

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
 Department of Electrical Engineering and Computer Science
Receivers, Antennas, and Signals – 6.661

Solutions -- Problem Set No. 8

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Problem 8.1

a) The threshold should be midway between the two alternative voltages, i.e. at zero volts.

b) Referring to Figure 4.2-2b and (4.2.10) in the text, the threshold should be:

$$\text{Thresh} = 0 + (N/A)\ln(P_2/P_1) = (N/2) \ln 3^{-1} = \text{span style="border: 1px solid black; padding: 2px;">-0.55 volts}$$

where N in this formula (4.2.10) is not WHz^{-1} but rather σ_n^2 , which is 1^2 here.

c) $\text{Thresh} = 0.5 + (N/A)\ln(P_2/P_1) = 0.5 + (1/1) \ln 3^{-1} = \text{span style="border: 1px solid black; padding: 2px;">-0.6 volts}$

This is below any voltage ever sent, but is nonetheless a correct answer because the noise is so large here relative to the separation between the two original signals that only when the received voltage is extremely negative would we favor the hypothesis that zero volts was sent.

d) The form of the solution is clearly $P\{\text{error}\} = 0.5 \text{ERFC}(z)$, so the problem is to find the value of z. Here we want $P\{\text{error}\} = \int_{-\infty}^{-b}(2\pi\sigma_y^2)^{-0.5} \exp(-y^2/2\sigma_y^2) dy$ (see (4.4.8)) where $b = -1$ and $\sigma_y = 1$. Referring to (4.4.10) and (4.4.12) we see that $z = 2^{-0.5}$ because $K = 1$. Therefore $P\{\text{error}\} = 0.5 \text{ERFC}(2^{-0.5})$.

Problem 8.2

See Example 4.2.2 on page ~4-10 of the text. The receiver takes the form of Figure 4.2-4 with three symbols rather than two. Using (4.2.18), the three matched filter impulse responses are $h_i(t) = a(t)s_i(1-t)$ for the three possible signals $s_i(t)$ [$i = 1,2,3$], where $a(t) = 1$ for $0 < t < 1$, and $a(t) = 0$ otherwise. The three biases b_i in (4.2.18) are

$$b_i = N \ln P_i - 0.5 \int_0^1 s_i^2(t) dt \text{ where}$$

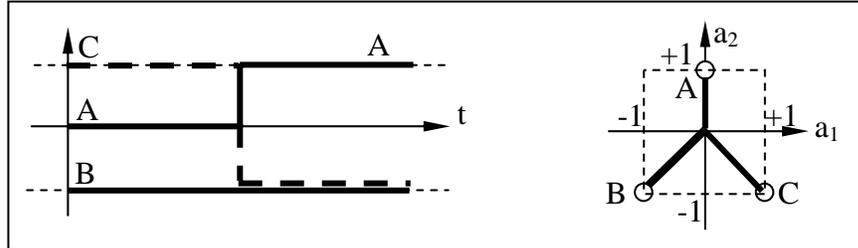
$$N = (kT_s/2) \int_{-\infty}^{\infty} |H(f)|^2 df = (kT_s/2) \int_0^1 a^2(t) dt = (kT_s/2) \text{ and where}$$

T_s is the noise temperature just prior to the matched filters, which is where $s_i(t)$ are defined.

The three biases b_i are therefore: 1) $-1.39(kT_s/2) - 0.5$,
 2) $-1.39(kT_s/2) - 0.25$, and 3) $-0.69(kT_s/2) - 0.25$.

Note that the kT factor is so small that it can be neglected here compared to the one-volt signals.

- b) If we divide the symbol period into M samples, then each symbol $s_i(t)$ can be represented as a vector in M -dimensional space. Consider 3-space. In this case the three symbols are farthest apart if they lie in a plane spaced at $\sim 120^\circ$. The same is true for any M . If we consider a 2-D case, the signals might each have two halves or samples of value a_{i1} and a_{i2} . They might be defined as illustrated below, where the more probable symbol A might be positioned at the top, and thus are separated more than the two symbols B and C at the bottom.



Problem 8.3

- a) Figure 4.4-3 suggests that $P_e = 10^{-4}$ corresponds to $E/N_o \cong 9.5$ dB for coherent BPSK. Therefore $E = 10^{\text{dB}/10}$ kT [J/bit] and $P[\text{W}] = RE = E = 10^{0.95} 1.38 \times 10^{-23} \times 100 = 1.23 \times 10^{-20}$ watts.
- b) $E = 1.23 \times 10^{-20}$ [J/bit]; it is independent of the data rate.