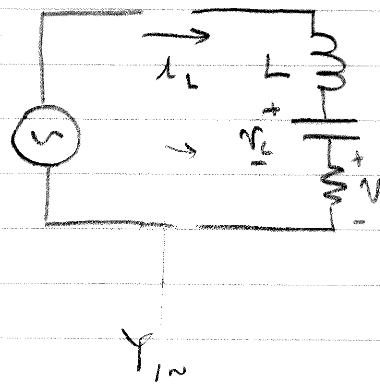
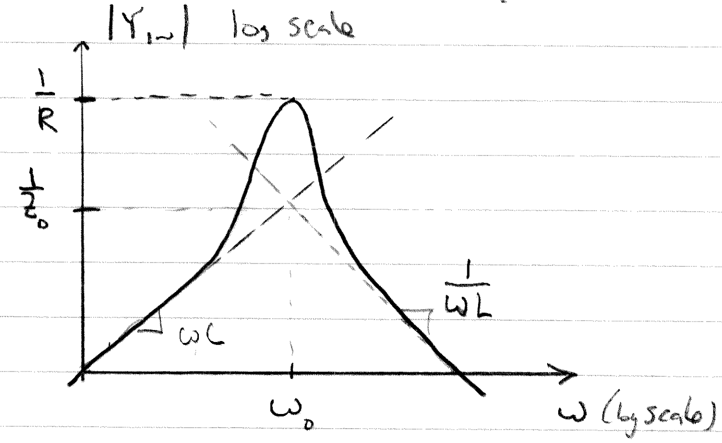
# 6.334 Lecture Notes — Resonant Power Conversion Cont'd

## REVIEW:

RESONANT NETWORKS PROVIDE STRONG IMPEDANCE VARIATION W/ FREQUENCY

$V_1 \cos(\omega t)$




$\omega_0 = \frac{1}{\sqrt{LC}}$

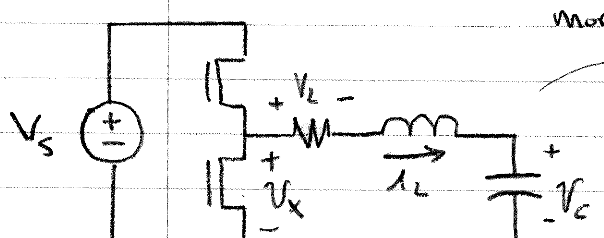
$Z_0 = \sqrt{\frac{L}{C}}$

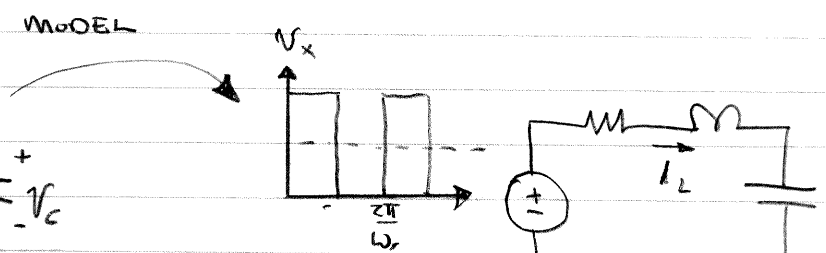
$Q @ \omega_0 = Z_0/R$

$\Rightarrow$ we can control power to load R by varying frequency!

## ★ "CLASS D" AMPLIFIER IN "SERIES RESONANT" CONVERTER

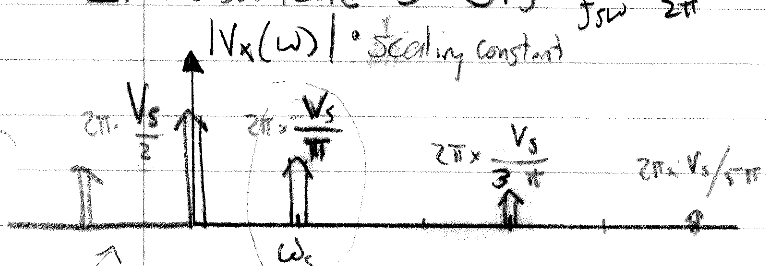
Suppose we drive the resonant tank (w/ resistive load R) from a half-bridge inverter:




$i_L(s) = V_x(s) \cdot Y_{in}(s)$

If we switch @ $D = 0.5$ $f_{sw} = \frac{\omega_s}{2\pi}$




@ high Q $\left\{ R << \sqrt{\frac{L}{C}} \right\}$

$\omega_s$ near $\omega_0$

$i_L \sim$ sinusoidal

Fourier xform based on exponential form of series, thus scaling on it ($\cdot 2\pi$) O+W p.298

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To control $i_L$, power to $R$, we might vary switching frequency above or below resonance

$$|i_L(j\omega_s)| = |V_x(j\omega_s) \cdot Y_{IN}(j\omega_s)|$$

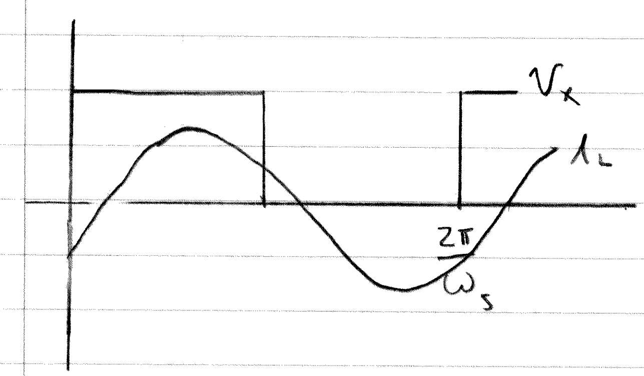
$\Rightarrow$ Above resonance $f_{sw} > \frac{\omega_o}{2\pi}$ network looks inductive

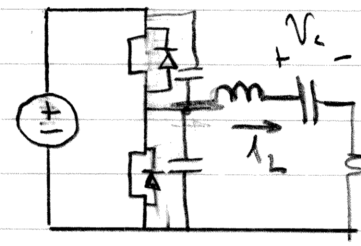
$\Rightarrow$ Below resonance $f_{sw} < \frac{\omega_o}{2\pi}$ network looks capacitive

(Keep: $3f_{sw} >> \omega_o$ to avoid amplifying harmonic voltage)

Fundamental Harmonic Approximation (FHA):

$\Rightarrow$ If harmonics well filtered (near resonance, high Q) $i_L \sim$ Sinusoidal




Above resonance $i_L$ lags $V_{x,1}$

$\Rightarrow$ If we add Capacitors across the switch we can always get ZVS turn on + turn off! — could be junction capacitance

0. A "nice" characteristic is that device + component currents change proportional to load current (good for efficiency vs. load)

1. Note: DEVICE voltages are clamped @ $V_s$, but $i_L$, $V_c$ can have large peak values. At resonance

$$\frac{\text{CAP voltage fundamental}}{\text{Drive fundamental}} = \left|\frac{V_c}{V_{x1}}\right|\Bigg|_{\omega=\omega_o} = \frac{\sqrt{\frac{L}{C}}}{R} = Q!$$

Can be $>> 1$

2. CHALLENGE: AS RESISTOR VARIES

   1. IF $R$ too small $V$'s $I$'s $\uparrow$ or must move away from resonance
   2. IF $R$ too large tank $Q\downarrow$, low selectivity (wide f var), nonsinusoidal waveforms, can lose soft switching

The load may vary naturally w/ system (e.g. induction cooking). Rectifiers provide loads, having equivalent resistance, that vary w/ power + output power.



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WITH THE SERIES CIRCUIT THERE ARE concerns w/ load Resistor variation

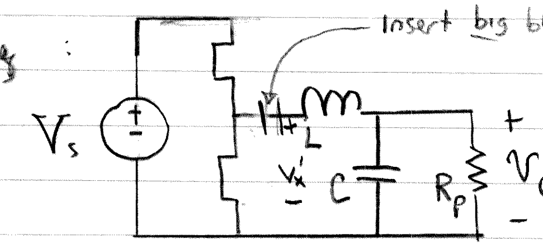
1. If R → too small we must move away from resonance or peak V's, I's become large
2. If R → too big selectivity of the tank becomes poor
   - → current not sinusoidal
   - → "soft" switching lost

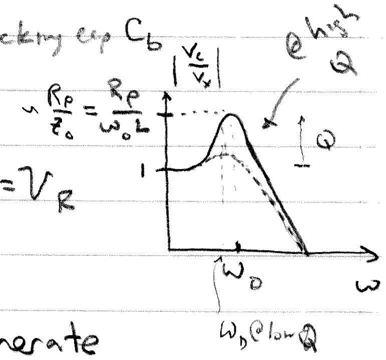
We can get different properties by loading the tank differently

e.g. parallel loading :

Neglecting $C_b$ (or from other side of $C_b$):

$$\frac{V_c}{V_x'} = \frac{1}{s^2LC + sL/R_p + 1}$$



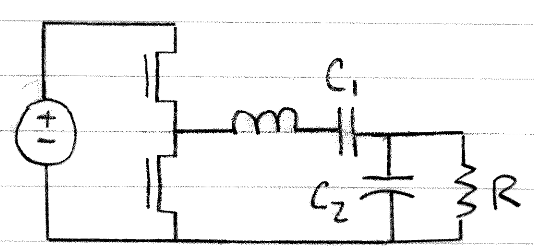

In this case tank Q becomes high as $R_p \uparrow$ (generate big V's I's in tank when $R_p \to \infty$.) and tank Q ↓ as $R_p \downarrow$.

A further impact of this is that device an component currents don't decline well as power decreases so efficiency at a light load may be a challenge

This is used to good effect in flourescent lamp ballasts

- → lamp $R \uparrow\uparrow$ until a big voltage is applied and lamp "strikes"
- → once lamp is struck, $R_p \downarrow$ and tank becomes more loaded, delivering appropriate power at lower voltages

To get a balance between series and parallel behavior, the "series parallel" approach is sometimes used: (also called "LCC")




- @ Low "R" values R "shorts out" $C_2$ and design acts "like" a series res conv
- @ high "R" values the equivalent resonant characteristics change [$C_{eq}$ goes from $C_1$ to $C_1 \| C_2$]

If we increase $f_{sw}$, we get behavior "like" parallel res.

R ↑↑

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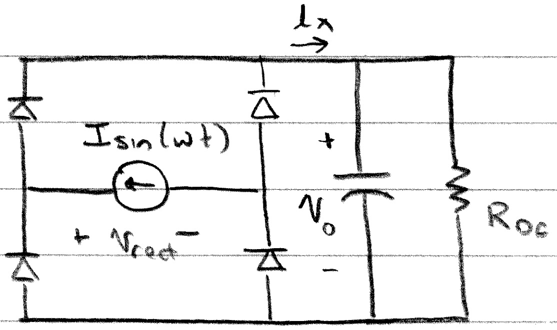
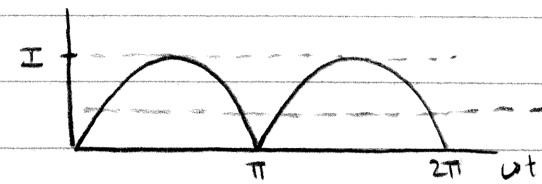
What about dc-dc conversion? $\Rightarrow$ model the rectifier!

We can also use the "Fundamental Harmonic Approximation" to model rectifiers

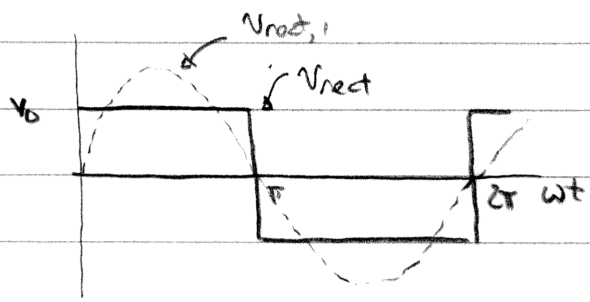



$$\langle i_x \rangle = \frac{1}{\pi}\int_0^{\pi} I\sin(\phi)\,d\phi = \frac{2}{\pi} I$$

$$\therefore \boxed{V_o = \frac{2}{\pi} I R_{DC}}$$

If $V_o$ is determined by a dc load resistor




Fundamental component of $V_{rect}$ is

$$\boxed{V_{rect,1} = \frac{4}{\pi} V_o \sin(\omega t)}$$

CASES:

① For a fixed output voltage $V_o$ (not determined by a dc resistor): $P_{AV} = \langle i_x \rangle \cdot V_o = \frac{2}{\pi} I V_o$ or $I = \frac{\pi}{2}\frac{P_{AV}}{V_o}$

So the FHA ac impedance posed by the rectifier is:

$$R_{ac,eff} = \frac{V_{rect,1}}{I\sin(\omega t)} = \frac{4}{\pi}\frac{V_o}{I} = \frac{4}{\pi}\frac{V_o}{\pi}\frac{2\cdot V_o}{P_{AV}}$$

$$\boxed{R_{ac,eff} = \frac{8}{\pi^2}\frac{V_o^2}{P_{AV}}}$$

{ General form for other rectifiers $k\frac{V_o^2}{P_{AV}}$ }

② If the rectifier is loaded with a dc resistor $R_{DC}$:

$$R_{ac,eff} = \frac{V_{rect,1}}{I\sin(\omega t)} = \frac{4V_o}{\pi I} = \frac{4}{\pi I}\cdot\frac{2 I R_{DC}}{\pi}$$

$$\boxed{R_{ac,eff} = \frac{8}{\pi^2}\cdot R_{DC}}$$

{ General form for other rectifiers $k' \cdot R_{DC}$ }

Similar expressions may be found for other rectifiers

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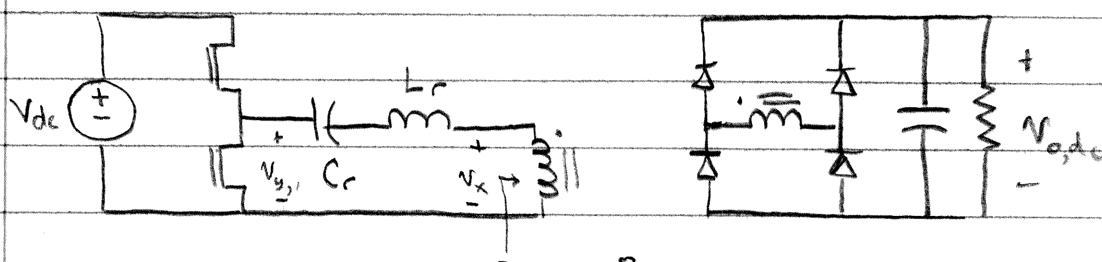
Consider a "series resonant" dc-dc converter



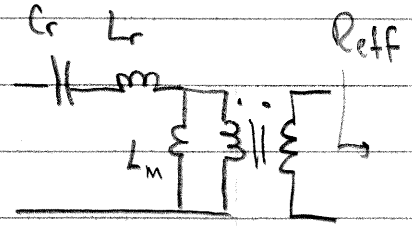

$R_{eff} = \frac{8}{\pi^2} R_{dc}$ or

- Operate above $\omega_0 = \frac{1}{L_r C_r}$ to get ZVS soft switching
- This works for a "dc gain" such that $|V_{x,1}| \leq |V_{y,1}|$
  - ⇒ we can change inverter +/or rectifier topology (full-bridge, half-bridge) to adjust the gain, as well as xformer turns ratio
  - ⇒ For very "light" loads ($R_{dc}$ big) tank selectivity / frequency range becomes a problem.
    - can "burst" converter on and off ⇒ $R_{eff} = \frac{8}{\pi^2} \frac{V_o^2}{P_{ON}}$ — power when burst is "on"
    - can use a different tank structure, such as parallel, LCC, LLC

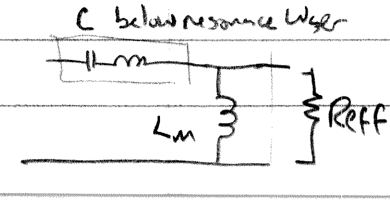
- we often pick a tank that meets requirements of an application + or absorbs parasitics in a useful way (e.g. "series-parallel"/LCC or prc can be used when a high-voltage secondary yields high Xformer capacitance +/or rectifier capacitance

  prc = parallel resonant converter

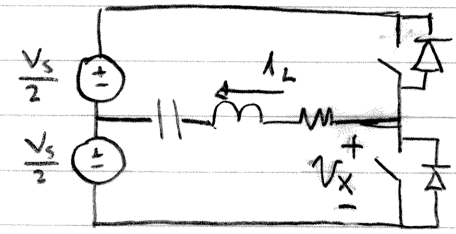
- by gapping the xformer, we can get an "LLC" design
  - above $\omega_{ser} = \frac{1}{\sqrt{L_r C_r}}$ we get behavior a bit like a series-resonant converter, but $L_M$ helps at low power + provides inductive ZVS current
  - below $\omega_{ser}$ we can still be inductively loaded and get additional voltage gain by resonance between $\hat{C}$ and $L_m$ "matching" over some range

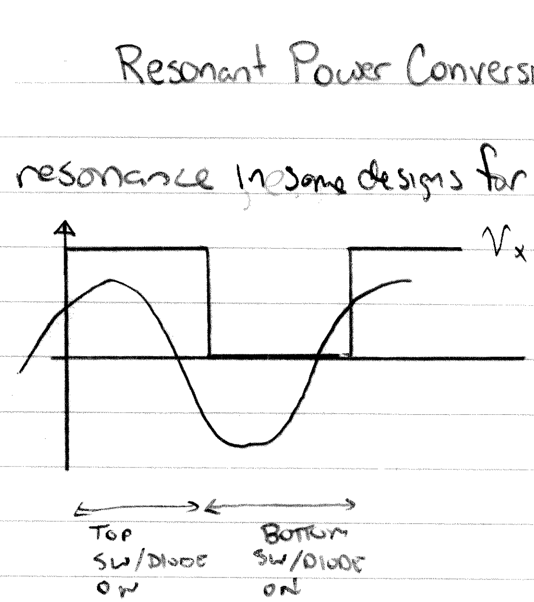



$\hat{C}$ below resonance $\omega_{ser}$



We can also operate below resonance in some designs for ZCS, e.g.



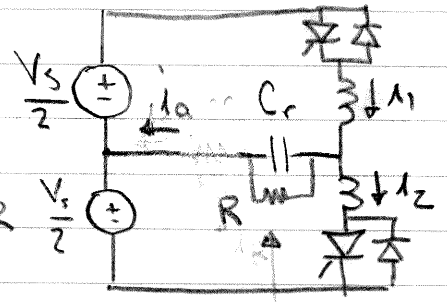



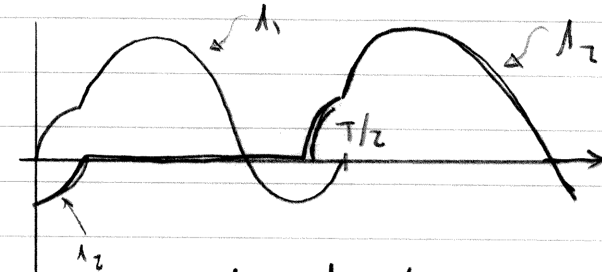

Since $i_L \to 0$

Since $i_L > 0$ before top switch turns on, bottom diode conducts + bottom switch off @ ZCS. Same w/ top switch

A variation of this trick uses multiple resonances to provide ZCS turn on for the switches also. Good for thyristors

MAPHAM INVERTER




$i_a = i_1 - i_2$

- DISTORTED SINE WAVE

★ re-position R so in parallel w/ C:

- $V_C$ filters, so $i_R$ IS sinusoidal