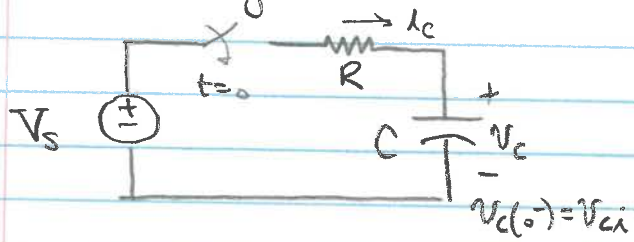
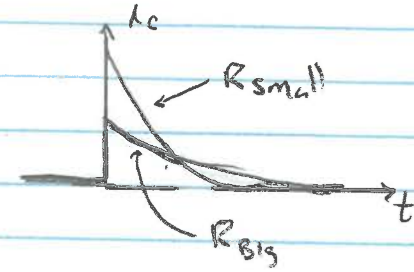
6.334 Lecture Notes — Switched-Capacitor Converters 2

Consider the loss encountered in "series" charging a capacitor from a dc voltage source V_s, starting at an initial voltage V_{ci}:





The charge delivered from the source to the capacitor is

$$\Delta q = \int dq = \int_{V_{ci}}^{V_s} C\,dv = C(V_s - V_{c,i})$$

The energy from the source is

$$\Delta E_{source} = V_s \Delta q = CV_s^2 - CV_sV_{ci}$$

The energy delivered to the cap is:

$$\Delta E_{cap} = \int v\,dq = \int_{V_{ci}}^{V_s} Cv\,dv = \tfrac{1}{2}CV_s^2 - \tfrac{1}{2}CV_{ci}^2$$

The energy lost in the series charging element (e.g. resistor) is:

$$\Delta E_{loss} = \Delta E_{source} - \Delta E_{cap} = \tfrac{1}{2}CV_s^2 - CV_sV_{ci} + \tfrac{1}{2}CV_{ci}^2$$

$$\boxed{\Delta E_{loss} = \tfrac{1}{2}C(V_s - V_{ci})^2}$$

- Note that this loss does NOT depend on the resistance value, or even if the resistor is linear or nonlinear!

- The charging efficiency is $\eta = \dfrac{\Delta E_{cap}}{\Delta E_{source}} = \dfrac{\frac{1}{2}CV_s^2 - \frac{1}{2}CV_i^2}{CV_s^2 - CV_sV_{ci}}$ $\quad \Delta V = V_s - V_{ci}$

  This reduces to

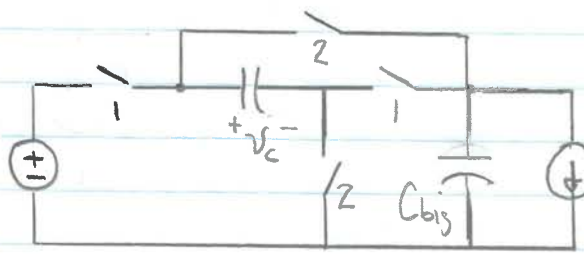
$$\eta = \tfrac{1}{2}\left(1 + \frac{V_{ci}}{V_s}\right) = 1 - \tfrac{1}{2}\left(\frac{\Delta V}{V}\right)$$

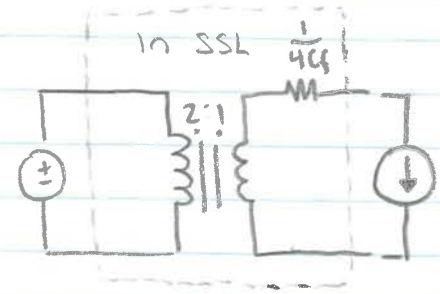
  50% @ $V_{ci}=0$ $\{\Delta V = V_s\}$
  → 100% as $V_{ci} \to V_s$ $\{\Delta V \to 0\}$

★ ⇒ we can make capacitor charging efficient by keeping $\Delta V = V_s - V_{ci}$ small!
(This is key to switched-capacitor converters)

- The only other way to make series charging from constant voltage more efficient is to introduce a series element (e.g. inductor) and use that to charge V_c to a final voltage $> V_s$. Otherwise the delivered charge limits us.

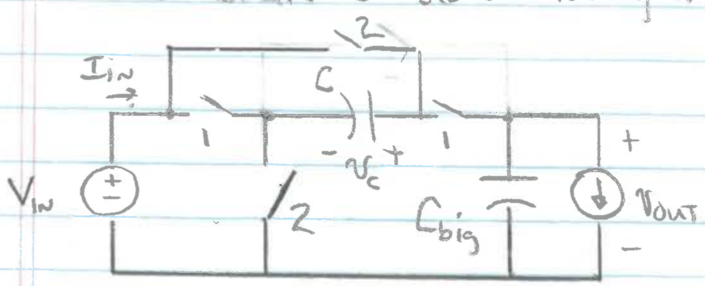
Last time we saw a simple 2:1 switched capacitor converter:



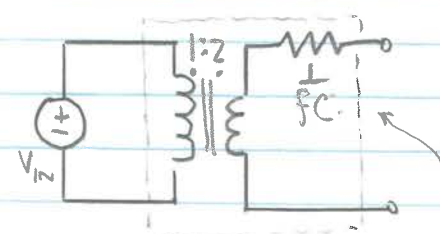


• The fact that the efficiency of v-source charging a capacitor improves with smaller $\Delta V/V$ $\{\eta = 1 - \frac{1}{2}\frac{\Delta V}{V_s}\}$ is reflected in the fact that $R_{eq} \propto 1/(Cf)$

Given appropriate switch implementations, SC converters are bidirectional. Consider "turning around" the converter above:



we get the following SSL model:

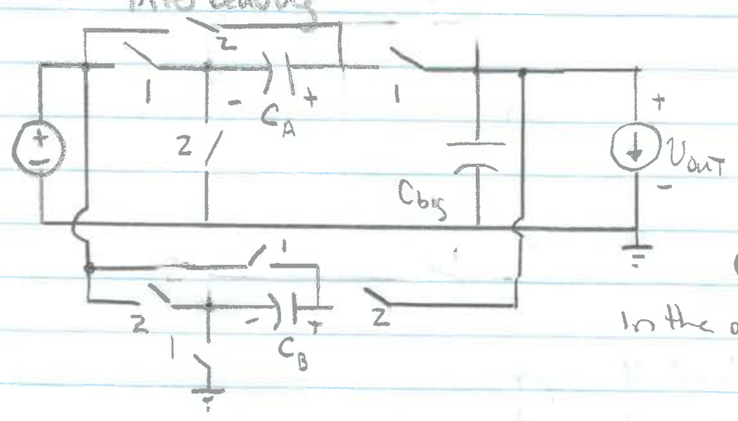


Notice how R_{eq} scales through the "transformer"

• STATE 1: Discharge $V_{IN} + V_c$ into C_{big} @ V_{OUT}
• STATE 2: Charge up V_c to V_{IN}

$$I_{OUT} = \frac{1}{2} I_{IN}$$
$$V_{OUT} = 2V_{IN} - \frac{I_{OUT}}{fC}$$

For this converter, C_{big} must "hold up" the output voltage as it only gets current pulses in 1 of 2 phases (phase 1). We can fix this with "interleaving"



• By interleaving we can greatly reduce C_{big} for a given allowed ripple. (In theory we can eliminate C_{big} if we can tolerate ripple in the output voltage.)

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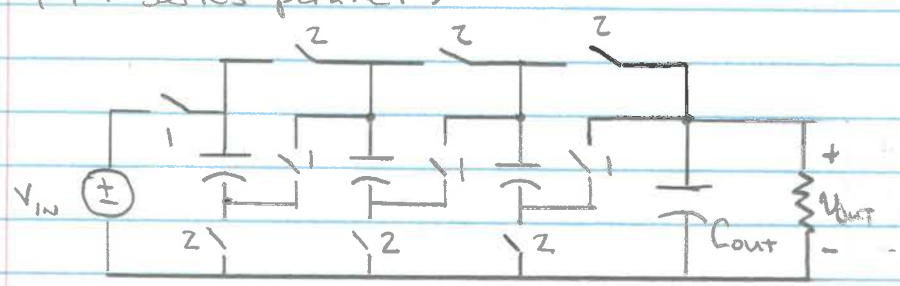
There are many other SC converter types, each with its own properties. One classic extension of the converter already shown is the "series-parallel"

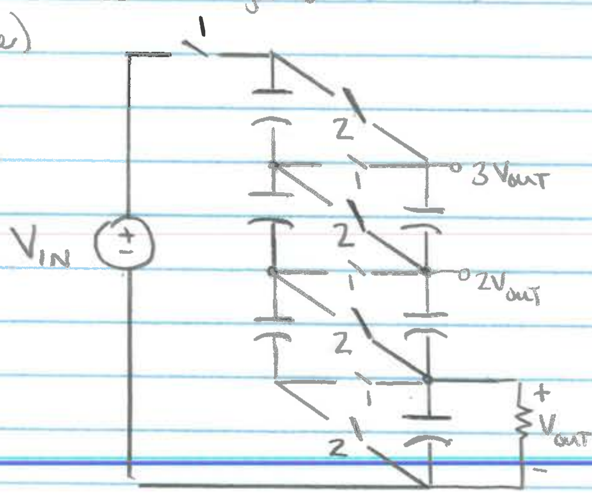
(4:1 Series-parallel)



* Capacitors charge (discharge) in series + discharge (charge) in parallel
* Capacitors see only $\min(V_{IN}, V_{OUT})$ (switches see more)
* N:1 has N-1 xfer caps + 1 output cap = N caps, 3N-2 switches

Another classic design is the "Ladder" converter

(4:1 ladder)



* STATE 1 rebalances "flying" cap set and delivers charge from V_{in} to stack
STATE 2 rebalances rest of cap set and delivers more charge to output
* N:1 converter has 2N-2 caps + 2N switches all rated for $\min(V_{IN}, V_{OUT})$ (This is a popular feature)
* Can generate several ratiometric voltages at once $\left(\frac{V_{IN}}{4}, \frac{2V_{IN}}{4}, \frac{3V_{IN}}{4}\right)$

* The interleaved version of this converter is very useful for multioutput conversion.

General achievable limits for 2-phase SC converters have been explored (see Makowski work).

For a number of capacitors K (including output cap but not input cap), we get a max conversion ratio $N = F(k)$, where $F(k)$ is the k^{th} fibonacci #, and require 3k-2 switches

# Capacitors k	K	0	1	2	3	4	5	6
max conversion ratio N	F(K)	1	1	2	3	5	8	13
min # of switches m	3k-2	-	0	4	7	10	13	16

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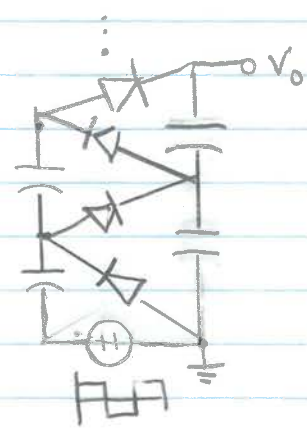
The component count limits do not tell us stresses on switches, capacitors. Different designs yield tradeoffs on how well they utilize switches, capacitors {some better for switches, some for caps}, how many components of what voltage/current, etc.

In each case, we can find an equivalent circuit model comprising a transformer with rational turns ratio & an equivalent output resistance. To find these, we merely find the charge xfers in each state. [see Seeman, et. al. IEEE Trans. Power Electron. March 2008]

Note: If we replace many of the active switches with diodes, we get "voltage multipliers", e.g. the cockroft-walton voltage multiplier is related to the ladder converter. (used to generate very high voltages from a low-voltage ac input)



There are likewise many ways to integrate modest amounts of magnetics to get hybrid SC/magnetic converters with interesting properties.

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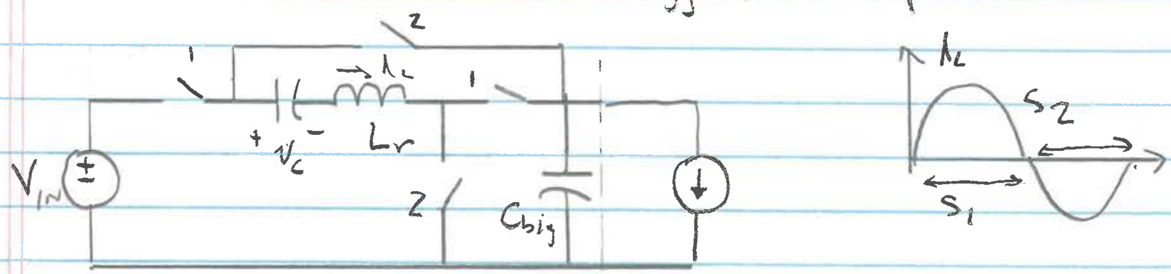
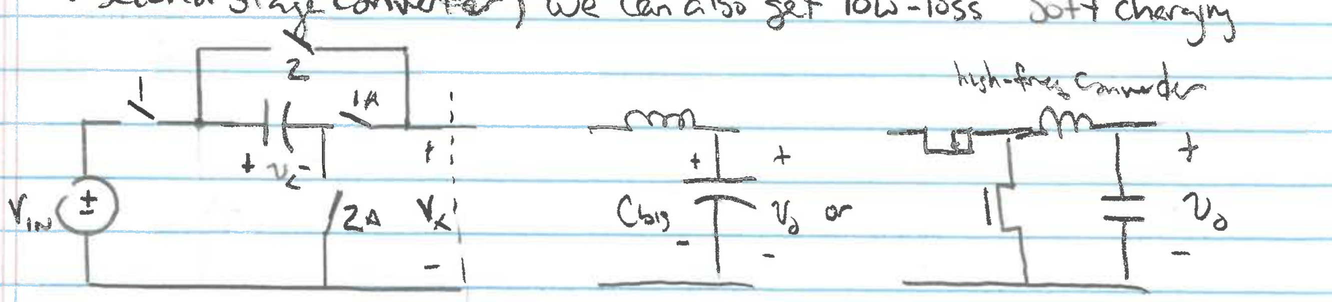
Resonant SC Conversion

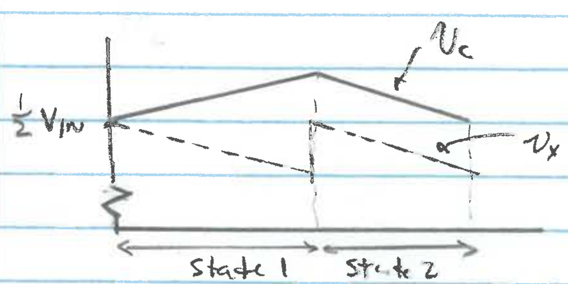
Consider our 2:1 step-down converter with a resonant inductor in series with the energy xfer capacitor:



This is known as a "resonant" switched-capacitor circuit. It provides the same conversion ratio as the nonresonant version. However instead of exponential pulses in the SSL we can switch this such that each phase (1 and 2) is ½ a resonant cycle (or an odd multiple) and get smooth currents with ~ZCS switching. This gives us FSL R_{eq} values while operating at low switching frequency, at the expense of adding an inductor and control timing.

If we eliminate C_{big} and make the load <u>inductive</u> (either by inserting an inductor at the output or by adding a second stage converter) we can also get low-loss "soft charging"





- we get "soft charging" of the capacitor with no charge sharing loss (similar to FSL limit @ low frequency)
- by using the high-frequency second stage or introducing another switch state (1A, 2A both on) we can regulate v_o as well!