Computing the Inverse Fourier Transform

As when we computed the Fourier transform of Ψ, we created a matrix:

$$\vec{\phi}_{\vec{x}}(\vec{q}) = \begin{pmatrix} e^{ix_1q_1} & e^{ix_1q_2} & \cdots & e^{ix_1q_m} \\ e^{ix_2q_1} & e^{ix_2q_2} & \cdots & e^{ix_2q_m} \\ \vdots & \vdots & \ddots & \vdots \\ e^{ix_nq_1} & e^{ix_nq_2} & \cdots & e^{ix_nq_m} \end{pmatrix}$$

so that

$$\Psi_t(x) = \int_{-\infty}^{\infty} e^{iqx} A_t(q)\, \frac{dq}{2\pi}$$

where

$$A_t(q) = A(q)e^{-iEt/\hbar}$$

Then, our wave function evaluated at time **t** is approximated discretely by

$$\Psi_t(x) = \sum_{\{q\}} \vec{\phi}(q)\vec{A}_t(q)\, \frac{\delta q}{2\pi} = \vec{\phi}(q) * \vec{A}_t\, \frac{\delta q}{2\pi}$$

$$= \begin{pmatrix} e^{iq_1x_1} & e^{iq_2x_1} & \cdots & e^{iq_mx_1} \\ e^{iq_1x_2} & e^{iq_2x_2} & \cdots & e^{iq_mx_2} \\ \vdots & \vdots & \ddots & \vdots \\ e^{iq_1x_n} & e^{iq_2x_n} & \cdots & e^{iq_mx_n} \end{pmatrix} * \begin{pmatrix} A_t(q_1) \\ A_t(q_2) \\ \vdots \\ A_t(q_m) \end{pmatrix} \frac{\delta q}{2\pi}$$

$$= \begin{pmatrix} A_t(q_1)e^{iq_1x_1} + A_t(q_2)e^{iq_2x_1} + \cdots + A_t(q_m)e^{iq_mx_1} \\ A_t(q_1)e^{iq_1x_2} + A_t(q_2)e^{iq_2x_2} + \cdots + A_t(q_m)e^{iq_mx_2} \\ A_t(q_1)e^{iq_1x_3} + A_t(q_2)e^{iq_2x_3} + \cdots + A_t(q_m)e^{iq_mx_3} \\ \vdots \\ A_t(q_1)e^{iq_1x_n} + A_t(q_2)e^{iq_2x_n} + \cdots + A_t(q_m)e^{iq_mx_n} \end{pmatrix} \frac{\delta q}{2\pi}$$

which is just

$$\Psi_t(x) = \begin{pmatrix} \Psi_t(x_1) \\ \Psi_t(x_2) \\ \vdots \\ \Psi_t(x_n) \end{pmatrix}$$