6.6220 HW6 Solutions*

6.1 Current Source Inverter

(a) The output RC filter introduces a phase-shift between the current output by the inverter and the voltage on the load – voltage will lag current. This means that when positive or negative current is delivered by the inverter, the switches that are off most block bipolar voltage. On the other hand, each switch only ever carries I_{dc} and so only needs to be carry unidirectional current. Thus, we can implement these bidirectional-voltage-blocking, unidirectional-current-carry switches using MOSFETs with series diodes as shown in Fig. 1.

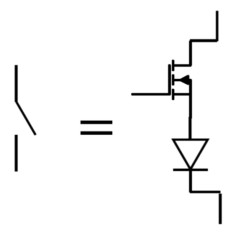

Figure 1: Example switch implementation for the current-source inverter in problem 8.22.

(b) An output current waveform with no third harmonic content, and the corresponding switching pattern, is shown in Fig. 2.

(c) One possibility for implementing a harmonic cancellation scheme with a second current-source inverter is to connect this new inverter to the same load using another transformer winding, as shown in Fig. 3. Note that the transformer is configured such that the current out of the output winding is the sum of the currents in the winding of each CSI while the voltage across each winding is equal.

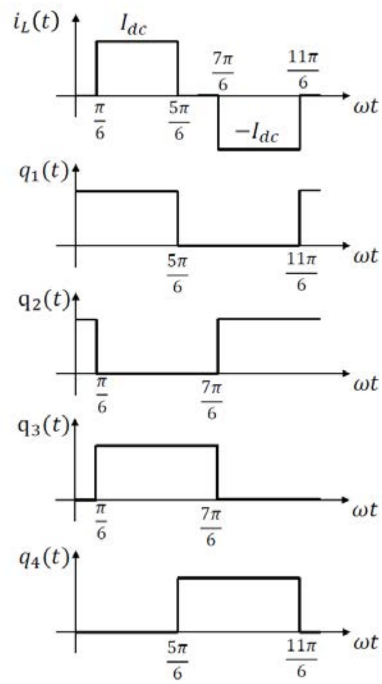

Figure 2: Output current and switching pattern of the current-source inverter in problem 8.22.

6.2 PWM THD

This problem is similar to KPVS Example 8.4, except now we're varying our PWM waveform with $d(t) = k|\sin\omega_a t|$. We are interested in $D(t) = k\,|\sin\omega_a t|$ where $k \leq 1$,

We assume our switching frequency is much greater than the ac output frequency so that $\sin(\omega_a t)$ is constant over a switching period. Since the height of each pulse is V_{dc}, the rms value squared of a pulse occurring at time t_o is

$$v_{\text{arms}}^2\,(t_o) = V_{\text{dc}}^2 D\,(t_o) = V_{\text{dc}}^2 k\,|\sin\omega_a t_o|$$

You can intuitively think about v_{arms}^2 as finding the squared average value of the pulse for one cycle, which is V_{dc}^2 with a duty ratio of $k\sin(\omega_a t_o)$.

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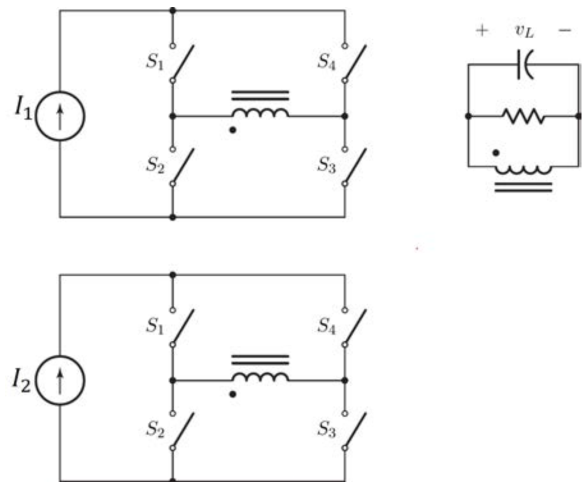

Figure 3: Illustration of how a pair of CSIs can be connected to implement a harmonic cancellation scheme in problem 8.22.

Now, we can find the rms value squared of the PWM pulse train, V_{arms}^2, by averaging the above over half a cycle of v_a.

$$V_{arms}^2 = \frac{1}{\pi}\int_0^\pi V_{dc}^2 k \sin(\omega_a t) d(\omega_a t) = \frac{2kV_{dc}^2}{\pi}$$

By scaling $d(t)$ by our depth of modulation k, our output v_{ac} now has a peak value of kV_{dc} and thus an rms value of $V_{acrms} = kV_{dc}/\sqrt{2}$. We can then find THD as follows:

$$\text{THD} = \sqrt{\frac{V_{arms}^2 - V_{acrms}^2}{V_{acrms}^2}} = \sqrt{\frac{4}{\pi k} - 1}$$

6.3 Harmonic Cancellation

The most direct way of answering this question is to compute the V_{a_n} such that

$$v_a = \sum_{n=1}^{\infty} V_{a_n} \sin \omega t$$

In order for v_a to be expressed only in terms of sines, the waveform given in Fig. 8.9 of the textbook must be shifted 18° to the left to produce the desired odd symmetry. Applying the definition,

$$V_{a_n} = \frac{2V_a}{\pi}\left(\int_{\frac{n\pi}{15}}^{\frac{4\pi}{15}} \frac{1}{2}\sin \omega t \, d(\omega t) + \int_{\frac{4\pi}{15}}^{\frac{11\pi}{15}} \sin \omega t \, d(\omega t) + \int_{\frac{11\pi}{15}}^{\frac{14\pi}{15}} \frac{1}{2}\sin \omega t \, d(\omega t)\right)$$

$$V_{a_n} = \frac{V_a}{n\pi}\left(\cos\frac{n\pi}{15} + \cos\frac{n4\pi}{15} - \cos\frac{n11\pi}{15} - \cos\frac{n14\pi}{15}\right)$$

Using some trig identities, this reduces to

$$V_{a_n} = \frac{4V_a}{n\pi}\left(\sin\frac{n\pi}{2}\right)\left(\sin\frac{n\pi}{3}\right)\left(\cos\frac{n\pi}{10}\right)$$

From this representation we can determine which values of n result in $V_{a_n} = 0$

$$\sin\frac{n\pi}{2} = 0 \text{ for } n \text{ even}$$

$\sin(n\pi/3) = 0$ for n a multiple of 3 $\cos(n\pi/10) = 0$ for n a multiple of 5. We see that v_a contains only the 7th, 11th, 13th, 17th, etc. harmonics. Given the number of harmonics eliminated, it is not surprising that the waveform has a relatively low THD.

6.4 Internal Energy Storage

Our microinverter is taking in dc power from the solar panel and converting it into ac power for the 50 Hz or 60 Hz ac grid. Since our microinverter is 100 % efficient, we can say that our dc input power is the same as our average ac output power, so $P_{dc} = \langle p_{ac} \rangle = P$.

From lecture or the textbook, we know that

$$\Delta E_{c,pp} = \int_{-T/8}^{T/8} P\cos(2\omega t)dt = \frac{P}{\omega}$$

Since more energy storage is needed for lower frequencies, the microinverter's energy storage requirement is thus limited by the lowest frequency specification, which is 50 Hz.

$$\Delta E_{c,pp} = \frac{250\,\text{W}}{2\pi 50\,\text{Hz}} = 0.80\,\text{J}$$

To meet the 60 Hz requirement, we would only need $\Delta E_{c,pp} = 0.66\,\mathrm{J}$, which would be insufficient for meeting the 50 Hz requirement.