

## 6.6220 HW3 Solution\*

### 3.1 Ferrocube ER core set

#### 3.1.A

One possible magnetic circuit model for the EM core shown in Fig. 1 is shown in Fig. 2 where the reluctances can be expressed as:

$$\mathcal{R}_1 = \frac{y}{\mu A_1}$$

$$\mathcal{R}_2 = \frac{x}{\mu A_2}$$

$$\mathcal{R}_3 = \frac{y}{\mu A_3}, \text{ where}$$

$$\mathcal{R}_{eq} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3/2$$

$$x = \frac{1}{2} \left( A - \frac{A - C}{2} \right) = \frac{A + C}{4}$$

$$y = 2 \left( F + \frac{E - F}{2} \right) = E + F$$

$$A_1 = \frac{\pi D_3^2}{4}$$

$$A_2 = B(E - F)$$

$$A_3 = B \frac{A - C}{2}$$

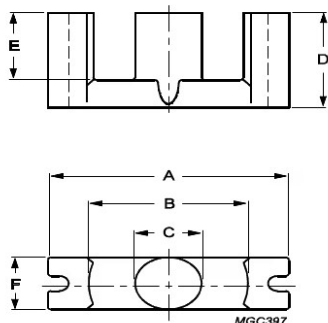


Figure 1: Ferrocube ER core

After plugging in the numbers from datasheet, we have equivalent reluctance

\*Mansi Joisher 2023 (adapted from Yang 2022, Boles 2021, Ranjram 2020)

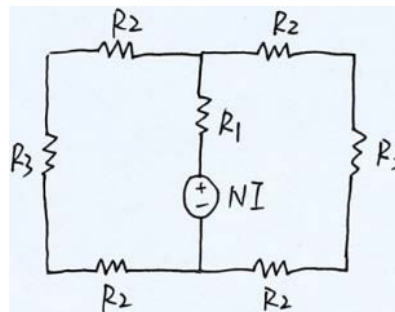


Figure 2: A Possible Magnetic Circuit Model

Alternatively, you can model the reluctance of the core using the effective length and area from the datasheet, which are pre-computed by the manufacturer so that you can have a simple single reluctance representation of the core. In that case, the magnetic circuit model would just be one source  $NI$  with one reluctance in series, having  $\mathcal{R} = \frac{l_e}{\mu A_e}$ .

#### 3.1.B

The specific inductance  $A_L$  can be expressed as

$$A_L = \frac{1}{\mathcal{R}_1 + \frac{2\mathcal{R}_2 + \mathcal{R}_3}{2}} \approx 2.8 \mu\text{H/turn}^2$$

(Or you can use the effective length and area from the datasheet:  $A_L \approx \frac{1}{\frac{l_e}{\mu A_e}}$ )

#### 3.1.C

We know the flux density  $B = \frac{\Phi}{A}$  and  $\Phi = Li/N = NA_L i$ . There are three possible flux densities in this magnetic core: in the middle leg,  $B_m = \frac{NA_L i}{A_1}$ ; in the top and bottom end plates,  $B_2 = \frac{NA_L i}{2A_2}$ ; in the outer legs,  $B_3 = \frac{NA_L i}{2A_3}$ . Since  $2A_2 > 2A_3 > A_1$ , the flux density in the center leg will be the highest at any time. Thus,

$$(Ni)_{max} = \frac{B_S A_1}{A_L} \approx 12.4 \text{ A} \cdot \text{turns}$$

Alternatively, you can use the listed minimum core area on the datasheet  $A_{min} = 100 \text{ mm}^2$  to get the same answer quickly without having to manually compute all the different areas.

### 3.1.D

For  $N = 10$ , the saturation current limit is  $I_S = (NI)_{max}/N \approx 1.24 \text{ A}$ .

### 3.1.E

With the addition of a  $250 \mu\text{m}$  gap, the air gap reluctance is  $\mathcal{R}_{gap} = \frac{l_g}{\mu_0 A_L} \approx 19.83 \times 10^5 \text{ A/Wb}$ , hence the new inductance value

$$L' \approx N^2 / (\mathcal{R}_{gap} + 1/A_L) \approx 42.8 \mu\text{H}$$

and the new maximum current is

$$I'_S = A_L B_S (\mathcal{R}_{gap} + 1/A_L) / N \approx 8.18 \text{ A}$$

The total energy storage is  $E_{gapped} = \frac{1}{2} L' I_S'^2 \approx 1.43 \text{ mJ}$ . For the ungapped case, the energy storage is  $E_{ungapped} = \frac{1}{2} L I_S^2 = \frac{1}{2} N^2 A_L I_S^2 \approx 0.22 \text{ mJ}$ . So, the gapped inductor's energy storage capability is substantially greater than the ungapped case by a factor of 6.6x.

## 3.2 Boost Inductor Design (KPVS 18.8)

### 3.2.A - Converter Parameters

The voltage conversion ratio of a boost converter is  $\frac{V_2}{V_1} = \frac{1}{1-D}$ . Here,  $V_{in} = 12\text{V}$  and  $V_{out} = 36\text{V}$ . Thus,  $D = 2/3$ .

When the switch is on, the voltage across the inductor is  $V_1$ . Thus the peak-to-peak ripple current through the inductor is

$$\Delta i_{L,pp} = \frac{V_1 D T}{L} = \frac{(12)(2/3)(1/500\text{k})}{3\mu} = 5.33 \text{ A}$$

The average input current  $I_1$  equals the average inductor current  $I_L$ . At full load we have  $I_L = I_1 = P_{in}/V_1 = 100/12 = 8.33 \text{ A}$ . Thus the ripple ratio and peak current are:

$$\mathcal{R}_L = \frac{\Delta i_{L,pp}/2}{I_L} = \frac{5.33/2}{8.33} = 0.32$$

$$i_{L,max} = I_L + \Delta i_{L,pp}/2 = 8.33 + 5.33/2 = 11 \text{ A}$$

### 3.2.B - Inductor Design

First, let's pick a core size. We do not want it to saturate. We know that

$$N B_{sat} A_{c,min} \geq L i_{L,max}$$

Thus, we find  $N \cdot A_{c,min} \geq 110 \text{ mm}^2 \cdot \text{turns}$ . If we use an RM10 core with  $A_{c,min} = 86.6 \text{ mm}^2$ , then  $N_{RM10} \geq 1.27$  (round up to 2, the nearest integer for a practical implementation). If we use an RM12 core with  $A_{c,min} = 121 \text{ mm}^2$ , then  $N_{RM12} \geq 0.909$  (round up to 1). Let's go with the RM10 core for now, since it would be great if we could meet our requirements in this smaller form-factor. (But for the purposes of this homework, you can also use RM12.)

For our example, let's pick  $N = 2$ . In this case,  $A_L = L/N^2 = 750 \text{ nH/turn}^2$ , which gives us a required effective core reluctance of  $\mathcal{R}_{eff} = \frac{1}{A_L} \approx 1.33 \cdot 10^6 \text{ A/Wb}$ . We can then solve for the gap length  $l_g$  that gives us our desired  $\mathcal{R}_{eff}$ . We can model  $\mathcal{R}_{eff}$  as the ungapped core reluctance in series with the gap reluctance. Assuming the air gap's effective cross-sectional area is the same as the core's, we get

$$\mathcal{R}_{eff} = \frac{l_e}{\mu_i \mu_0 A_e} + \frac{l_g}{\mu_0 A_e}$$

We then get that the required air gap length is  $l_g \approx 142 \mu\text{m}$ . (Note: In general, when designing an inductor, it's possible to pick  $N$  such that you need a negative gap length to achieve the desired effective core reluctance. In this case, the design is not practical, and you should pick a larger  $N$ .)

Now, let's pick a wire. We know it will have to supply a peak current of  $11 \text{ A}$  and the current density has to be less than  $500 \text{ A/cm}^2$ . Thus,  $A_{wire} \geq \frac{i_{L,max}}{500} = 2.2 \text{ mm}^2$  and the wire diameter  $D_{wire} = \sqrt{\frac{4A_{wire}}{\pi}} \geq 1.67 \text{ mm}$ .

Using KPSV Table 18.1 (or an online reference), we see that AWG 13 or below meets this spec. Let's pick AWG11 which has a diameter of 2.31 mm.

Now we need to make sure we can fit 2 turns of AWG11 on the core. The window area of the core is  $A_{window} = 11.9 * 4.25 = 50.6 \text{ mm}^2$  (obtained from example 18.6). The net wire area is  $A_{Cu} = N\pi D_{wire}^2/4 = 0.5\pi(2.31^2) = 8.4\text{mm}^2$ . We are only using 17% of the window, so we should have no problem fitting our winding.

Our final design is:

- $L = 3 \mu\text{H}$
- RM10 core
- $N = 2$  turns
- AWG11 wire ( $D_{wire} = 2.31 \text{ mm}$ )
- 17% of window area used
- $142\mu\text{m}$  center-post gap
- $B_{pk} = 0.19 \text{ T}$
- $J = 263 \text{ A/cm}^2$

### 3.2.C - Inductor Loss

Winding loss arises from the wire resistance which is related to the wire length  $l_{wire} = Nl_t = 104 \text{ mm}$ . From Table 18.1, AWG11 has a per-length resistance of  $5.03 \Omega/\text{km}$ , so our wire has a dc resistance of  $R_{wire} = 104 \times 10^{-6}\text{km} \times 5.04\Omega/\text{km} = 0.524\text{m}\Omega$ .

The rms current through the inductor is  $I_{L,rms} = \sqrt{I_L^2 + \Delta i_{L,pp}^2/12} = 8.47\text{A}$ . Thus  $P_{wire} = I_{L,rms}^2 R_{wire} = 37.6\text{mW}$ .

For the given core loss model, core loss depends on the *ac* flux only. In particular, we care about the *peak* ac flux, i.e. the amplitude (not the peak-to-peak) of the ac flux. Therefore,

$$\Delta B_{ac,pk} = \frac{L\Delta i_{L,pp}/2}{A_e N} \approx 46.2 \text{ mT}$$

Now, using  $V_{core} = 4.3 \text{ cm}^3$ , we can find core loss directly from the provided formula:

$$P_{core} = 3.9 \times 10^{-9} (500 \cdot 10^3)^{2.4} (0.0462)^{2.25} (4.3) = 728.6 \text{ mW}$$

So, the total loss is  $P_{tot} = 37.6 \text{ mW} + 728.6 \text{ mW} = 766.2 \text{ mW}$ .

A few things you would definitely adjust in a good design:

1. There is an unavoidable "fill factor" that you must include to account for the fact that you need to account for unavoidable spacing between the turns (e.g. due to wire insulation), so you may actually use 50% or less of the available bobbin winding area. If you use round wire, you also lose some area compared to rectangular/square wire.
2. It is essential to account for skin and proximity effect in a real design. For example, skin depth of the wire at  $25^\circ\text{C}$  is  $\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \approx 93.5 \mu\text{m}$ . This means any ac current in our wire is going to flow through a tiny fraction of the available wire area and this can be catastrophic in terms of loss! Note however that the dc current in the winding will happily use the available conductor area.
3. Core loss and winding loss (often called copper loss) are competing loss components in a magnetic component and a good design minimizes the sum of these components. For example, as the number of turns increases, winding loss increases (both because of the longer length of wire required, and because a thinner wire must be used to fill the same area) but core loss decreases (since flux density is reduced for the same designed inductance, peak ac current, and core cross-sectional area). In many cases, a design that aims to minimize only one of these components is not an optimal design!

### 3.3 Permanent magnet KPVS 18.13)

(a) We are told  $H_c = 920 \text{ kA/m}$ ,  $B_r = 1.25 \text{ T}$ . The magnet is  $l_M = 1 \text{ mm}$  in length and has a cross-sectional area  $A_M = 9 \text{ mm}^2$ . Referencing the reluctance model, the MMF is  $F_C = H_c l_M = 920 \text{ A}$

and the reluctance is

$$R_M = \frac{l_M}{\mu_M A_M} = \frac{F_C}{B_r A_M} = \frac{920 \text{ A}}{1.25 \text{ T} \cdot 9 \cdot 10^{-6} \text{ m}^2} = 8.18 \cdot 10^7 \text{ A}/(\text{T} \cdot \text{m}^2).$$

(b) If a 0.5 T magnetic flux density is desired, we have  $\Phi = BA_M = F_C/(R_M + R_{\text{ext}})$ , where  $R_{\text{ext}}$  is the external reluctance of the magnetic path. Setting  $B = 0.5 \text{ T}$ , we get  $R_{\text{ext}} = 1.23 \cdot 10^8 \text{ A/Wb}$ .

### 3.4 Gapped transformer (KPVS 19.1)

#### 3.4.A - Magnetic Circuit Model

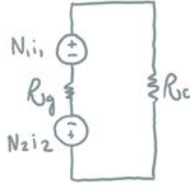


Figure 3: Magnetic circuit model.

One possible magnetic circuit model is shown in Fig. 3. It has the following parameter values:

$$\mathcal{R}_c = \frac{2w + 2h - g}{\mu_c A_c}$$

$$\mathcal{R}_g = \frac{g}{\mu_o A_c}$$

#### 3.4.B - Magnetizing Inductance

We can derive the magnetizing inductance seen from port 1 by assuming  $i_2 = 0$ . Thus,

$$L_{\mu 1} = L_{11} = \frac{\lambda_1}{i_1} \Big|_{i_2=0} = \frac{N_1 \phi}{i_1} \Big|_{i_2=0} \quad (1)$$

With the magnetic circuit model, we can calculate  $\phi$  for  $i_2 = 0$  to be:

$$\phi = \frac{N_1 i_1}{\mathcal{R}_c + \mathcal{R}_g} \quad (2)$$

Inserting into  $L_{\mu 1}$  gives:

$$L_{\mu 1} = \frac{N_1 \phi}{i_1} \Big|_{i_2=0} = \frac{N_1^2}{\mathcal{R}_c + \mathcal{R}_g} \quad (3)$$

Intuitively, in this ideal scenario where you leakage fields are neglected, you can think about the magnetizing inductance as the inductance for an inductor using just winding  $N_1$ , and solve for it accordingly.

#### 3.4.C - Maximum Current

We can examine the maximum current through winding  $N_1$  from the perspective of flux linkage for  $i_2 = 0$ :

$$\lambda_1 = N_1 B A_c = L_{11} i_1 \rightarrow i_{1 \max} = \frac{N_1 B_{\text{sat}} A_c}{L_{11}} \quad (4)$$

If the core reluctance is negligible,  $L_{11}$  can be calculated as (like part B):

$$L_{11} = \frac{N_1 \phi}{i_1} \Big|_{i_2=0} = \frac{N_1^2}{\mathcal{R}_g} = \frac{N_1^2 \mu_o A_c}{g} \quad (5)$$

Thus

$$i_{1 \max} = \frac{B_{\text{sat}} g}{N_1 \mu_o} \quad (6)$$

Again, with winding  $N_2$  being open, you can also treat this situation as an inductor with just one winding  $N_1$  and solve for the maximum current this way.

#### 3.4.D - V2 Polarity

Because the  $N_1$  and  $N_2$  windings have been wound in opposite directions on the core based on how their voltages are defined,  $v_2$  will be negative when  $v_1$  is positive. This makes sense in the context of Faraday's Law; if all turns see the same flux (and change in flux), the voltage induced on each turn will be the same. Because of how the turns in winding  $N_2$  are attached, their voltage will add such that  $v_2$  is negative if  $v_1$  is positive.

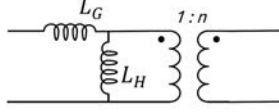


Figure 4: Cantilever model in Problem 19.2

### 3.4.E - Inductance Matrix

We already calculated  $L_{11}$ , so all that remains is  $L_{22}$  and  $L_m$ . If the core reluctance is negligible,  $L_{22}$  can be calculated as

$$L_{22} = \frac{N_2 \phi}{i_2} \Big|_{i_1=0} = \frac{N_2^2}{\mathcal{R}_g} = \frac{N_2^2 \mu_o A_c}{g} \quad (7)$$

Let's also derive  $L_m$ . From the perspective of port 1, the mutual inductance  $L_m$  equals:

$$L_m = \frac{\lambda_1}{i_2} \Big|_{i_1=0} = \frac{N_1 \phi}{i_2} \Big|_{i_1=0} \quad (8)$$

With the magnetic circuit model, we can calculate  $\phi$  for  $i_1 = 0$  to be:

$$\phi = -\frac{N_2 i_2}{\mathcal{R}_g} \quad (9)$$

Inserting into  $L_m$  gives:

$$L_m = \frac{N_1 \phi}{i_2} \Big|_{i_1=0} = -\frac{N_1 N_2}{\mathcal{R}_g} \quad (10)$$

Thus, our inductance matrix is

$$L = \begin{bmatrix} \frac{N_1^2 \mu_o A_c}{g} & -\frac{N_1 N_2 \mu_o A_c}{g} \\ -\frac{N_1 N_2 \mu_o A_c}{g} & \frac{N_2^2 \mu_o A_c}{g} \end{bmatrix} H \quad (11)$$

The mutual inductances being negative aligns with their MMFs being in opposite flux directions.

### 3.5 Cantilever model KPVS 19.2

The cantilever model is shown in Fig. 4 for reference. Let's compare the result of the measurements done in example 20.2 to the result of those measurements performed on the model in Fig. 4.

First, the inductance of the primary is measured with the secondary open-circuited. Thus,

$$L_{11} = L_G + L_H. \quad (12)$$

Second, the inductance of the secondary is measured with the primary open-circuited. Thus,

$$L_{22} = n^2 L_H. \quad (13)$$

Finally, an ac voltage of 1.047V is applied to the primary with the secondary open-circuited and a secondary voltage of 0.2082V is measured. The primary voltage will split on  $L_G$  and  $L_H$  such that

$$v_2 = n v_1 \frac{L_H}{L_H + L_G} \quad (14)$$

Using equations (12) and (13), we have

$$v_2 = v_1 \frac{L_{22}}{n L_{11}} \quad (15)$$

Thus

$$n = \frac{v_1 L_{22}}{v_2 L_{11}} \quad (16)$$

Solving (12), (13), and (16) for  $L_G$ ,  $L_H$ , and  $n$ , we obtain

$$n = 0.2024 \quad (17)$$

$$L_H = 1952.0 \mu H \quad (18)$$

$$L_G = 34.99 \mu H \quad (19)$$

Because sometimes the extended cantilever model is also called the 'cantilever model', the results for the extended model are also provided. These parameters are directly obtained from the transformer parameters as described in the book chapter. Namely,

$$l_{11} = L_{11} = 1987 \mu H, \quad (20)$$

and

$$n_2 = \frac{L_{12}}{L_{11}} = \frac{395.1}{1987} = 0.1988 \quad (21)$$

are extracted directly from the measured parameters. To determine  $l_{12}$  we must compute the inverse inductance matrix,

$$\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} 0.0284 & -0.14438 \\ -0.14438 & 0.7055 \end{bmatrix}. \quad (22)$$

Thus,

$$\boxed{l_{12} = -\frac{1}{n_1 n_2 B_{12}} = 34.84 \mu\text{H}} \quad (23)$$

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