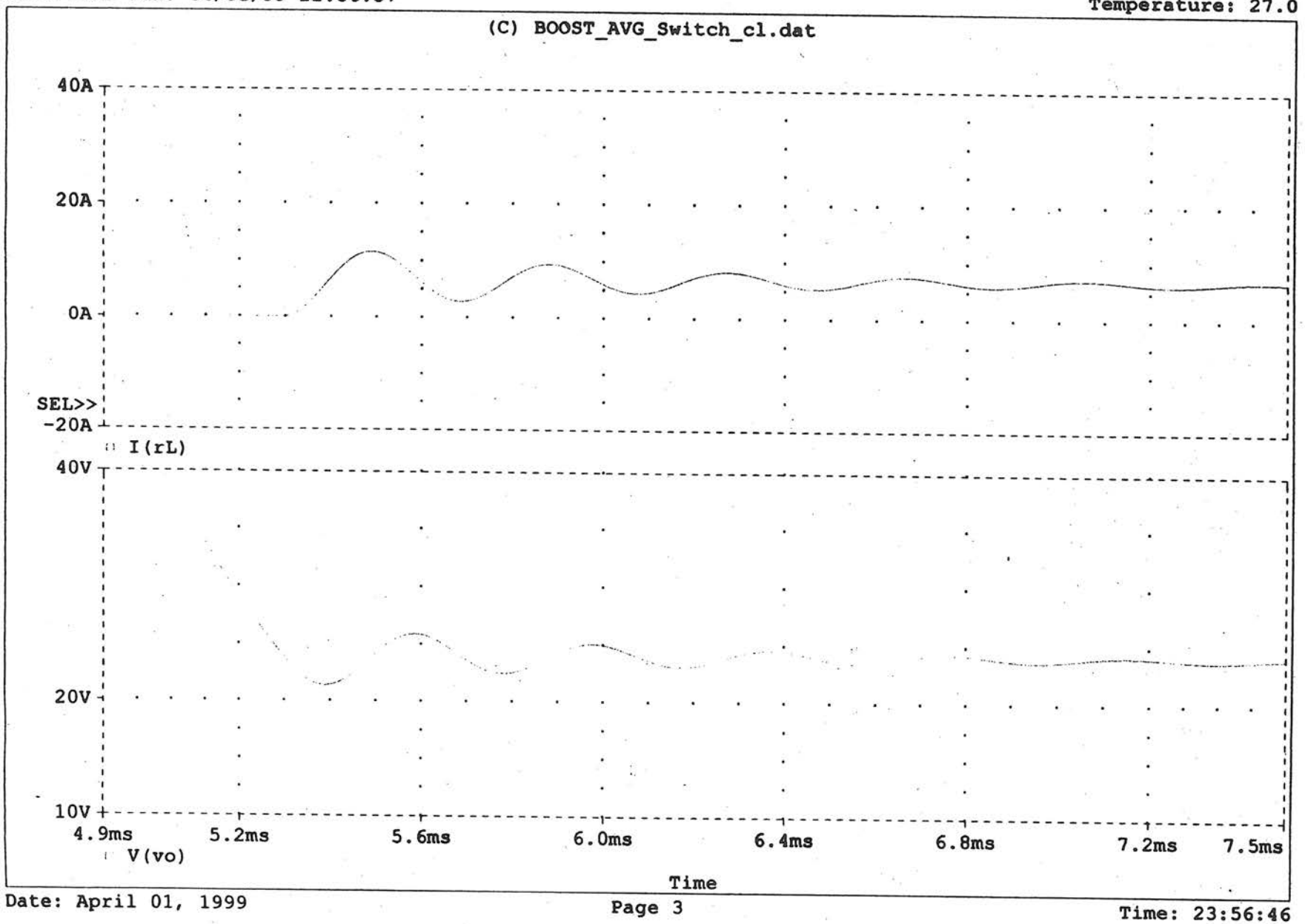
Temperature: 27.0



* C:\MSimEv_8\Projects\BOOST_AVG_Switch_cl.sch

Date/Time run: 04/01/99 22:36:37

Temperature: 27.0



Date: April 01, 1999

Time: 23:56:46

6.622 Power Electronics

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