6.6220 HW10 Solutions*

10.1 KPVS 26.1: Input Filter for Buck Converter

Proceeding as in Example 26.1, the range of duty cycles is

$$D = \frac{V_2}{V_1} \in \left[\frac{10}{25}, \frac{10}{20}\right] = [0.4, 0.5]$$

while the range of the average inductor current is

$$I_L = \frac{P_o}{V_2} \in \left[\frac{50}{10}, \frac{100}{10}\right] = [5, 10]$$

Assuming zero inductor ripple and that the first harmonic of I_x is most important for filter design, we approximate the drive waveform as a sinusoid at $\omega_s = 2\pi(500k) = 3.15\times10^6$ rad/s having worst case amplitude at D=0.5 and $I_L = 10, I_{x,1} = 6.36$ A. As in the example, we'll take a conservative (though arbitrary) factor of three in our ripple requirement to account forour simplifications. So, we require $I_{y,1} = 5 \times 10^{-3}$ A. Since our worst-case is identical to that of Example 26.1, the rest of this solution proceeds identically.

10.2 KPVS 26.4: EMI Noise Seperator

(a) Applying KVL: $v_1 - v_p - v_R = 0$ and $v_2 + v_P - v_R = 0$, where v_R is the voltage across the resistor and v_p is the voltage on the winding of the transformer between v_1 and v_{CM}. Since the transformer is 1:1, this voltage is the same on all four ports of the transformer. Summing these expressions yields $v_1 + v_2 = 2v_R$, as expected.

(b) Applying KVL:

$$v_{DM} - 2v_p = 0$$

We have seen that $v_p = v_1 - v_R = (v_1 - v_2)/2$. Thus $v_{DM} = v_1 - v_2$ as expected.

(c) Calling the current out of port 1 i_1, the current out of port 2 i_2, and the current into the DM resistor i_{DM}, and noting that the transformer windings must be magnetically in-series (the voltage at every port is identical), we can write:

$$i_1 - i_2 = 2i_{DM} = \frac{v_1 - v_2}{50}$$

Similarly, we can connect these currents to the v_{CM} voltage as:

$$i_1 + i_2 = \frac{v_1 + v_2}{50}$$

Summing these equations yields:

$$2i_1 = \frac{v_1 - v_2}{50} + \frac{v_1 + v_2}{50} = \frac{v_1}{25}$$

Thus, $Z_1 = v_1/i_1 = 50\ \Omega$, as expected.

10.3 KPVS 26.5: Filter Capacitor Parasitics

(a) We can derive the asymptotes by considering the impedance of the RLC branch, which is

$$Z_{RLC} = \frac{1}{j\omega C_f} + j\omega L_c + R_c$$

At low frequencies, the capacitance's impedance dominates, and the branch is effectively an open-circuit. Thus, $\tilde{i}_x = \tilde{i}_y$, and the gain is 1 as expected. As the frequency increases, the dominantly C_f branch interacts with the series L_f inductance and once we are past the resonance of L_f and C_f, this LC filter rolls-off at -40dB/dec. This continues to happen until the impedance of C_f drops low enough that R_c begins to control the series impedance. We assume this happens at the frequency where their impedance magnitude is equal, $\omega = 1/(R_c C_f)$. After this, the branch is considered dominantly resistive, and forms an RL filter between L_f and R_c, rolling off at -20dB per decade. This continues as frequency is increased until the branch becomes inductive, which we again assume starts to happen around the frequency where the impedance magnitude of L_c is equal to R_c, $\omega =$

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R_c/L_c. Beyond this value, the dominantly inductive branch forms a simple current divider with L_f which results in constant gain between $\tilde{i}_y$ and $\tilde{i}_x$ of $L_c/(L_c+L_f)$ (this can be straightforwardly derived by equating $-i_y Z_{L_f} = (i_y - i_x) Z_{L_c}$).

(b) At high frequencies, the current gain approaches the final asymptote

$$|H_i| \rightarrow \frac{L_c}{L_c + L_f} \approx \frac{L_c}{L_f} = 10^{-3}$$

The figures in the chapter just show the base-10 log of the value, which would be -3 dB. On a conventional bode plot, we'd show this as 20 times the base-10 log, which is -60 dB.

(c) In this case, the resistance never dominates the branch and we transition directly from a capacitive impedance to an inductive impedance. So, this asymptote would occur at the resonant frequency $\omega = 1/\sqrt{L_c C_f}$.