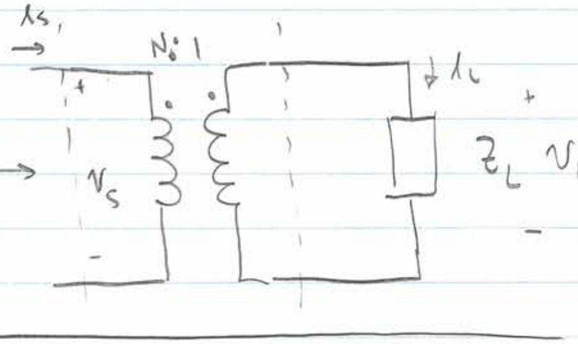
6.334 Lecture — RESONANT CONVERTERS: MATCHING NETWORKS

Matching Networks: In power conversion we often want to scale voltage + current to "match" a load network to a source

One way to change a load impedance value is with a transformer

$Z_{IN} = N^2 Z_L \longrightarrow$

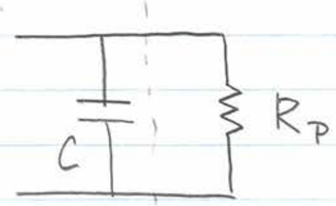



Voltage steps up by N
current steps down by N
$\Rightarrow$ Impedance $\frac{V}{I}$ by $N^2$!

If we are concerned with sinusoidal waveforms over a narrow frequency range (as in resonant converters, RF communications, etc.) we can use a reactive <u>matching network</u> to effect impedance transformation

Simple example: suppose we have a known load $R_p$, and want to transform it to appear as a smaller resistor $R_s$. At a single frequency $\omega$. We can do this with an LC tank

$Z_{eq1} \longrightarrow$




Place a capacitor in parallel with $R_p$

$$Z_{eq1} = \frac{R_p / j\omega C}{R_p + 1/j\omega C} = \frac{R_p}{1 + j\omega R_p C} \left(\frac{1 - j\omega R_p C}{1 - j\omega R_p C}\right)$$

$$= \frac{R_p}{1 + (\omega R_p C)^2} - j\,\frac{\omega R_p^2 C}{1 + (\omega R_p C)^2}$$

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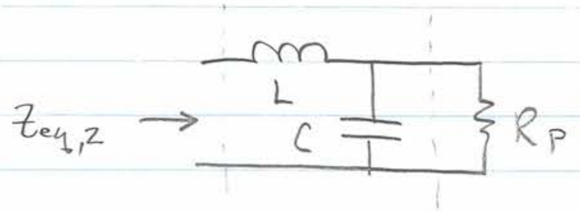
Now add an inductance in series to eliminate reactive term :




This is called an "L-section" matching network

$$Z_{eq,2} = \frac{R_P}{1+(\omega R_P C)^2} + j\left[\omega L - \frac{\omega R_P^2 C}{1+(\omega R_P C)^2}\right]$$

$\therefore$ By Picking $C$ we can make $Re\{Z_{eq,2}\} = R_s @ \omega$ (desired value)
Picking $L$ we can make $Im\{Z_{eq,2}\} = 0 @ \omega$ (desired value)

- we effectively <u>transform</u> $R_P$ into an apparent resistance $R_S$ at a single frequency $\omega$
- This is achieved through the action of the resonant tank $L, C$
- Must know $R_P, \omega$ in design. Practical transformations limited by → achievable component values, parasitics/losses

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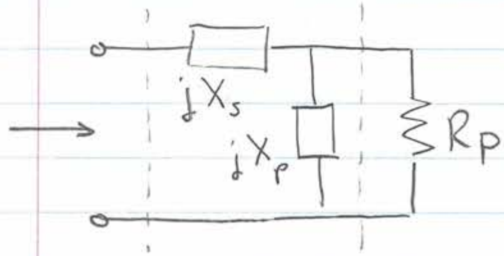
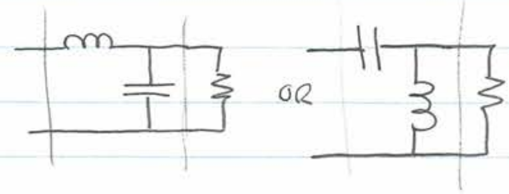
In general, to "step down" resistances, we need two reactances $jX_p$ and $jX_s$ <u>which have opposite sign</u>, configured as follows :




e.g.




We may choose how to implement the reactances based on (e.g.)
- How dc, harmonic frequencies, etc. are transformed
- how convenient (or reasonable) the values are
- to "absorb" one or both reactances into other circuitry

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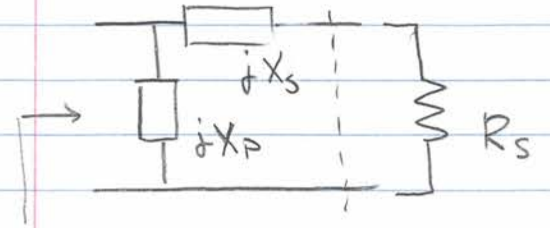
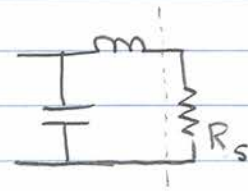
To "step up" $R_S$ to a higher value $R_P$, we can use the same resonant network "backwards". Again, we need reactances w/ opposite signs:

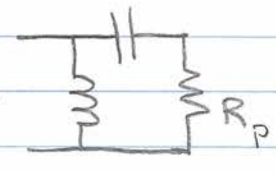



$R_P$ @ $\omega$

e.g.




OR




These networks work in <u>both</u> directions: If we transform <u>down</u> from $R_P$ to $R_S$ in one direction, we transform <u>up</u> from $R_S$ to $R_P$ in the other direction!

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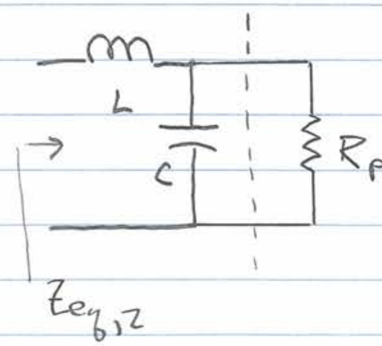
Easy ways to identify values: If we want to "<u>reduce</u>" an apparent resistance, place a reactance in <u>parallel</u> with it. To increase, place a reactance in <u>series</u> with it.

Lets look at the values we get in our original example:




$$Z_{eq,2} = \frac{R_P}{1+(\omega R_P C)^2} + j\left[\omega L - \frac{\omega R_P^2 C}{1+(\omega R_P C)^2}\right]$$

To achieve $Z_{eq,2} = R_S$, we require:

$$R_S = \frac{R_P}{1+(\omega R_P C)^2} \Rightarrow \boxed{\omega R_P C = \sqrt{\frac{R_P}{R_S} - 1}}$$

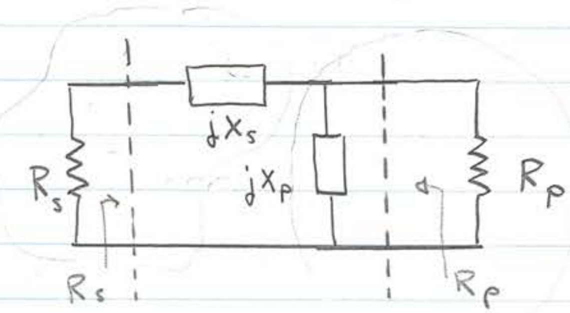
and

$$\omega L = \omega R_P C \frac{R_P}{1+(\omega R_P C)^2} = \sqrt{\frac{R_P}{R_S} - 1}\; R_S$$

OR $$\boxed{\frac{\omega L}{R_S} = \sqrt{\frac{R_P}{R_S} - 1}}$$

# 6.334 Lecture — Resonant Converters: Matching Networks

Defining a "Transformation Q" $Q_T$




$$Q_T \triangleq \sqrt{\frac{R_p}{R_s} - 1}$$

Result only: shown in handout

we need for matching:

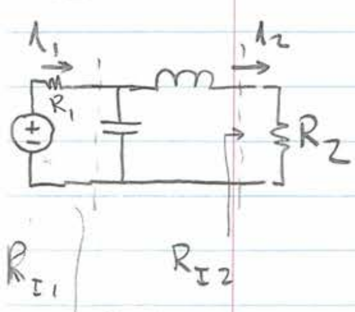
$$\frac{X_s}{R_s} = \frac{R_p}{X_p} = Q_T$$

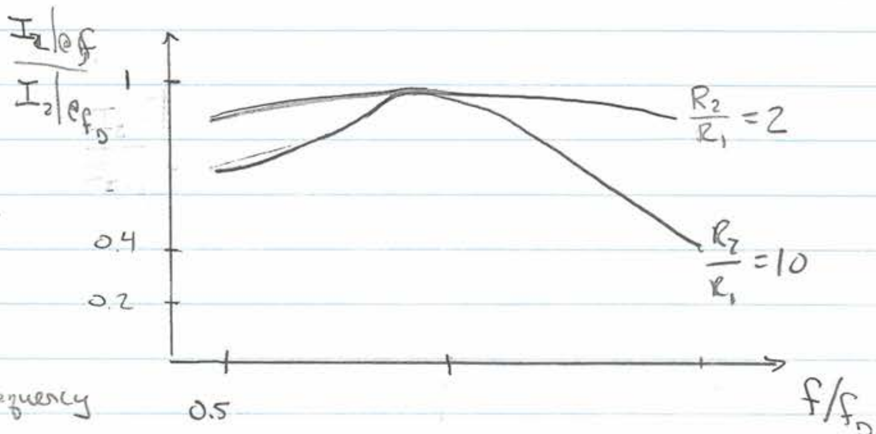
If we implement one element as an inductor (quality factor $Q_L = \frac{\omega L}{R_{L,ESR}}$) and one element as a capacitor (quality factor $Q_C = \frac{1}{\omega C R_{C,ESR}}$) we get a matching network efficiency:

$$\boxed{\eta \simeq 1 - \frac{Q_T}{Q_L} - \frac{Q_T}{Q_C}}$$

* The larger the transformation ratio, the lower the efficiency!

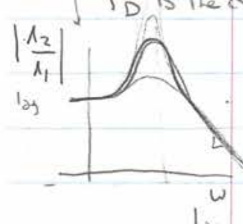
Also * Note: the larger the transformation ratio we want, the more narrowband the transformation becomes (high Q)




See Everitt + Anner "Communication Engineering" 3rd Ed p. 412

$f_D$ is the design frequency




Moreover, the transformation requires increasingly high Q components to remain efficient.

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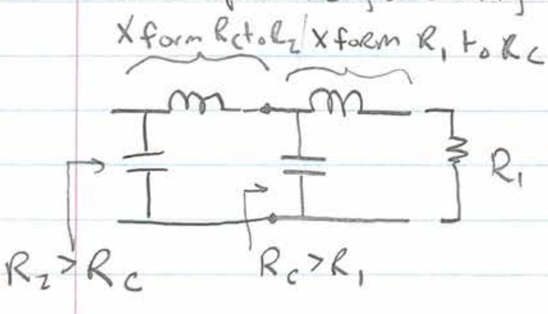
For limited available size + Q of components, or for wider-band operation, we may choose a multi-stage design, e.g.:




e.g., select $R_C = \sqrt{R_1 R_2}$

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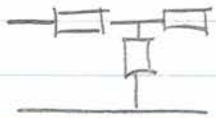
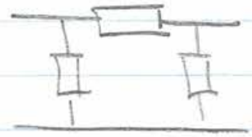
Sometimes other "forms" of matching networks are used



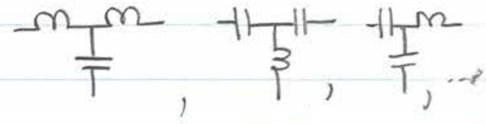
"T"

or



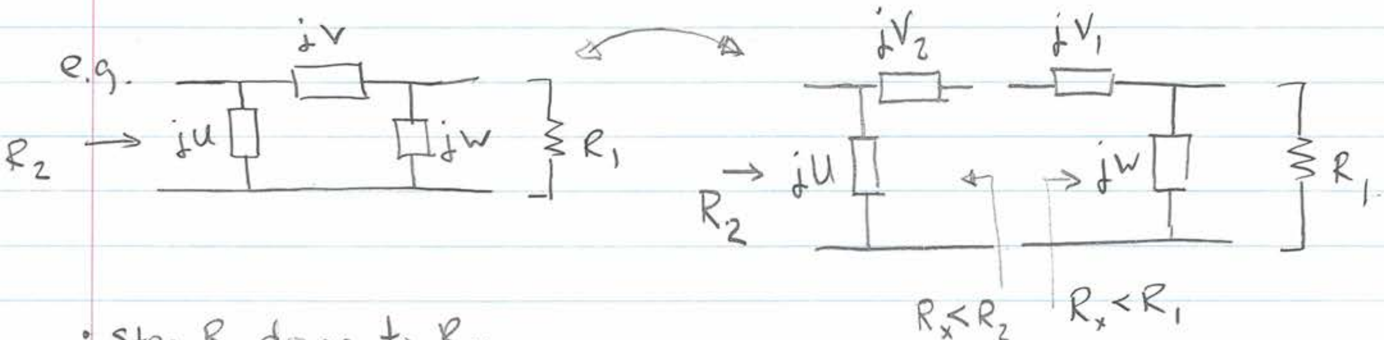
"π"

e.g.



These can be thought of as "back-to-back" L-section networks

e.g.




- Step $R_1$ down to $R_x$
- Step $R_x$ up to $R_2$

⇒ L-section networks are broader-band and more efficient than T or π networks, but are fully determined by the required transformation ratio + frequency

⇒ T + π sections allow an additional degree of freedom ($R_x$) which can be used to
- provide narrower bandwidth (selectable)
- use more desirable component values
- control phase shift of waveforms at input + output

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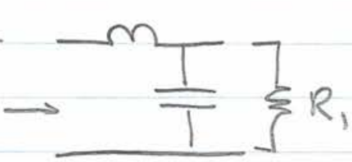
It is also useful to note how variations affect our designs

e.g.

$Z_{IN}$ →




$$Re\{Z_{IN}\} = \frac{R_1}{1+\omega^2 R_1^2 C} = R_2$$

- as $R_1 \uparrow$ $R_2 \downarrow$ and vice versa "inversion"!
- T + π networks double invert, so as $R_1 \uparrow$, $R_2 \uparrow$ also