

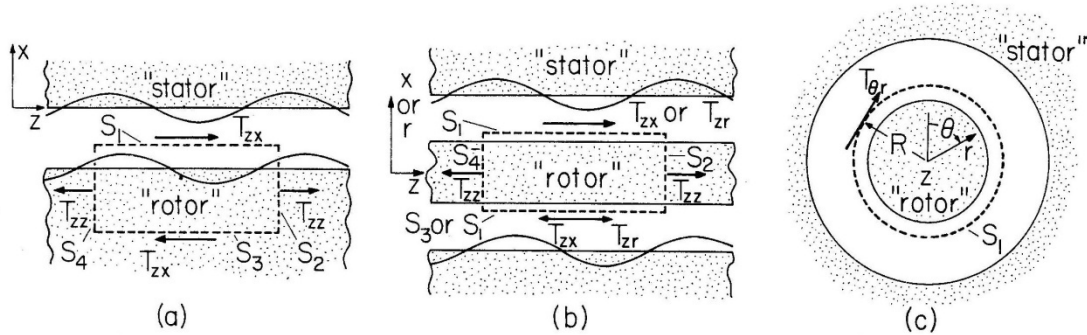
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6.642 Continuum Electromechanics
Fall 2008

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Lecture 3: Continuum Electromechanics (Melcher) – Section 4.1-4.3

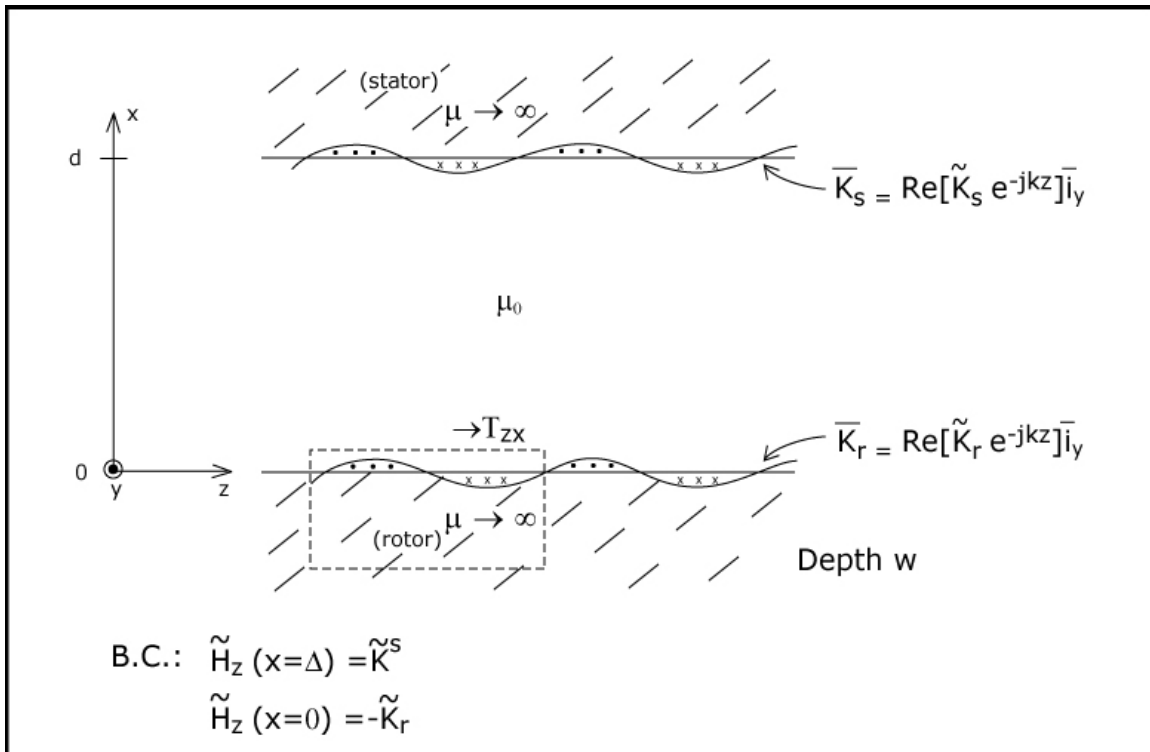
I. Air-Gap Magnetic Machines



Typical "air-gap" configurations in which a force or torque on a rigid "rotor" results from spatially periodic sources interacting with spatially periodic excitations on a rigid "stator." Because of the periodicity, the force or torque can be represented in terms of the electric or magnetic stress acting at the air-gap surfaces S_1 : (a) planar geometry or developed model; (b) planar or cylindrical beam; (c) cylindrical rotor.

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A. Generalized Description



$$f_z = \oint_S T_{zx} n_x dz dy = w \int_0^{2\pi/k} \mu_0 H_z H_x|_{x=0} dz = w \int_0^{2\pi/k} \mu_0 H_z^r H_x^r dz$$

force on a wavelength

$$a(z) = \text{Re} \left[\tilde{A} e^{-jkz} \right]$$

$$b(z) = \text{Re} \left[\tilde{B} e^{-jkz} \right]$$

$$\frac{k}{2\pi} \int_0^{2\pi/k} a(z) b(z) dz = \frac{1}{2} \text{Re} \left[\tilde{A} \tilde{B}^* \right] = \frac{1}{2} \text{Re} \left[\tilde{A}^* \tilde{B} \right]$$

$$f_z = \frac{2\pi w}{k} \frac{\mu_0}{2} \text{Re} \left[\tilde{H}_z^r \tilde{H}_x^{r*} \right]$$

$$= \frac{\pi w \mu_0}{k} \text{Re} \left[-\tilde{K}_r \tilde{H}_x^{r*} \right]$$

$$\begin{bmatrix} \tilde{B}_x^s \\ \tilde{B}_x^r \end{bmatrix} = \mu_0 k \begin{bmatrix} -\coth kd & \frac{1}{\sinh kd} \\ -\frac{1}{\sinh kd} & \coth kd \end{bmatrix} \begin{bmatrix} \tilde{\chi}^s \\ \tilde{\chi}^r \end{bmatrix}$$

$$\tilde{H}_z = +jk\tilde{\chi} \Rightarrow \tilde{\chi}^s = \frac{1}{jk} \tilde{H}_z^s = \frac{\tilde{K}^s}{jk}$$

$$\tilde{\chi}^r = \frac{\tilde{H}_z^r}{jk} = -\frac{\tilde{K}_r^r}{jk}$$

$$\mu_0 \tilde{H}_x^r = \mu_0 k \left[\frac{-\tilde{\chi}^s}{\sinh kd} + \tilde{\chi}^r \coth kd \right]$$

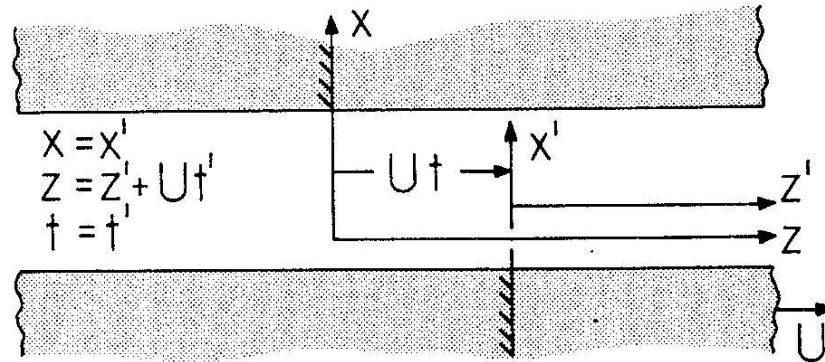
$$= \mu_0 k \left[\frac{-\tilde{K}^s}{jk \sinh kd} - \frac{\tilde{K}_r^r}{jk} \coth kd \right]$$

$$\text{Re} \left[-\tilde{K}_r^* \tilde{H}_x^r \mu_0 \right] = -\text{Re} \left[+j\mu_0 \left(\frac{\tilde{K}_r^* \tilde{K}^s}{\sinh kd} + \tilde{K}_r^* \tilde{K}_r^r \coth kd \right) \right]$$

$$= \text{Re} \left[-\mu_0 j \tilde{K}_r^* \tilde{K}^s / \sinh kd \right]$$

$$f_z = -\frac{\pi w}{k} \frac{\mu_0}{\sinh kd} \text{Re} \left[j \tilde{K}_r^* \tilde{K}_s^s \right] \text{ (force on each wavelength)}$$

B. Synchronous Interaction



Rotor and stator reference frames z' and z .

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$$K^s = K_0^s \sin[\omega_s t - kz] = \text{Re}[-jK_0^s e^{j(\omega_s t - kz)}]$$

$$\begin{aligned} K^r &= K_0^r \sin[\omega_r t - k(z' - \delta)]; z' = z - Ut \\ &= K_0^r \sin[(\omega_r + kU)t - k(z - \delta)] \\ &= \text{Re}[-jK_0^r e^{j(\omega_r + kU)t} e^{-jk(z - \delta)}] \end{aligned}$$

$$\tilde{K}^s = -jK_0^s e^{j\omega_s t}$$

$$\tilde{K}^r = -jK_0^r e^{jk\delta} e^{j(\omega_r + kU)t}$$

$$\begin{aligned} f_z &= -\frac{\pi W}{k} \frac{\mu_0}{\sinh kd} \text{Re} \left[j(-jK_0^s) e^{j\omega_s t} (jK_0^r e^{-jk\delta}) e^{-j(\omega_r + kU)t} \right] \\ &= -\frac{\pi W}{k} \frac{\mu_0}{\sinh kd} K_0^s K_0^r \text{Re} \left[j e^{-jk\delta} e^{j(\omega_s - \omega_r - kU)t} \right] \end{aligned}$$

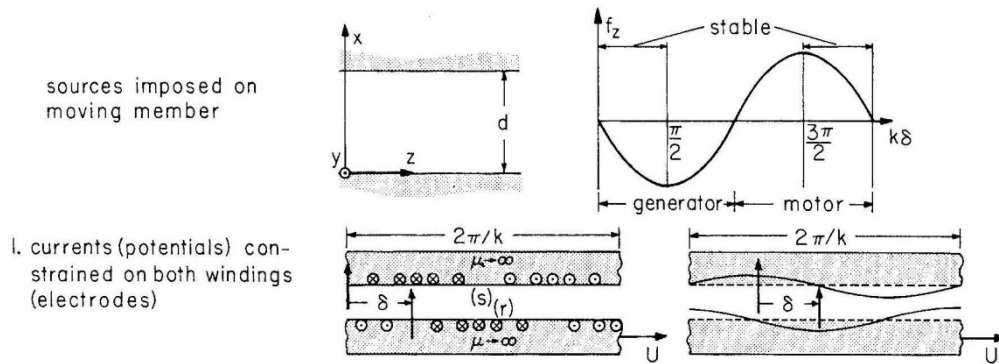
For time average force

$$\Rightarrow \omega_s = \omega_r + kU \quad (\text{synchronous condition})$$

$$\text{Usually } \omega_r = 0 \Rightarrow \omega_s = kU$$

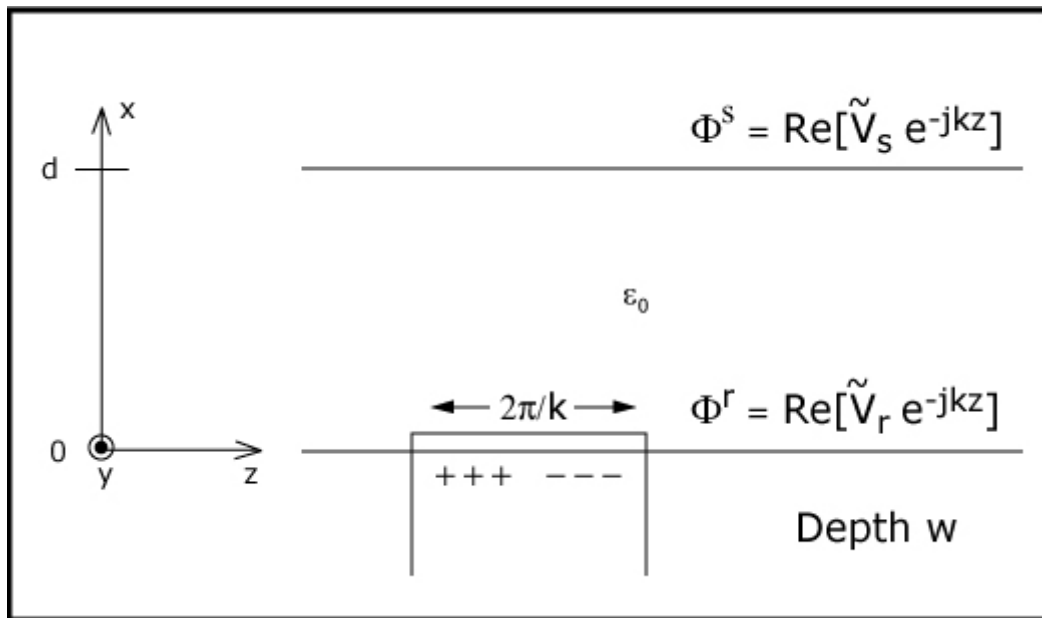
$$\langle f_z \rangle = -\frac{\pi W}{k} \frac{\mu_0}{\sinh kd} K_0^s K_0^r \sin k\delta$$

Basic configurations illustrating classes of electromechanical interactions and devices. MQS and EQS systems respectively in left and right columns.



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II. Electrostatic Machine



$$f_z = w \frac{2\pi}{k} \int_0^{2\pi/k} T_{zx}|_{x=0} dz = \frac{2\pi w}{k} \int_0^{2\pi/k} \epsilon_0 E_z E_x|_{x=0} dz$$

$$\tilde{E}_z^r = jk \tilde{V}_r$$

$$f_z = \frac{\pi w}{k} \text{Re} \left[\epsilon_0 \tilde{E}_z^{r*} \tilde{E}_x^r \right]$$

$$= \frac{\pi w}{k} \text{Re} \left[\epsilon_0 \left(-jk \tilde{V}_r \right) \tilde{E}_x^r \right]$$

$$\begin{bmatrix} \tilde{D}_x^s \\ \tilde{D}_x^r \end{bmatrix} = \epsilon_0 k \begin{bmatrix} -\coth kd & \frac{1}{\sinh kd} \\ -\frac{1}{\sinh kd} & \coth kd \end{bmatrix} \begin{bmatrix} \tilde{V}_s \\ \tilde{V}_r \end{bmatrix}$$

$$\epsilon_0 \tilde{E}_x^r = \epsilon_0 k \left[\frac{-\tilde{V}_s}{\sinh kd} + \tilde{V}_r \coth kd \right]$$

$$\begin{aligned} \operatorname{Re} \left[-jk \epsilon_0 \tilde{V}_r^* \tilde{E}_x^r \right] &= \operatorname{Re} \left[-jk^2 \epsilon_0 \tilde{V}_r^* \left(\frac{-\tilde{V}_s}{\sinh kd} + \tilde{V}_r \coth kd \right) \right] \\ &= \operatorname{Re} \left[+jk^2 \epsilon_0 \tilde{V}_s \tilde{V}_r^* / \sinh kd \right] \end{aligned}$$

$$f_z = \pi w \frac{k \epsilon_0}{\sinh kd} \operatorname{Re} \left[j \tilde{V}_s \tilde{V}_r^* \right]$$

$$V_s = V_0^s \cos(\omega_s t - kz)$$

$$V_r = -V_0^r \cos(\omega_r t - k(z' - \delta)); z' = z - Ut$$

$$\tilde{V}_r = -V_0^r e^{j(\omega_r + kU)t} e^{jk\delta}$$

$$\tilde{V}_s = V_0^s e^{j\omega_s t}$$

$$\langle f_z \rangle = \frac{\pi w k \epsilon_0}{\sinh kd} \operatorname{Re} \left[-j V_0^s V_0^r e^{-jk\delta} e^{j(\omega_s - \omega_r - kU)t} \right]$$

$$\omega_s = \omega_r + kU$$

$$\langle f_z \rangle = -\frac{\pi w k \epsilon_0}{\sinh kd} V_0^s V_0^r \sin k\delta$$