6.622 Power Electronics *Prof. David Perreault*

Lecture 35 - Resonant Power Conversion 1

1 Resonant Power Conversion

In many power converter applications, one converts between dc and high-frequency sinusoidal ac (either as an intermediate or final waveform). Applications include:

1. RF Power amplifiers for communications, radar, etc.
2. High frequency inverters for induction heating
3. “Resonant” dc/dc converters using high-frequency sinusoidal intermediate waveforms & energy storage
4. Electronic ballasts for lighting
5. Power converters using resonant energy storage/transformer structures (e.g., piezoelectric, “electromechanical” transformers)

Here we consider topologies, designs + control methods that are suited to these applications. These applications share certain objectives:

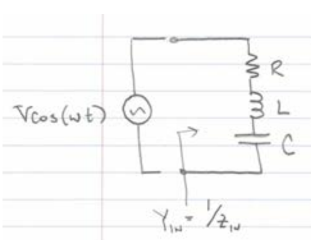
1. High-frequency sinusoidal ac waveforms
2. A desire for high-efficiency power conversion

<u>But</u>

Different applications have different requirements

1. Load characteristics (narrow, well-known range vs wide load variations
2. Need for or absence of regulation + control of output
3. high sinusoidal waveform purity (required or not)

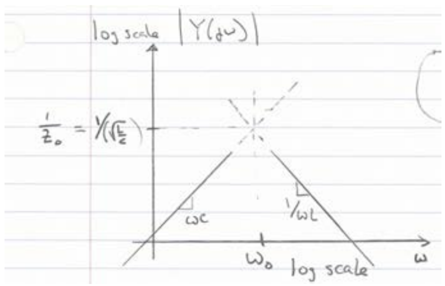
2 Review of Resonant Circuits



$$Y_{in} = \frac{1}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{R + j(\omega L - \frac{1}{\omega C})}$$

$$|Y_{in}| = \frac{1}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$$\angle = -tan^{-1}(\frac{\omega L - \frac{1}{\omega C}}{R})$$



- @ low frequency, capacitor limits admittance $Y_{in} \approx j\omega C$
- @ high frequency, inductor limits admittance $Y_{in} = \frac{1}{j\omega L} = \frac{j}{\omega L}$

Both are imaginary with opposite sign.

@ some intermediate frequency, the impedance of L, C and equal and opposite

$$j\omega L = -\frac{j}{\omega C} = 0$$

$\boxed{\Rightarrow \omega = \omega_0 = \frac{1}{\sqrt{LC}}}$

ω_0: "undamped natural frequency"

$@\omega_0 : |z_L| = \omega_0 L = \frac{L}{C}, |z_C| = \frac{\omega_0}{C} = \frac{L}{C}$

"Characteristic impedance" $\boxed{z_0 \triangleq \sqrt{\frac{L}{C}}}$

$$\star \rightarrow \text{ Note } |V_c(\omega_0)| = \frac{|I_L|}{\omega_0 C} = \sqrt{\frac{L}{C}} \cdot |I_L| = z_0 |I_L| \Rightarrow z_0 \text{ relates } |v_c| \text{ to } |i_L| @\omega_0$$

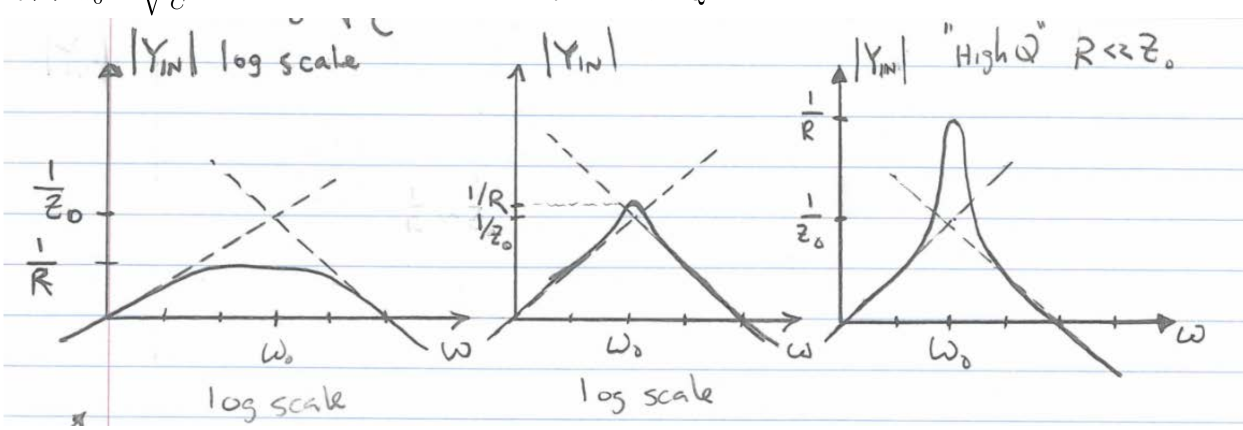
What happens near $\omega = \omega_0$?

$@\omega = \omega_0, Y_{in} = \frac{1}{R}$ (L, C impedances cancel)

∴ Three possible cases

$\boxed{\text{The behavior depends on relative values of } z_0, R}$

$R >> z_0 = \sqrt{\frac{L}{C}}$ $\qquad$ $R \sim Z$ "low Q"



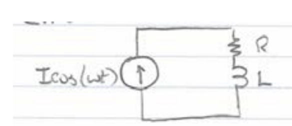
We often discuss "Q" or "quality factor" of a system. The quality factor is proportional to the peak ac energy stored to the energy dissipated in an ac cycle

$\boxed{Q \triangleq 2\pi \frac{\text{Peak energy stored}}{\text{Energy dissipated}}}$

Example: a series RL circuit

$$E_{stored} = \frac{1}{2} L I^2$$

$$E_{diss} = (\frac{1}{2} I^2 R)(T) = (\frac{1}{2} I^2 R) \cdot \frac{2\pi}{\omega} = \frac{\pi I^2 R}{\omega}$$

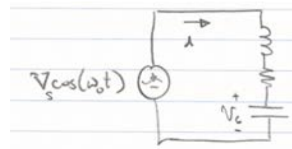


$$\Rightarrow Q = 2\pi \cdot \frac{LI^2\omega}{RI^2 2\pi}$$

$\boxed{\Rightarrow Q = \frac{\omega L}{R}}$

Let's consider the Q of the series LRC $\underline{@\omega = \omega_0}$

$$i(t) = \frac{V_s}{R} cos(\omega_0 t)$$



From before @ $I = \frac{V}{R}$

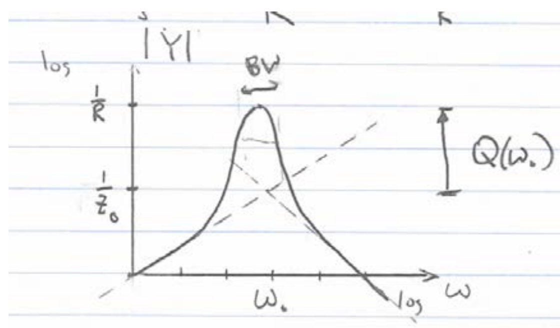
$$E_{diss} = \frac{\pi V_s^2}{\omega_0 R}$$

@ ω_0, i_L, v_c out of phase $\therefore i_{L,P} @ v_c = 0$

$$E_{stored} = \frac{1}{2}L(\frac{V_s}{R})^2 = \frac{1}{2}\frac{LV_s^2}{R^2}$$

$$\therefore Q(\omega_0) = 2\pi \frac{LV_s^2}{2R^2} \cdot \frac{\omega_0 R}{\pi v_s^2} = \frac{\omega_0 L}{R} = \frac{\sqrt{\frac{L}{C}}}{R} = \frac{z_0}{R}$$

So, considering our plot:



1. The resonance peaks up the admittance (due to L, C cancellation) by a factor $Q(\omega_0) = \frac{z_0}{R}$!
2. Consider the <u>voltage gain</u> from source voltage V_s to peak capacitor voltage $|v_c|$ at drive frequency $\omega = \omega_0$

$$V_{c,pk} = \frac{I}{\omega_0 C} = \frac{V_s}{R \cdot \omega_0 C} = V_s \frac{z_0}{R} = V_s \cdot Q(\omega_0)$$

$\therefore$ for large Q(@ω_0) we can get <u>large</u> voltage gain from resonance.
This is often used in practice (e.g.) for "striking" the lamp in a lamp ballast)

3. Consider the "bandwidth" over which large resonant peaking occurs. We will consider the bandwidth (or frequency range) over which the power delivered to R is at least <u>half</u> that delivered at ω_0 ("half-power bandwidth")

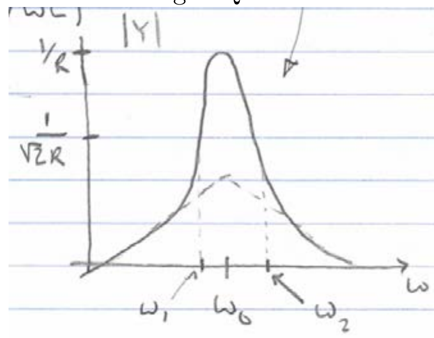
$$@\omega_0, |I| = \frac{V_s}{R}$$

For half-power we get $|I| = \frac{V_s}{\sqrt{2}R}$

$$\text{or } |Y| = \frac{1}{\sqrt{2}R} = |\frac{1}{R + j(\omega L - \frac{1}{\omega C})}|$$

$$\therefore \frac{1}{\sqrt{2}R} = \frac{1}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$\frac{1}{2}$ power BW is <u>not</u> centered on ω_0, but close @ big L Q



$$\therefore |\omega L - \frac{1}{\omega C}| = R$$

Happens @ two frequencies, $\omega_1 < \omega_0, \omega_2 > \omega_0$

$$@\omega_1 : \omega_1 L - \frac{1}{\omega_1 C} = -R$$

$$\omega_1^2 LC = \omega_1 RC - 1 = 0$$

$$\omega_1 = \frac{-RC + \sqrt{(RC)^2 + 4LC}}{2LC} = -\frac{R}{2L} + \sqrt{\frac{1}{LC} + (\frac{R}{2L})^2}$$

$$@\omega_2; \omega_2 L - \frac{1}{\omega_2 C} = R \Rightarrow \omega_2^2 LC - \omega_2 RC - 1 = 0$$

$$\therefore = \frac{RC}{2LC} + \sqrt{(\frac{RC}{2LC})^2 + \frac{1}{LC}} = \sqrt{\frac{1}{LC} + (\frac{R}{2L})^2} + \frac{R}{2L}$$

Finding the half-power bandwidth BW = $\omega_2 - \omega_1$

$$\omega_2 - \omega_1 = \frac{R}{L}$$

Normalizing this to ω_0, we get:

$$\boxed{\frac{BW}{\omega_0} = \frac{R\sqrt{LC}}{L} = \frac{R}{\sqrt{\frac{L}{C}}} = \frac{R}{z_0} = \frac{1}{Q(\omega_0)}}$$

So we find that the half-power bandwidth (over which the power delivered to R is at least 1/2 that at ω_0) normalized to ω_0 is 1/Q!

$\Rightarrow$ higher Q gives larger peaking over a narrower bandwidth