

6.728 APPLIED QUANTUM AND STATISTICAL PHYSICS

QUIZ 2

November 13, 2006

PLEASE WRITE ALL ANSWERS IN THE BLANK BOOKLET AND NOT ON THE EXAM ITSELF. HOWEVER, PLEASE RETURN THE EXAM, THE BOOKLET, AND THE FORMULA SHEETS.

NAME:

Problem	Possible	Score
1	20	
2	20	
3	20	
4	40	
Total	100	

Problem 1 (20 Points)

In terms of the basis states $u_1(x)$ and $u_2(x)$, the two-level system has its Hamiltonian given by

$$\hat{H} = \begin{pmatrix} E_1 & -V \\ -V & E_2 \end{pmatrix}$$

where $E_2 > E_1$ and $V > 0$.

(a) Find the eigen wavefunctions $\phi_1(x, t)$ and $\phi_2(x, t)$. Write your answer explicitly in terms of $u_1(x)$ and $u_2(x)$ and be sure to include the full time dependence.

(b) If a general wavefunction of the system $\Psi(x, t)$ is initially in the state $\phi_1(x)$, what is the probability as a function of time that the system will be in the state described by $\phi_2(x)$?

Problem 2 (20 Points)

Consider a two-level system described by the Hamiltonian

$$\hat{H} = \begin{pmatrix} -\Delta & 0 \\ 0 & \Delta \end{pmatrix}$$

where $\Delta > 0$. The system is initially in the state

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

What is the probability, as a function of time, that the system will be in the state

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Problem 3 (20 Points)

A two-level system is described initially by the density matrix

$$\rho(t = 0) = \begin{pmatrix} \frac{1}{2} & \frac{-i}{2} \\ \frac{i}{2} & \frac{1}{2} \end{pmatrix}$$

and it evolves after a long time to

$$\rho(t \rightarrow \infty) = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

- (a) What is the initial wavefunction that $\rho(t = 0)$ describes?
- (b) What is the final wavefunction that $\rho(t \rightarrow \infty)$ describes?
- (c) How does the probability of finding the system in one of its two states change at these two times?
- (d) Describe what physically happens for the system to evolve from its initial density matrix to its final density matrix.

Problem 4 (40 Points)

Consider a particle of mass m confined to a box of length d in one dimension. Also consider a circuit that is a parallel combination of an inductor L and capacitance C , which has a characteristic frequency $\omega = 1/\sqrt{LC}$.

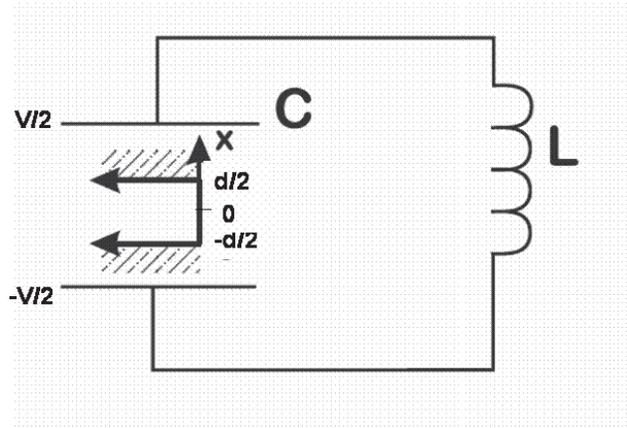


Figure 1: LC circuit and particle-in-a-box. Note that the particle-in-a-box is simply drawn 90° from the usual orientation.

For this uncoupled system, the lowest three energies are

$$\begin{aligned} E_1 &= \frac{\hbar^2 \pi^2}{2m d^2} + \frac{1}{2} \hbar \omega \\ E_2 &= \frac{\hbar^2 \pi^2}{2m d^2} + \frac{3}{2} \hbar \omega \\ E_3 &= \frac{\hbar^2 4\pi^2}{2m d^2} + \frac{1}{2} \hbar \omega \end{aligned}$$

(a) Find the three time-independent eigen functions $\psi_1(x, v)$, $\psi_2(x, v)$ and $\psi_3(x, v)$ which corresponds to the above energies. NOTE: v in this problem is the voltage across the capacitor.

(b) If the initial state of the system is

$$\Psi(x, v, t = 0) = \frac{1}{\sqrt{2}} (\psi_1(x, v) + \psi_2(x, v)),$$

find $\langle x(t) \rangle$ and $\langle v(t) \rangle$. Again, v is the voltage across the capacitor.

The problem continues on the next page

(c) Now consider a coupling term between the two systems,

$$\hat{H}_{\text{int}} = -\frac{e}{d}\hat{v}\hat{x}.$$

The Hamiltonian Matrix using the above three basis states has the following form

$$\hat{H} = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & -A \\ 0 & -A & E_3 \end{pmatrix}$$

where the constant $A > 0$. Estimate the new eigen energies for the ground state, and the first and second excited states, assuming that only the three states in part (a) are needed in the finite basis set expansion. (*Note: You do NOT need to evaluate the constant A.*)