Power Electronics Notes - D. Perreault

★★ RESONANT POWER CONVERSION

In many power converter applications, one converts between dc and high-frequency sinusoidal ac (either as an intermediate or final waveform). Applications include:

1. RF Power amplifiers for communications, radar, etc.
2. High frequency inverters for induction heating
3. "Resonant" dc/dc converters using high-frequency sinusoidal intermediate waveforms + energy storage
4. Electronic ballasts for lighting
5. power converters using resonant energy storage/transformer structures (e.g. piezoelectric "electromechanical" transformers)

Here we consider topologies, designs + control methods that are suited to these applications. These applications share certain objectives:

1. High-frequency sinusoidal ac waveforms
2. A desire for high-efficiency power conversion.

But

Different applications have different requirements:

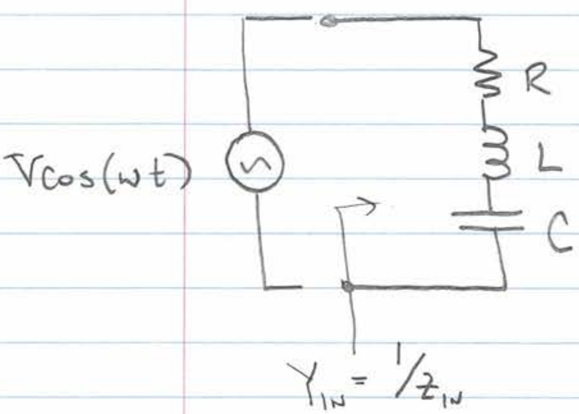
1. Load characteristics (narrow, well-known range vs. wide load variations)
2. Need for or absence of regulation + control of output
3. high sinusoidal waveform purity (required or not)

NOTE: THIS YEAR, FIRST LECTURE COVERS ONLY RESONANT CIRCUITS REVIEW.

6.334 Lecture Notes — Resonant Power Conversion # 1

REVIEW OF RESONANT CIRCUITS

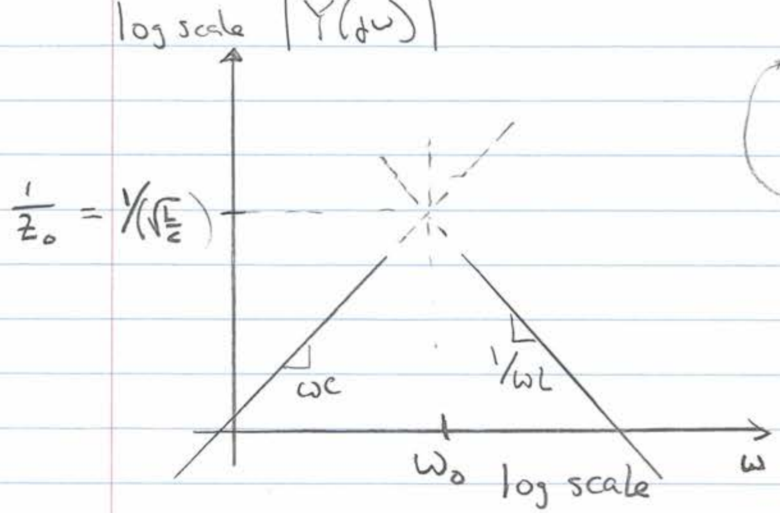
Consider a second-order series-resonant tank



$$Y_{IN} = \frac{1}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\left| Y_{IN} \right| = \frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\measuredangle Y_{IN} = -\mathrm{ATAN}\left(\frac{\omega L - 1/\omega C}{R}\right)$$



(imaginary w/ opposite sign)

@ low frequency capacitor limits admittance $Y_{IN} \simeq j\omega C$

@ high frequency inductor limits admittance $Y_{IN} \simeq \frac{1}{j\omega L} = -\frac{j}{\omega L}$

@ some intermediate frequency the impedances of L, C are equal and opposite

$$j\omega L - \frac{j}{\omega C} = 0 \quad \Rightarrow \quad \boxed{\omega = \omega_0 = \frac{1}{\sqrt{LC}}}$$

ω_0: "Undamped natural frequency"

"CHARACTERISTIC IMPEDANCE"

$$@\,\omega_0: \ |Z_L| = \omega_0 L = \sqrt{\frac{L}{C}}, \quad |Z_C| = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}} \qquad \boxed{Z_0 \triangleq \sqrt{\frac{L}{C}}}$$

★ → NOTE $|V_C(\omega_0)| = \frac{|I_L|}{\omega_0 C} = \sqrt{\frac{L}{C}} \cdot |I_L| = Z_0 |I_L| \Rightarrow Z_0$ Relates $|V_C|$ to $|I_L|$ @ ω_0!

6.334 Lecture Notes — Resonant Power Conversion #1

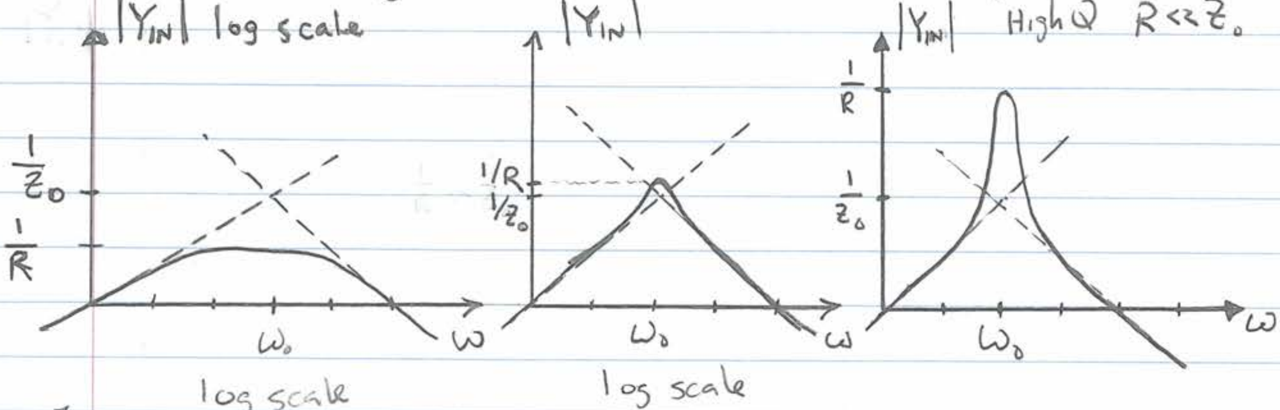
what happens near $\omega = \omega_0$?

$@\ \omega = \omega_0 \quad Y_{IN} = \frac{1}{R}$ (L, C impedances cancel)

$\therefore$ Three possible cases

THE BEHAVIOR DEPENDS ON RELATIVE VALUES OF Z_0, R

$R \gg Z_0 = \sqrt{\frac{L}{C}}$ — $R \sim Z_0$ "low Q" — "High Q" $R \ll Z_0$

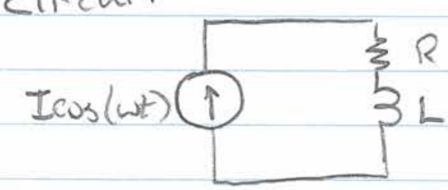


(DRAW ON ONE PLOT IN DIFFERENT COLORS)

WE OFTEN DISCUSS THE "Q" or "quality factor" of a system. The quality factor is proportional to the peak (ac) energy stored to the energy dissipated in an ac cycle

$$\boxed{Q \triangleq 2\pi \frac{\text{PEAK ENERGY STORED}}{\text{ENERGY DISSIPATED}}}$$

example: a series RL circuit



$$E_{STORED} = \frac{1}{2} L I^2$$

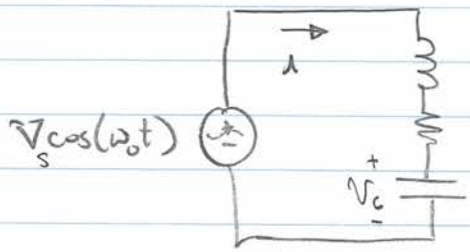
$$E_{DISS} = \left(\frac{1}{2} I^2 R\right)(T) = \left(\frac{1}{2} I^2 R\right) \cdot \frac{2\pi}{\omega} = \frac{\pi I^2 R}{\omega}$$

$$\Rightarrow Q = 2\pi \cdot \frac{L I^2 \omega}{R I^2 2\pi} \Rightarrow \boxed{Q = \frac{\omega L}{R}}$$

6.334 Lecture Notes Resonant Power Conversion #1

lets consider the Q of the series LRC @ $\omega = \omega_0$

$$i(t) = \frac{V_s}{R}\cos(\omega_0 t)$$



from before @ $I = V/R$

$$E_{DISS} = \frac{\pi V_s^2}{\omega_0 R}$$

@ ω_0 i_L, V_c $90°$ out of phase $\therefore$ $i_{L,pk}$ @ $V_c = 0$

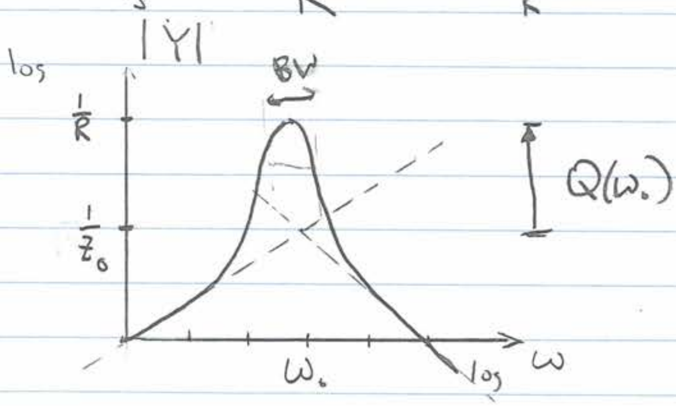
$$E_{STORED} = \frac{1}{2}L\left(\frac{V_s}{R}\right)^2 = \frac{1}{2}\frac{LV_s^2}{R^2}$$

$\omega_0 = \frac{1}{\sqrt{LC}}$

ENERGY STORED TO ENERGY LOST @ $\omega = \omega_0 = Z_0/R$

$$\therefore Q(\omega_0) = 2\pi \frac{LV_s^2}{2R^2} \cdot \frac{\omega_0 R}{\pi V_s^2} = \frac{\omega_0 L}{R} = \frac{\sqrt{L/C}}{R} = \frac{Z_0}{R}!$$

So, considering our plot:



① THE RESONANCE PEAKS UP THE ADMITTANCE (DUE TO L,C cancellation) by a factor $Q(\omega_0) = \frac{Z_0}{R}$!

② Consider the voltage gain from source voltage V_s to peak capacitor voltage $|V_c|$ at drive frequency $\omega = \omega_0$.

$$V_{c,pk} = \frac{I}{\omega_0 C} = \frac{V_s}{R \cdot \omega_0 C} = V_s \frac{Z_0}{R} = V_s \cdot Q(\omega_0)$$

$\therefore$ for large Q (@ ω_0) we can get large voltage gain from resonance.

This is often used in practice (e.g. for "striking" the lamp in a lamp ballast.)

(3) Consider the "Bandwidth" over which large resonant peaking occurs. We will consider the bandwidth (or frequency range) over which the power delivered to R is at least <u>half</u> that delivered at ω_o ("half-power bandwidth")

@ ω_o, $|I| = \frac{V_s}{R}$

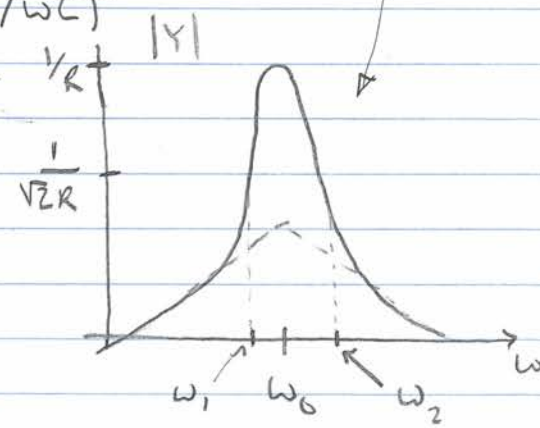
for half-power we get $|I| = \frac{V_s}{\sqrt{2}R}$

or $|Y| = \frac{1}{\sqrt{2}R} = \left| \frac{1}{R + j(\omega L - 1/\omega C)} \right|$

$\frac{1}{2}$ power BW is <u>not</u> centered on ω_o, but close @ high Q

$\therefore \frac{1}{\sqrt{2}R} = \frac{1}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$

$\therefore \left| \omega L - \frac{1}{\omega C} \right| = R$



happens @ two frequencies, $\omega_1 < \omega_o$, $\omega_2 > \omega_o$

@ ω_1: $\omega_1 L - \frac{1}{\omega_1 C} = -R$

$\omega_1^2 LC + \omega_1 RC - 1 = 0$

$\omega_1 = \frac{-RC + \sqrt{(RC)^2 + 4LC}}{2LC} = -\frac{R}{2L} + \sqrt{\frac{1}{LC} + \left(\frac{R}{2L}\right)^2}$

@ ω_2: $\omega_2 L - \frac{1}{\omega_2 C} = R \Rightarrow \omega_2^2 LC - \omega_2 RC - 1 = 0$

$\therefore \omega_2 = \frac{+RC}{2LC} + \sqrt{\left(\frac{RC}{2LC}\right)^2 + \frac{1}{LC}} = \sqrt{\frac{1}{LC} + \left(\frac{R}{2L}\right)^2} + \frac{R}{2L}$

Finding the half-power bandwidth $BW = \omega_2 - \omega_1$

$$\omega_2 - \omega_1 = R/L$$

Normalizing this to ω_0, we get:

$$\boxed{\frac{BW}{\omega_0} = \frac{R\sqrt{LC}}{L} = \frac{R}{\sqrt{L/C}} = \frac{R}{Z_0} = \frac{1}{Q(\omega_0)}!}$$

So we find that the half-power bandwidth (over which the power delivered to R is at least ½ that at ω_0) normalized to ω_0 is $1/Q$!

⇒ higher Q gives larger peaking over a narrower bandwidth