6.622 Power Electronics *Prof. David Perreault*

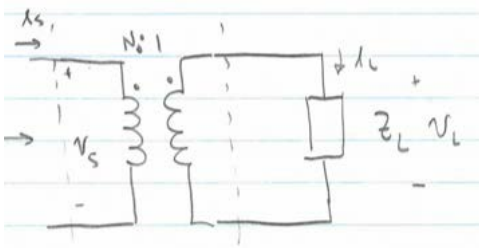
Lecture 37 - Resonant Converters: Matching Networks

1 Matching Networks

In power conversion we often want to scale voltage and current to "match" a load network to a source.

One way to change a load impedance value is with a transformer

$Z_{in} = N^2 Z_L \rightarrow$

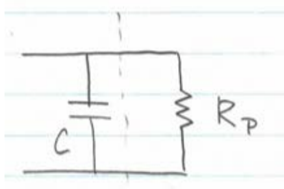


Voltage steps up by N
Current steps down by N
$\Rightarrow$ Impedance $\frac{V}{I}$ by N^2!

If we are concerned with sinusoidal waveforms over a narrow frequency range (as in resonant converters, RF communications, etc.), we can use a reactive matching network to effect impedance transformation.

Simple example: Suppose we have a known load R_p, and want to transform it to appear as a smaller resistor R_s. At a single frequency ω, we can do this with an LC tank.

$Z_{eq1} \rightarrow$



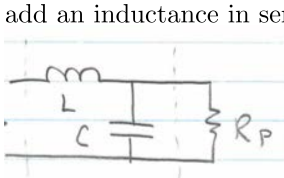
Place a capacitor in parallel with R_p

$$Z_{eq1} = \frac{R_p/j\omega C}{R_p + 1/j\omega C} = \frac{R_p}{1 + j\omega R_p C}\frac{1 - jwR_pC}{1 - jwR_pC}$$

$$= \frac{R_p}{1 + (\omega R_p C)^2} - j\frac{\omega R_p^2 C}{1 + (\omega R_p C)^2}$$

Now add an inductance in series to eliminate reactive term:

$Z_{eq,2} \rightarrow$



This is called an "L-section" matching network

$$Z_{eq,2} = \frac{R_p}{1 + (\omega R_p C)^2} + j[\omega L - \frac{\omega R_p^2 C}{1 + (\omega R_p C)^2}]$$

- $\therefore$ By picking C, we can make $\text{Re}\{Z_{\text{eq},2}\} = R_s @ \omega$ (desired value).
- Picking L, we can make $\text{Im}\{Z_{\text{eq},2}\} = 0 @ \omega$ (desired value).
- We effectively transform R_P into an apparent resistance R_s at a single frequency ω.

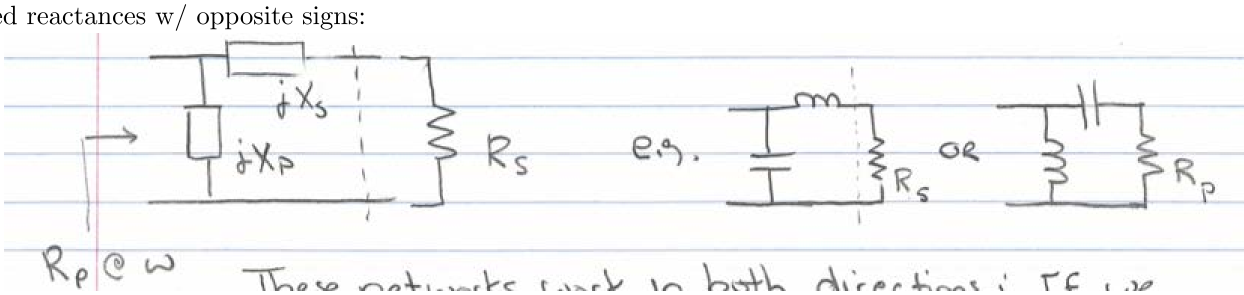
- This is achieved through the action of the resonant tank L, C.
- Must know R_P, ω in design. Practical transformations are limited by $\rightarrow$ achievable component values, parasitics/losses.

In general, to "step down" resistances, we need two reactances jX_P and jX_S which have opposite signs, configured as follows:

We may choose how to implement the reactances based on (e.g.)

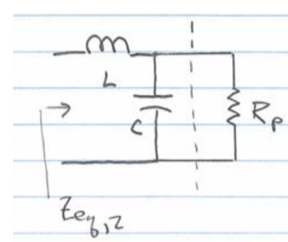
- How dc, harmonic frequencies, etc. are transformed
- how convenient (or reasonable) the values are
- To "absorb" one or both reactancs into the circuitry

To "step up" R_s to a higher value R_p, we can use the same resonant network "backwards". Again, we need reactances w/ opposite signs:



These networks work in both directions: If we transform down from R_p to R_s in one direction, we transform up from R_s to R_p in the other direction!

Easy ways to identify values: If we want to "reduce" an apparent resistance, place a reactance in parallel with it. To increase, place a reactance in series with it.



Lets look at the values we get in our original example:

$$Z_{eq,2} = \frac{R_p}{1+(\omega R_p C)^2} + j[\omega L - \frac{\omega R_p^2 C}{1+(\omega R_p C)^2}]$$

To achieve $Z_{eq,2} = R_s$, we require:

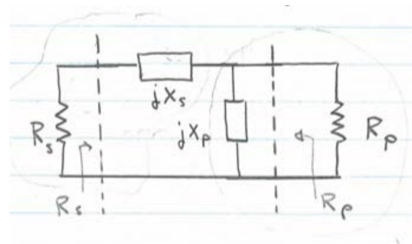
$$R_s = \frac{R_p}{1+(\omega R_p C)^2} \Rightarrow \boxed{\omega R_p C = \sqrt{\frac{R_p}{R_s} - 1}}$$

and

$$\omega L = \omega R_p C \frac{R_p}{1+(\omega R_p C)^2} = R_s \sqrt{\frac{R_p}{R_c} - 1}$$

$$\text{or} \quad \boxed{\frac{\omega L}{R_s} = \sqrt{\frac{R_p}{R_s} - 1}}$$

Defining a "transformation Q" Q_T



$$Q_t \overset{\Delta}{=} \sqrt{\frac{R_p}{R_s} - 1}$$

We need for matching:

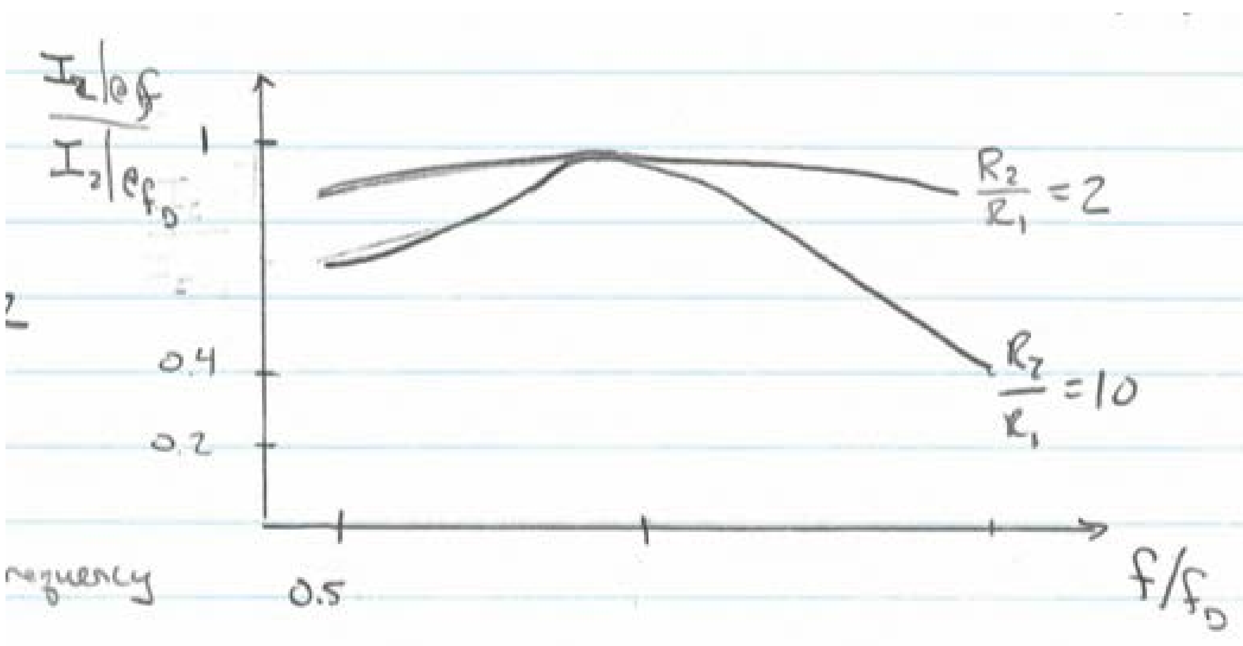
$$\frac{X_s}{R_s} = \frac{R_p}{X_p} = Q_T$$

Result only: shown in handout $\rightarrow$

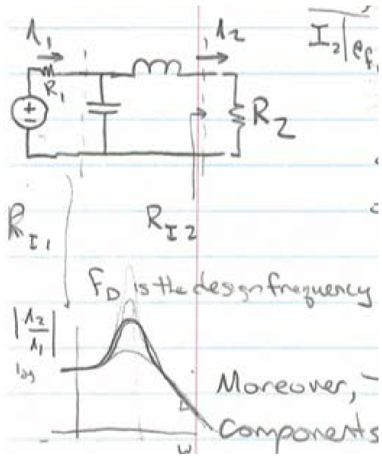
If we implement one element as an inductor (quality factor $Q_L = \frac{\omega L}{R_{L,ESR}}$) and one element as a capacitor (quality factor $Q_C = \frac{1}{\omega C R_{C,ESR}}$), we get a matching network efficiency,

$$\boxed{\eta \approx 1 - \frac{Q_T}{Q_L} - \frac{Q_T}{Q_C}}$$

- The larger the transformation ratio, the lower the efficiency!
- Note: The larger the transformation ratio we want, the more narrowband the transformation becomes (high Q).

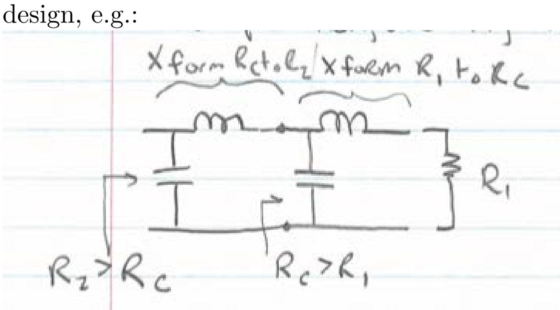


See Everitt + Anner "Communication Engineering" 3rd Edition p. 412



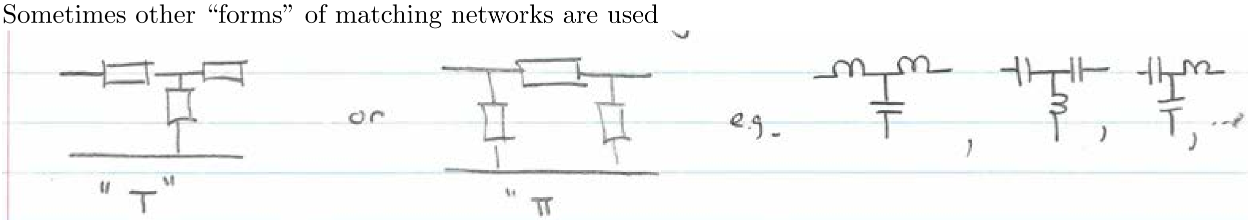
Moreover, the transformation requires increasingly high Q components to remain efficient.

For limited available size +Q of components or for wider-band operation, we may choose a multi-stage design, e.g.:

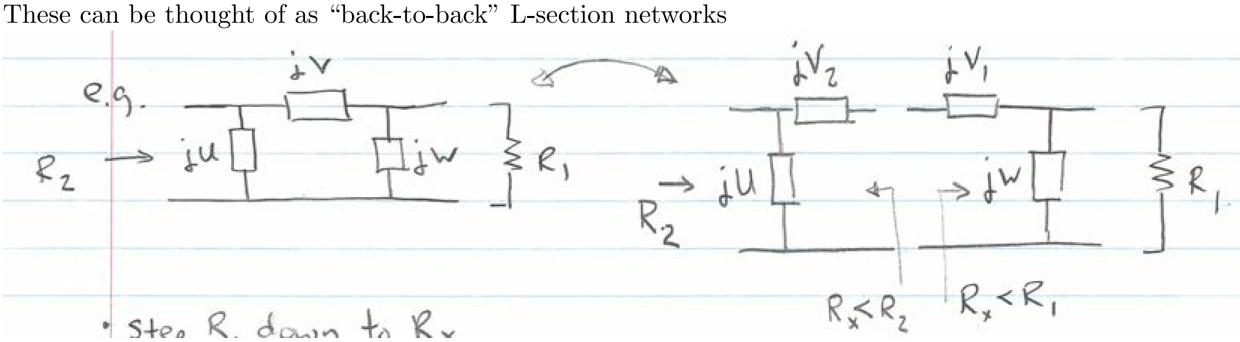


e.g., select $R_c = \sqrt{R_1 R_2}$

Sometimes other "forms" of matching networks are used



These can be thought of as "back-to-back" L-section networks



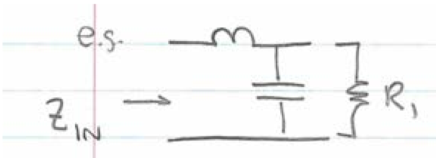
- Step R_1 down to R_x
- Step R_x up to R_2

⇒ L-section networks are <u>broader-band</u> and <u>more efficient</u> than T or Π networks, <u>but</u> are fully determined by the required transformation ratio and frequency.

⇒ T + Π sections allow an additional degree of freedom (R_x), which can be used to:

- provide narrower bandwidth (selectable),
- use more desirable component values,
- control phase shift of waveforms at input and output.

It is also useful to note how variations affect our designs



$$Re\{Z_{in}\} = \frac{R_1}{1+\omega^2 R_1^2 C} = R_2$$

As $R_1 \uparrow, R_2 \downarrow$ and vice versa "inversion"!

T and Π networks double invert, so as $R_1 \uparrow, R_2 \uparrow$ also