6.6220 HW7 Solutions*

7.1 KPVS 9.6: 3-Phase Bridge Power Factor

3-Phase Bridge

Because the three phases are symmetric, we only need to find the power factor of one phase to find the power factor of the 3-phase bridge. In particular, we'll focus on finding the power factor of source v_a with voltage and current waveforms shown in Fig. 1.

To find the power factor, we need to find the average power and the rms voltage and current.

$$\langle P\rangle = \frac{1}{\pi}\int_{\pi/6}^{5\pi/6} V_s \sin(\omega t) I_d \; d(\omega t) = \frac{\sqrt{3}}{\pi} V_s I_d$$

$$V_{a,rms} = \frac{V_s}{\sqrt{2}}$$

$$I_{a,rms} = \sqrt{\frac{2}{3} I_d^2} = I_d \sqrt{\frac{2}{3}}$$

Power factor for the 3-phase bridge is then

$$PF_{3\phi} = \frac{\langle P\rangle}{V_{a,rms} I_{a,rms}} = \frac{3}{\pi} \approx 0.955$$

Single-Phase Bridge

For a single-phase bridge, the circuit and waveforms are shown in Fig. 2.

In this case, our average power and rms voltage and current are

$$\langle P\rangle = \frac{1}{\pi}\int_{0}^{\pi} V_s \sin(\omega t) I_d \; d(\omega t) = \frac{2}{\pi} V_s I_d$$

$$V_{rms} = \frac{V_s}{\sqrt{2}}$$

$$I_{rms} = I_d$$

So, our power factor is

$$PF_{1\phi} = \frac{\langle P\rangle}{V_{rms} I_{rms}} = \frac{2\sqrt{2}}{\pi} \approx 0.900$$

The 3-phase bridge therefore has a greater power factor than the single-phase bridge.

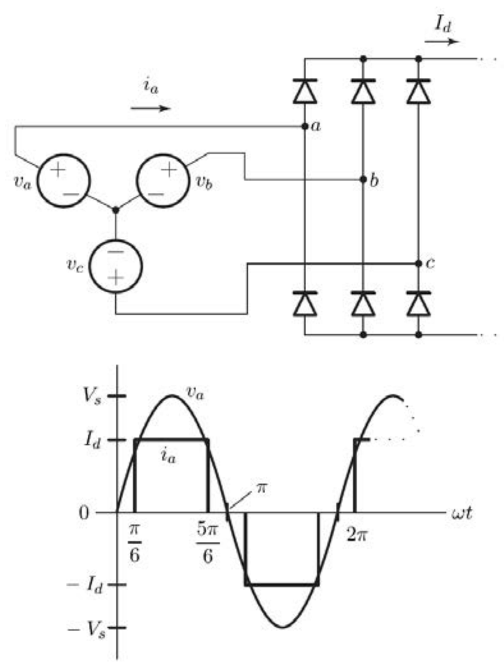

Figure 1: 3-Phase Bridge (reproduced KPVS Fig. 9.8a)

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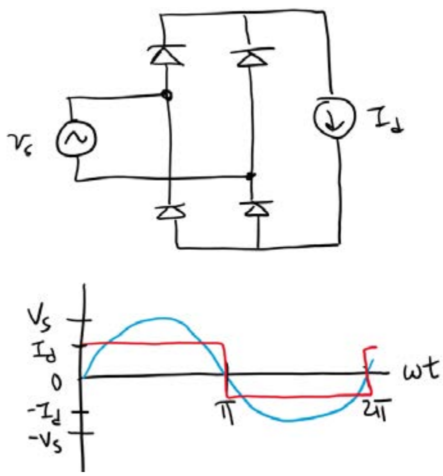

Figure 2: Single-Phase Bridge

7.2 KPVS 9.2 (a), (b): 12-Pulse Rectifier

We're interested in analyzing different currents in a 12-pulse rectifier with transformer connections as shown in Fig. 3

7.2.A Line current $i_{a'}$

Looking at the Δ/Y transformer block, from KCL, the line current $i_{a'}$ is the difference of the two currents going into the primary of the top and bottom transformers, or $i_{a'} = i_{a'p} - i_{c'p}$ as labeled in Fig. 3. So, to find $i_{a'}$, we must first find the secondary-side currents $i_{a's}$ and $i_{c's}$ and then transform them into their respective primary-side currents.

To the right of our Δ/Y transformer (not shown here) is a 6-pulse rectifier. The secondary-side currents will thus have the form of a 6-pulse rectifier, with each phase shifted by 120°. In particular, $i_{c's}$ will lag behind $i_{a's}$ by 240°. We then reflect the secondary-side currents across the $\sqrt{3}$: 1 transformer, which scales the current magnitudes by a factor of $1/\sqrt{3}$, giving us the primary-side waveforms shown in Fig. 4. (Here, we started our waveforms at angle $-\pi/6$ to make subsequent analysis easier.) Taking the difference between $i_{a'p}$ and $i_{c'p}$ gives us the line current $i_{a'}$, shown in Fig. 4.

7.2.B Primary-side line current i_A

From KCL, the primary-side line current i_A is the sum of the primary-side currents going into each transformer block, or $i_A = i_a + i_{a'}$. We know what $i_{a'}$ is from the previous part. For i_a, we know that the outputs of the Y/Y and Δ/Y transformer blocks are shifted by $\pi/6$ from each other. Now, we need to figure out which direction to shift the waveforms.

Looking at the top transformer in the Δ/Y transformer block, we can see that the secondary-side current $i_{a's}$ is driven by the primary-side voltage v_{ab}. In the Y/Y transformer block, the secondary-side current i_{as} is driven by the primary-side voltage v_a. As shown in lecture, we know that v_{ab} lags behind v_a by $\pi/6$. Therefore, $i_{a's}$ should also lag behind i_a by $\pi/6$, giving us the current waveform for i_a shown in Fig. 4. To get our primary-side line current i_A, we add the two primary-side currents for each transformer block, giving us the waveform in Fig. 4.

7.3 KPVS 9.15: 3-Phase Bridge Inverter

The voltage v_{nr} is the average of the three phase voltages and is plotted in Fig. 9.20(d) of the text. The phase-to-neutral quantities can be computed as $v_{an} = v_{ar} - v_{nr}$, $v_{bn} = v_{br} - v_{nr}$, and $v_{cn} = v_{cr} - v_{nr}$. We add these quantities as shown in Table 2 (see last page).

7.4 KPVS 9.18: 3-Phase Inverter PWM Modulation

7.4.A Local-average line-to-line output voltages

In this problem, our duty ratios are

$$d_1 = 1 - d_4 = \frac{1}{2} + \frac{m}{2}\sin(\omega t) + \frac{m}{12}\sin(3\omega t)$$
$$d_3 = 1 - d_6 = \frac{1}{2} + \frac{m}{2}\sin(\omega t - \frac{2\pi}{3}) + \frac{m}{12}\sin(3\omega t)$$
$$d_5 = 1 - d_2 = \frac{1}{2} + \frac{m}{2}\sin(\omega t + \frac{2\pi}{3}) + \frac{m}{12}\sin(3\omega t)$$

To find the local-average line-to-line voltages, we must first find the local-average line-to-reference voltages. For now, let's look at voltage v_{ar}; the other two voltages have analogous analysis. When switch S1 is on and switch S4 is off, v_a is tied to V_{dc} so that $v_{ar} = V_{dc}/2$. When switch S1 is off and switch S4 is on, v_a is tied to ground so that $v_{ar} = -V_{dc}/2$. The local-average line-to-reference voltage $\overline{v}_{ar}$ is thus

$$\begin{aligned}
\overline{v}_{ar} &= d_1 \frac{V_{dc}}{2} + (1 - d_1)\frac{-V_{dc}}{2} \\
&= (\frac{1}{2} + \frac{m}{2}\sin(\omega t) + \frac{m}{12}\sin(3\omega t))\frac{V_{dc}}{2} \\
&\quad +(1 - \frac{1}{2} - \frac{m}{2}\sin(\omega t) - \frac{m}{12}\sin(3\omega t))\frac{-V_{dc}}{2} \\
&= \frac{m}{2}V_{dc}\sin(\omega t) + \frac{m}{12}V_{dc}\sin(3\omega t)
\end{aligned}$$

We can similarly find the other two local-average line-to-reference voltages.

$$\begin{aligned}
\overline{v}_{br} &= \frac{m}{2}V_{dc}\sin(\omega t - \frac{2\pi}{3}) + \frac{m}{12}V_{dc}\sin(3\omega t) \\
\overline{v}_{cr} &= \frac{m}{2}V_{dc}\sin(\omega t + \frac{2\pi}{3}) + \frac{m}{12}V_{dc}\sin(3\omega t)
\end{aligned}$$

The local-average line-to-line voltages can be found by taking the differences between the line-to-reference voltages and then simplifying using the following two trig identities:

$$\begin{aligned}
\sin u - \sin v &= 2\sin(\frac{1}{2}(u - v))\cos(\frac{1}{2}(u + v)) \\
\cos(u - \frac{\pi}{2}) &= \sin u
\end{aligned}$$

Using these trig identities, we can find $\overline{v}_{ab}$.

$$\begin{aligned}
\overline{v}_{ab} &= \overline{v}_{ar} - \overline{v}_{br} \\
&= \frac{m}{2}V_{dc}\sin(\omega t) - \frac{m}{2}V_{dc}\sin(\omega t - \frac{2\pi}{3}) \\
&= \frac{m}{2}V_{dc}\left(2\sin\left(\frac{1}{2}(\frac{2\pi}{3})\right)\cos\left(\frac{1}{2}(2\omega t - \frac{2\pi}{3})\right)\right) \\
&= \frac{m}{2}V_{dc}\left(\sqrt{3}\cos(\omega t - \frac{\pi}{3})\right) \\
&= m\frac{\sqrt{3}}{2}V_{dc}\sin(\omega t + \frac{\pi}{6})
\end{aligned}$$

Similarly, we can find the other two local-average line-to-line voltages.

$$\begin{aligned}
\overline{v}_{bc} &= m\frac{\sqrt{3}}{2}V_{dc}\sin(\omega t - \frac{\pi}{2}) \\
\overline{v}_{ca} &= m\frac{\sqrt{3}}{2}V_{dc}\sin(\omega t + \frac{5\pi}{6})
\end{aligned}$$

7.4.B Duty ratio extremae

To find the locations of extremae of d_1, we want to take its first derivative with respect to ωt and set it equal to zero.

$$\begin{aligned}
\frac{d(d_1)}{d\omega t} &= \frac{d}{d\omega t}(\frac{1}{2} + \frac{m}{2}\sin(\omega t) + \frac{m}{12}\sin(3\omega t)) \\
&= \frac{m}{2}\cos(\omega t) + \frac{m}{4}\cos(3\omega t)
\end{aligned}$$

Setting the above to zero and multiplying by $4/m$, we get that the local extremae occur when

$$2\cos(\omega t) + \cos(3\omega t) = 0$$

7.4.C Distortion boundary

We want to show that for $m = \frac{2}{\sqrt{3}}$, d_1 has a maximum of 1 at $\omega t = 60°$ and $120°$ and a minimum of zero at $\omega t = 240°$ and $300°$.

From part b, we know where the extremae of d_1 are. Now, we just need to show whether the extremae at these specific points are maxima or minima. To do this, we take the second derivative of d_1 with respect to ωt. If $\frac{d(d_1)^2}{d^2\omega t} < 0$, we have a maximum. If $\frac{d(d_1)^2}{d^2\omega t} > 0$, we have a minimum.

$$\frac{d(d_1)^2}{d^2\omega t} = \frac{-m}{2}\sin(\omega t) - \frac{3m}{4}\sin(3\omega t)$$

To get the value of d_1 at each point, we plug ωt back into our equation for d_1. A summary of the specific points we're interested in is in Table 1.

ωt	$\frac{d(d_1)^2}{d^2\omega t}$	extrema type	d_1
60°	$-\frac{\sqrt{3}}{4}$	maximum	1
120°	$-\frac{\sqrt{3}}{4}$	maximum	1
240°	$\frac{\sqrt{3}}{4}$	minimum	0
300°	$\frac{\sqrt{3}}{4}$	minimum	0

Table 1: d_1 extremae types and values

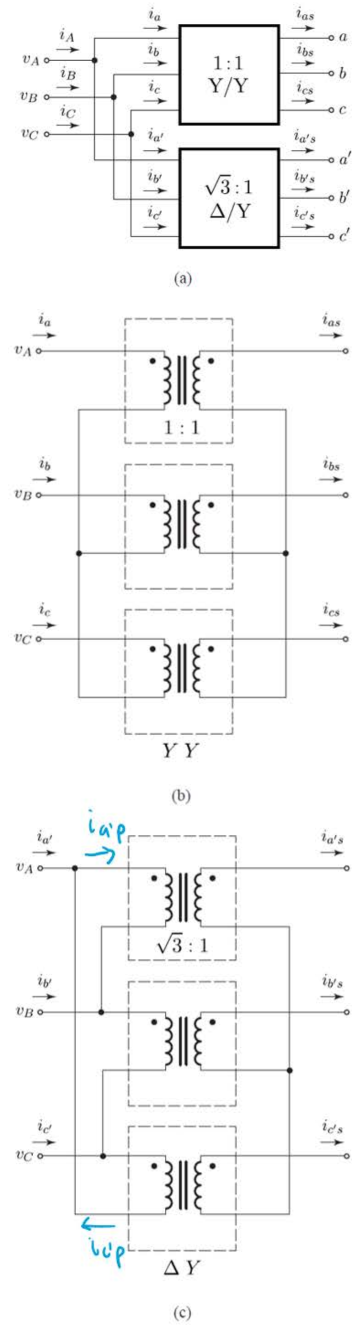

Figure 3: Transformer connections for the 12-pulse rectifier (reproduced KPVS Fig. 9.28)

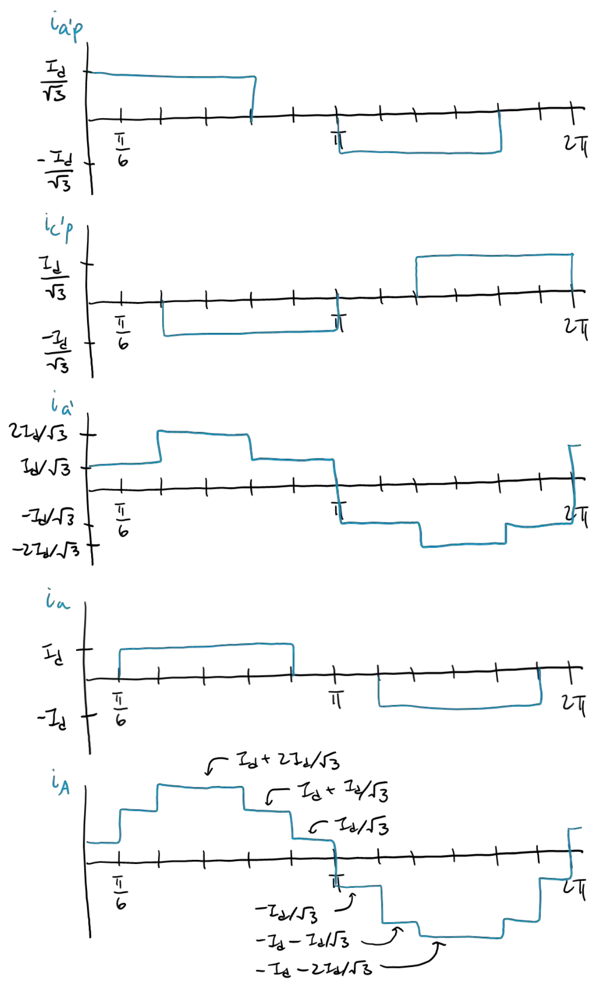

Figure 4: Primary-side Δ line currents

Vector {a b c}	Sw. on	v_{ar}	v_{br}	v_{cr}	v_{ab}	v_{bc}	v_{ca}	v_{an}	v_{bn}	v_{cn}	v_{nr}
$S_0 = \{000\}$	4, 6, 2	$-1/2$	$-1/2$	$-1/2$	0	0	0	0	0	0	$-1/2$
$S_1 = \{100\}$	1, 6, 2	$+1/2$	$-1/2$	$-1/2$	$+1$	0	-1	$+2/3$	$-1/3$	$-1/3$	$-1/6$
$S_2 = \{110\}$	1, 3, 2	$+1/2$	$+1/2$	$-1/2$	0	$+1$	-1	$+1/3$	$+1/3$	$-2/3$	$+1/6$
$S_3 = \{010\}$	4, 3, 2	$-1/2$	$+1/2$	$-1/2$	-1	$+1$	0	$-1/3$	$+2/3$	$-1/3$	$-1/6$
$S_4 = \{011\}$	4, 3, 5	$-1/2$	$+1/2$	$+1/2$	-1	0	$+1$	$-2/3$	$+1/3$	$+1/3$	$+1/6$
$S_5 = \{001\}$	4, 6, 5	$-1/2$	$-1/2$	$+1/2$	0	-1	$+1$	$-1/3$	$-1/3$	$+2/3$	$-1/6$
$S_6 = \{101\}$	1, 6, 5	$+1/2$	$-1/2$	$+1/2$	$+1$	-1	0	$+1/3$	$-2/3$	$+1/3$	$+1/6$
$S_7 = \{111\}$	1, 3, 5	$+1/2$	$+1/2$	$+1/2$	0	0	0	0	0	0	$+1/2$

Table 2: Update of Table 9.1 in the text with line-neutral quantities added for problem 9.15