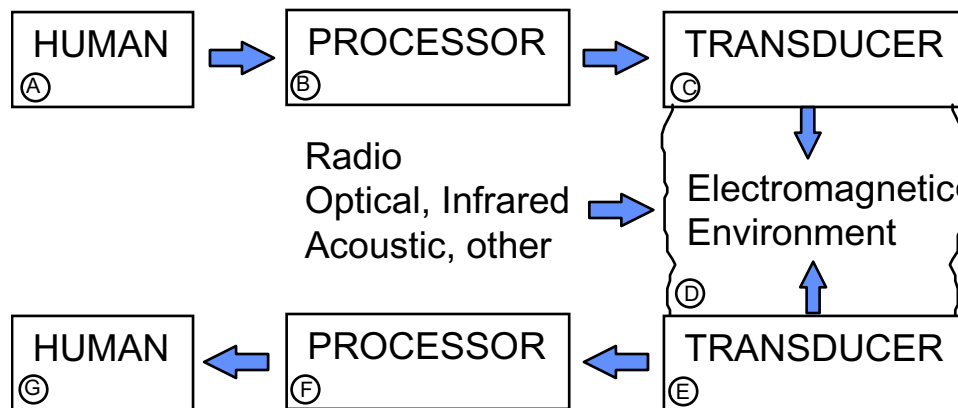


# CHAPTER 1: INTRODUCTION

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## 1.1 SCOPE AND CONTENT

Communications and sensing systems are ubiquitous. They are found in military, industrial, medical, consumer, and scientific applications employing radio frequency, infrared, visible, and shorter wavelengths. Even acoustic systems operate under similar principles. Communications examples range from optical fiber or satellite systems to wireless radio. Radio astronomy, radar, lidar, and sonar systems probe the environment and have counterparts in analytical instruments and memory systems used for a wide variety of purposes.



**Figure 1.1:** Architecture of communications and sensing systems.

Figure 1.1 characterizes the major electromagnetic and signal processing elements of such systems, not all which of are necessarily involved in any particular case. For example, in *communication systems* a human (A) (or computer counter-part) typically generates signals which are first processed (B) and then coupled to an electromagnetic environment (D) by a transducer or antenna (C). After propagating through the environment (D) the signals are intercepted by another transducer (E) which usually consists of an antenna followed by a detector which converts these electromagnetic signals into voltages and currents. The signals from the transducer (E) are then generally manipulated in processor (F) before transmission to the human or computer recipient (G). Communications and active systems that probe the environment generally involve all seven elements (A)-(G). Passive systems generally involve only the last four, from the environment (D) to the user (G). Examples of the latter include environmental or astronomical observations, medical systems seeking spectral or thermal signatures of disease, and the readout of information from memory systems such as compact disks.

To completely analyze such a broad range of systems would require several textbooks. Here the fundamentals for each of the elements in Figure 1.1 are presented in a generally complete

way but, for efficiency, only few of their possible combinations are presented in any detail. For example, the probability of symbol detection error is analyzed for communications systems, but this analysis is not repeated for other systems. It is hoped that readers of this book will acquire sufficient understanding of the elements of Figure 1.1 to be able to conceive, design, and analyze a wide variety of electromagnetic signal-based systems by combining these elements appropriately.

The chapters of this book can be divided into three groups. First Chapter 1 defines the basic notation and surveys briefly some of the basic notation fundamental to signal processing and electromagnetic waves.

The second group of chapters focuses on the fundamental elements of communications and sensing systems. Chapter 2 discusses basic noise processes and the devices used for detection of radio, infrared, and visible signals, including those first-stage signal processing operations that yield the desired signal, energy, or power spectral density estimates. Chapter 3 then discusses the transducers and antennas that link these detectors to the electromagnetic environment, including wire antennas, apertures, simple optics, common propagation phenomena, and how the transmitting and receiving properties of systems are related in a simple way.

The third group of chapters deals with complete systems applied to communications (Chapter 4) and both active and passive sensing (Chapter 5). Estimation techniques for both sensing and communication systems are then discussed separately in Chapter 6.

## 1.2 MATHEMATICAL NOTATION

Because physical signals are generally analog, we rely in this text more heavily on continuous functions and operators than on discrete signals and the  $z$  transform. Physical signals in time or space are generally represented by lower case letters followed by their arguments in parentheses, whereas their transforms are generally represented by capital letters, again followed by their arguments in parentheses. Complex quantities are generally indicated by underbars. For example, the *Fourier transform* relating a voltage pulse  $v(t)$  to its spectrum  $\underline{V}(f)$  is:

$$\underline{V}(f) \triangleq \int_{-\infty}^{\infty} v(t) e^{-j2\pi ft} dt \quad [\text{volts/Hz=voltsec}] \quad (1.2.1)$$

$$\underline{v}(t) = \int_{-\infty}^{\infty} \underline{V}(f) e^{+j2\pi ft} df \quad [\text{volts}] \quad (1.2.2)$$

$$v(t) \leftrightarrow \underline{V}(f) \quad (1.2.3)$$

where frequency is generally represented by  $f(\text{Hz})$  or  $\omega = 2\pi f$  (radians/second). We abbreviate this Fourier relationship as:  $v(t) \leftrightarrow \underline{V}(f)$ . These relations apply for pulses of finite energy, i.e.:

$$\int_{-\infty}^{\infty} |v(t)|^2 dt < \infty \quad (1.2.4)$$

The energy spectrum  $S(f) = |\underline{V}(f)|^2$  and has units [volts<sup>2</sup>/Hz] for the case where  $v(t)$  has units [volts]. This *energy density spectrum* is the Fourier transform of the voltage *autocorrelation function*  $R(\tau)$ , where:

$$|\underline{V}(f)|^2 = S(f) \leftrightarrow R(\tau) \triangleq \int_{-\infty}^{\infty} v(t) v^*(t-\tau) dt \quad [v^2 \text{ sec}] \text{ or [J], etc.} \quad (1.2.5)$$

*Parseval's theorem*, which says that the integral of power over time equals the integral of energy spectral density  $S(f)$  over frequency, follows easily from Equation (1.2.5) and the definition of a Fourier transform (1.2.2) for  $t = 0$ :

$$R(0) = \int_{-\infty}^{\infty} v^2(t) dt = \int_{-\infty}^{\infty} S(f) df. \quad (1.2.6)$$

These relationships for analytic pulse signals can be represented compactly by the following notation:

$$\begin{array}{ccc} v(t) \leftrightarrow \underline{V}(f) & & \\ \downarrow \quad \downarrow & & \\ R(\tau) \leftrightarrow |\underline{V}(f)|^2 \triangleq S(f) & & \end{array} \quad (1.2.7)$$

The single-headed arrows pointing downward indicate that the transformations from  $v(t)$  to  $R(\tau)$ , and from  $\underline{V}(f)$  to  $|\underline{V}(f)|^2$  are irreversible. The units of these quantities depends on the units associated with  $v(t)$ . For example, if  $v(t)$  represents volts as a function of time, then the units in clockwise order in (1.2.7) for these four quantities are: volts, volts/Hz, (volts/Hz)<sup>2</sup>, and volts<sup>2</sup> seconds. If this voltage  $v(t)$  is across a 1-ohm resistor, then we can associate the autocorrelation function  $R(\tau)$  with the units Joules, and the energy density spectrum  $S(f)$  with the units Joules/Hz.

Another important operator is *convolution*, represented by an asterisk, where:

$$a(t) * b(t) \triangleq \int_{-\infty}^{\infty} a(\tau) b(t-\tau) d\tau = c(t). \quad (1.2.8)$$

Note that a unit impulse convolved with any function yields the original function, where we define the unit impulse  $\delta(t)$  as a function which is zero for  $|t| > 0$ , and has an integral of value unity.

Periodic signals with finite energy in each period  $T$  can be reversibly characterized by their Fourier series and irreversibly characterized by their autocorrelation function  $R(\tau)$  and its Fourier transform, the energy density spectrum  $\Phi_m$ . These are related as suggested in Equation 1.2.9

$$\begin{array}{ccc} v(t) \leftrightarrow V_m \text{ (volts)} & & \\ \downarrow & \quad \downarrow & \\ R(\tau) \leftrightarrow \Phi_m \text{ (watts Hz}^{-1}\text{)} \text{ or } S(f) \text{ (Joules)} & & \end{array} \quad (1.2.9)$$

The Fourier series  $V_m$  can be simply computed from the original waveform  $v(t)$  as:

$$\underline{V}_m = T^{-1} \int_{-T/2}^{+T/2} v(t) e^{-jm2\pi f_0 t} dt \quad (1.2.10)$$

where  $T$  equals  $f_0^{-1}$  and:

$$v(t) = \sum_{m=-\infty}^{\infty} \underline{V}_m \bullet e^{jm2\pi f_0 t} \quad (1.2.11)$$

$$R(\tau) = \int_{-T/2}^{+T/2} v(t) v^*(t-\tau) dt = \sum_{m=-\infty}^{+\infty} |\underline{V}_m|^2 e^{jm2\pi f_0 \tau} \quad (1.2.12)$$

$$\Phi_m = |\underline{V}_m|^2 = T^{-1} \int_{-T/2}^{+T/2} R(\tau) e^{-jm2\pi f_0 \tau} d\tau \quad (1.2.13)$$

Random signals  $x(t)$  can often be characterized by their autocorrelation function:

$$\phi_x(\tau) = E[x(t)x(t-\tau)]. \quad (1.2.14)$$

Such signals are called “*wide-sense stationary*” stochastic signals. For the special case where the signal  $x(t)$  is the voltage across a 1-ohm resistor, the autocorrelation function  $\phi(\tau)$  for  $\tau = 0$  may

be regarded as the average power dissipated in the resistor, and the Fourier transform of the autocorrelation function can be regarded as the power spectral density  $\Phi(f)$  (watts/Hz) where:

$$\begin{array}{ccc} v(t) & \leftrightarrow & (?) \\ \downarrow & & \downarrow \\ \phi_v(\tau) & \leftrightarrow & \Phi(f) \end{array} \quad (1.2.15)$$

In many cases we shall encounter *Gaussian noise*  $n(t)$ , where the probability distribution of  $n$  is:

$$P\{n\} = \frac{1}{\sigma\sqrt{2\pi}} e^{-(n/\sigma)^2/2} \quad (1.2.16)$$

$$E[n^2] = \int_{-\infty}^{\infty} p(n)n^2 dn = \sigma^2. \quad (1.2.17)$$

Band-limited *Gaussian white noise*, which is defined as having a uniform power spectral density  $\Phi(f)$  over a band of width  $B$  (Hz), can be characterized by the noise power spectral density  $N_o$ , where:

$$E[n^2] = \sigma^2 = N_o B \quad (1.2.18)$$

Signal or wave powers are often characterized in terms of decibels, where if a signal increases its power of  $P_1$  to  $P_2$  we say there has been a gain of:

$$\text{dB gain} \triangleq 10 \log_{10} (P_2/P_1) \quad (1.2.19)$$

Thus an amplifier having power output equal to the power input exhibits 0 dB gain, where power gains of a factor of 10 or 100 correspond to 10 dB and 20 dB, respectively.

### 1.3 ELECTROMAGNETIC NOTATION

We characterize electromagnetic phenomena in terms of the *electric field*  $\bar{E}$  (volts/meter) and *magnetic field*  $\bar{H}$  (amperes/meter), where these fields have both a magnitude and direction at each point in space and time. We represent the *electric displacement* by  $\bar{D}$  (coulombs/meter<sup>2</sup>), where for simple media  $\bar{D} = \epsilon\bar{E}$  and the *permittivity*  $\epsilon$  for vacuum is  $\epsilon_0 = 8.854 \times 10^{-12}$  farads/meter; F/m. We represent the *magnetic flux density* by  $\bar{B}$  (Tesla = Weber/m<sup>2</sup> =  $10^4$  Gauss), where for simple media  $\bar{B} = \mu\bar{H}$  and the *permeability*  $\mu$  for

vacuum is  $\mu_0 = 4\pi \times 10^{-7}$  henries/meter; H/m. The *electric current density* is represented by  $\bar{\mathbf{J}}$  (amperes / meter<sup>2</sup>; A / m<sup>2</sup>) and the *electric charge density* by  $\rho$  (coloumbs/meter<sup>3</sup>; C/m<sup>3</sup>).

This text uses SI (mks) units throughout, in which case *Maxwell's equations* become:

$$\nabla \times \bar{\mathbf{E}} = -\frac{\partial \bar{\mathbf{B}}}{\partial t} \quad (1.3.1)$$

$$\nabla \times \bar{\mathbf{H}} = \bar{\mathbf{J}} + \frac{\partial \bar{\mathbf{D}}}{\partial t} \quad (1.3.2)$$

$$\nabla \cdot \bar{\mathbf{D}} = \rho \quad (1.3.3)$$

$$\nabla \cdot \bar{\mathbf{B}} = 0 \quad (1.3.4)$$

$$\nabla \triangleq \hat{x} \partial / \partial x + \hat{y} \partial / \partial y + \hat{z} \partial / \partial z \quad (1.3.5)$$

where  $\hat{x}$ ,  $\hat{y}$ , and  $\hat{z}$  are *unit vectors* in Cartesian coordinates.

In general these field quantities are functions of both space and time and can be represented in different ways. For example, the x component of a *monochromatic electric field* at frequency  $\omega$  and position  $\bar{\mathbf{r}}$  can be represented as:

$$E_x(\bar{\mathbf{r}}, t) = \text{Re} \left\{ E_x(\bar{\mathbf{r}}) e^{j(\omega t - \phi_x(\bar{\mathbf{r}}))} \right\} = \text{Re} \left\{ \underline{E}_x(\bar{\mathbf{r}}) e^{j\omega t} \right\} \quad (1.3.6)$$

where the operator  $\text{Re} \{ \}$  extracts the real part of its argument, and the phasor can be represented as:

$$\underline{E}_x(\bar{\mathbf{r}}) = E_x(\bar{\mathbf{r}}) e^{-j\phi_x(\bar{\mathbf{r}})} \quad (1.3.7)$$

In general, we may combine all three vector components of  $\bar{\mathbf{E}}(\bar{\mathbf{r}}, t)$  in a *phasor representation* to yield:

$$\bar{\mathbf{E}}(\bar{\mathbf{r}}, t) = \text{Re} \left\{ \underline{\bar{\mathbf{E}}}(\bar{\mathbf{r}}) e^{j\omega t} \right\} \quad (1.3.8)$$

where we note that  $\underline{\bar{\mathbf{E}}}(\bar{\mathbf{r}})$  has six numbers associated with it (three vectors, each with magnitude and phase). The other variables are also expressible as phasors when the signals are monochromatic.

Maxwell's equations can be therefore rewritten in terms of phasors as:

$$\nabla \times \bar{\mathbf{E}} = -j\omega\bar{\mathbf{B}} \quad (1.3.9)$$

$$\nabla \times \bar{\mathbf{H}} = j\omega\bar{\mathbf{D}} + \bar{\mathbf{J}} \quad (1.3.10)$$

$$\nabla \cdot \bar{\mathbf{D}} = \rho \quad (1.3.11)$$

$$\nabla \cdot \bar{\mathbf{B}} = 0 \quad (1.3.12)$$

Many waves in communications or sensing systems travel on wires or transmission lines where they may be characterized in terms of voltages  $V(z, t)$  and currents  $I(z, t)$  as a function of position  $z$  and time  $t$ . Most commonly such signals travel on *transverse-electromagnetic-field (TEM) transmission lines* for which the voltage is measured between the two conductors at a particular position  $z$  and the currents  $I(z, t)$  in the two wires are equal and opposite at any position  $z$ . Such transmission lines can be characterized by their inductance  $L$  per unit length (H/m) and their capacitance  $C$  per unit length (C/m). In general,  $LC = \mu\epsilon$ , where  $\mu\epsilon$  is a function of the medium between and around the conductors.

In general such transmission lines satisfy the *wave equation*:

$$\frac{\partial^2 v}{\partial z^2} = LC \frac{\partial^2 v}{\partial t^2} \quad (1.3.13)$$

The general *wave equation solution* is the linear superposition of an arbitrarily shaped *forward moving wave*  $v_+(z - t/\sqrt{LC})$  and a backward moving wave where:

$$v(z, t) = v_+(z - t/\sqrt{LC}) + v_-(z + t/\sqrt{LC}) \quad (1.3.14)$$

which leads to:

$$i(z, t) = \sqrt{C/L} \left[ v_+(z - t/\sqrt{LC}) - v_-(z + t/\sqrt{LC}) \right] \quad (1.3.15)$$

The instantaneous power at any position  $z$  on the transmission line is simply the product of the voltage  $v$  and current  $i$  at that point in space and time. The phasor equivalents of equations (1.3.14) and (1.3.15) are:

$$\underline{V}(z) = \underline{V}_+ e^{-jkz} + \underline{V}_- e^{+jkz} \quad (1.3.16)$$

and

$$\underline{I}(z) = Y_0 [\underline{V}_+ e^{-jkz} - \underline{V}_- e^{+jkz}] \quad (1.3.17)$$

where the *wave number* or *propagation constant*  $k = \omega\sqrt{LC} = \omega\sqrt{\mu\epsilon}$ , and the *characteristic admittance* of the transmission line  $Y_0 = Z_0^{-1} = \sqrt{C/L}$ , and where  $Z_0$  (ohms) is called the *characteristic impedance* of the TEM transmission line.

Waves are also characterized by their *wavelength*  $\lambda$ , where  $\lambda = c/f$  and  $c$  is the phase velocity of the electromagnetic waves in that medium.