Electronics A

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Outline

- Elements of circuit analysis ⟵ **TODAY**
- Elements of semiconductor physics
  - Semiconductor diodes and resistors
  - The MOSFET and the MOSFET inverter/amplifier
- Op-amps

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Elements of circuit analysis

- **There are many ways to analyze circuits**
- **Here we'll go over a few of them**
  - **Elements laws, connection laws and KVL/KCL**
  - **Nodal analysis**
  - **Intuitive approaches**
  - **Superposition**

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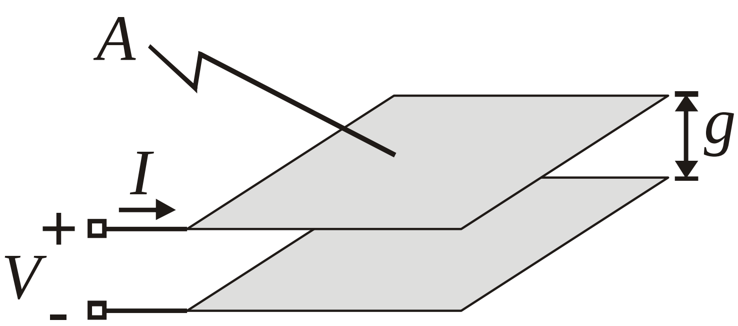
Lumped elements in circuits

> **Circuit elements (R, L, C) are lumped <u>approximations</u> of complex devices**

> **The electrical capacitor**

- **What is the relation between Q and V?**



$$\oint_{\substack{closed\\surface}} \varepsilon \mathbf{E} \cdot d\mathbf{a} = Q$$

$$\varepsilon E A = Q$$

$$\nabla \times \mathbf{E} = 0 \quad \Rightarrow \quad \mathbf{E}(\mathbf{r},t) = -\nabla V(\mathbf{r},t)$$

$$V(b) - V(a) = -\int_a^b \mathbf{E} \cdot d\mathbf{l}$$

$$V(b) - V(a) = V = Eg \quad \Rightarrow \quad E = V/g$$

$$Q = \varepsilon A V/g = \frac{\varepsilon A}{g} V = CV$$

$$C = \frac{\varepsilon A}{g}$$

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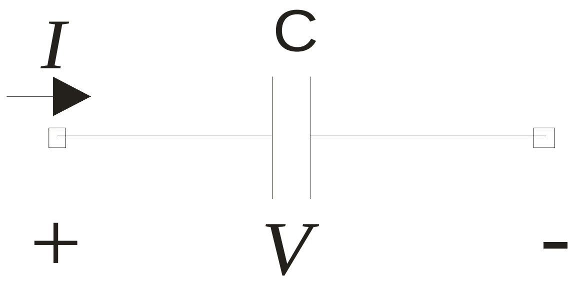
Lumped elements in circuits

> The electrical capacitor

- **We can replace all of field theory with terminal relations**
- **And introduce an element with an element law**
- **As long as capacitor size << wavelength of electrical signal**
  - » **In general, MEMS are small**

    e.g., λ=50 μm ➔ 600 GHz



$$Q = CV$$

$$I = \frac{dQ}{dt} = \frac{d}{dt}(CV)$$

$$I = C\frac{dV}{dt}$$

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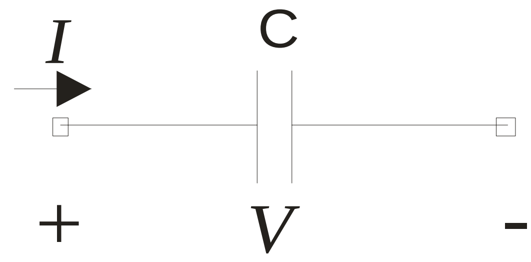
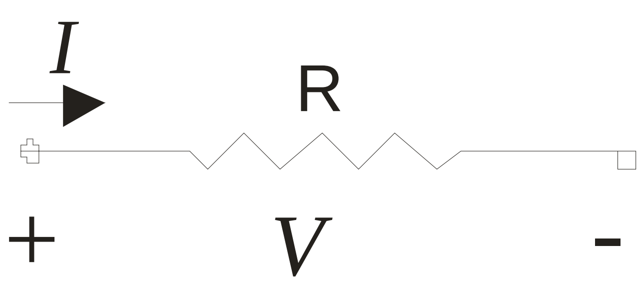
Elements and element laws

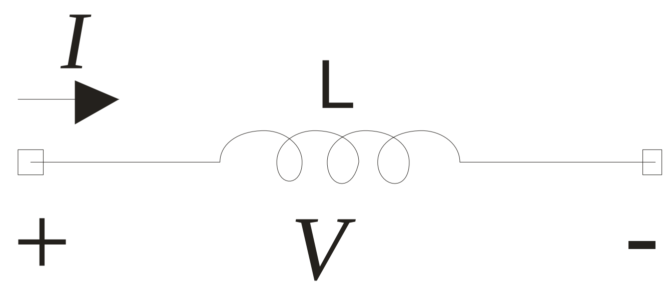
- Do this with all three basic elements
- Resistor
- Capacitor
- Inductor



$$I = C\frac{dV}{dt}$$



$$V = RI$$



$$V = L\frac{dI}{dt}$$

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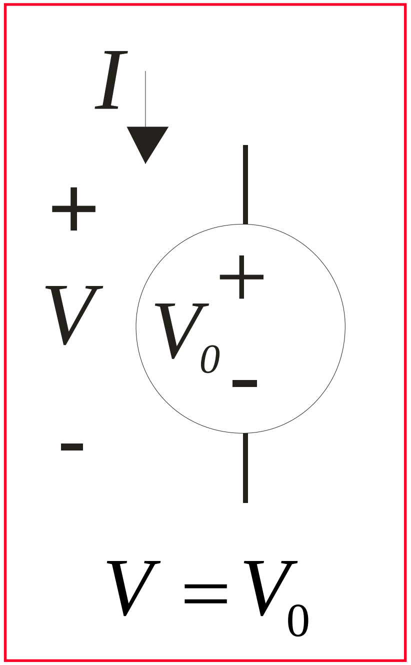
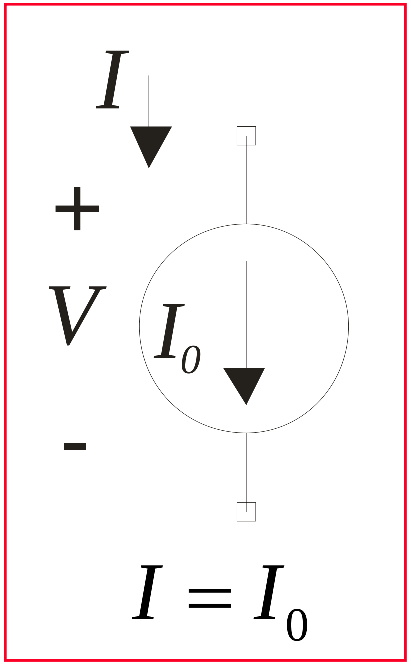
Source elements

- > **We need elements to provide energy into the circuit**
- > **Two common ones are voltage source and current source**





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KVL and KCL

- These are continuity laws that allow us to solve circuits
- Kirchhoff's voltage law
  - ***The oriented sum of voltages around a loop is zero***
- Kirchhoff's voltage law
  - ***The sum of currents entering a node is zero***

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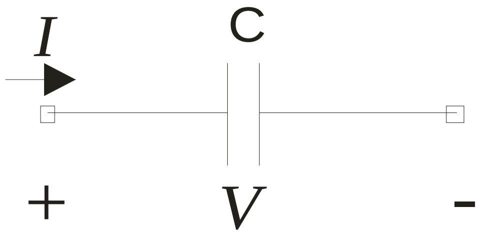
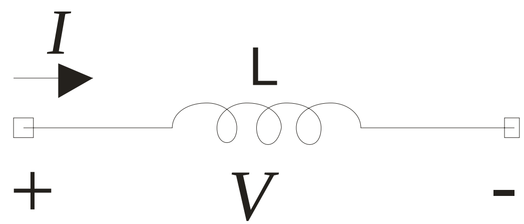
Complex impedances

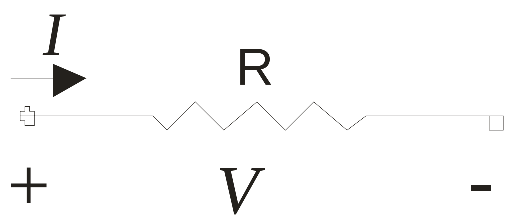
- **For LTI systems, use complex impedances instead**
  - **Implicitly working in frequency domain**
- **Much easier circuit analysis**
- **All elements treated the same, like "resistors"**



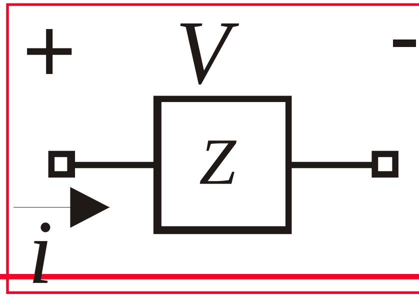
$$V = IZ_C = I\frac{1}{Cs}$$



$$V = IZ_L = ILs$$



$$V = IZ_R = IR$$



$$V(s) = I(s)Z(s)$$

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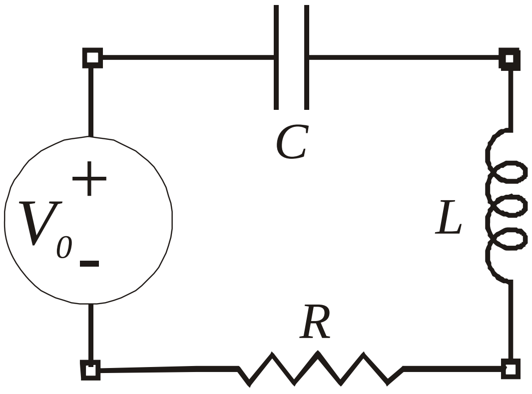
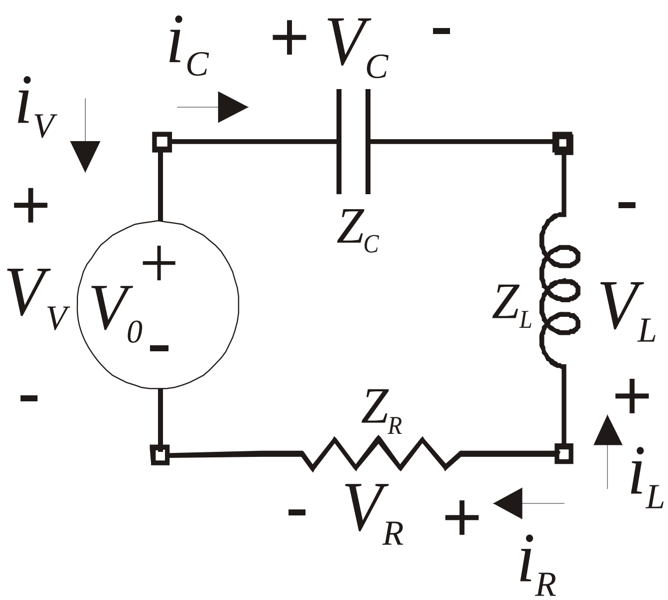
Let's analyze a circuit

1. Figure out what are you trying to determine
2. Replace elements with complex impedances
3. Assign across and through variables
4. Use KVL
5. Substitute in element laws
6. Solve





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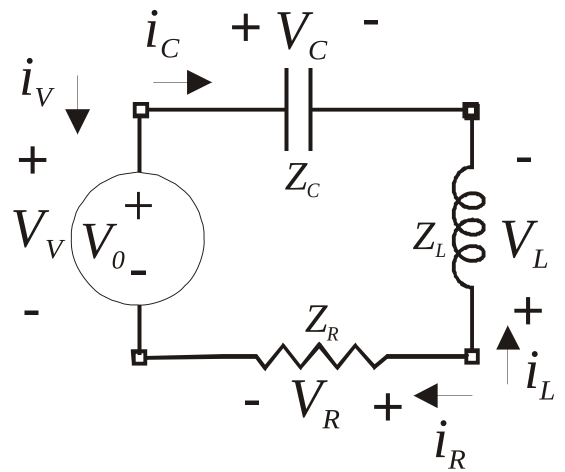

$$V_V - V_C + V_L - V_R = 0$$

$$V_0 - i_C Z_C + i_L Z_L - i_R Z_R = 0$$

$$i_C = -i_L = i_R$$

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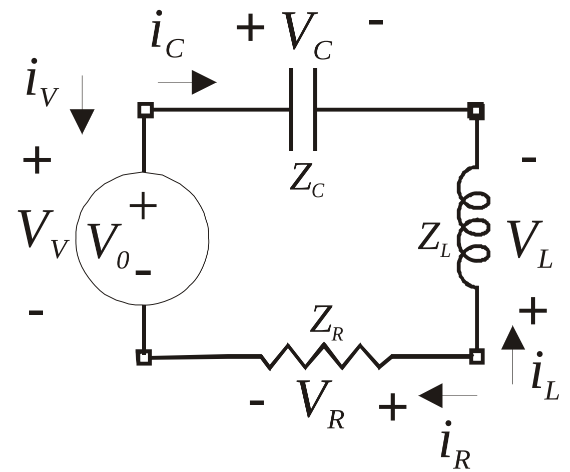

$$V_0 - i_R Z_C - i_R Z_L - i_R Z_R = 0$$

$$i_R = \frac{V_0}{Z_C + Z_L + Z_R} = \frac{V_0}{1/Cs + Ls + R}$$

$$i_R = \frac{Cs}{LCs^2 + RCs + 1} V_0$$

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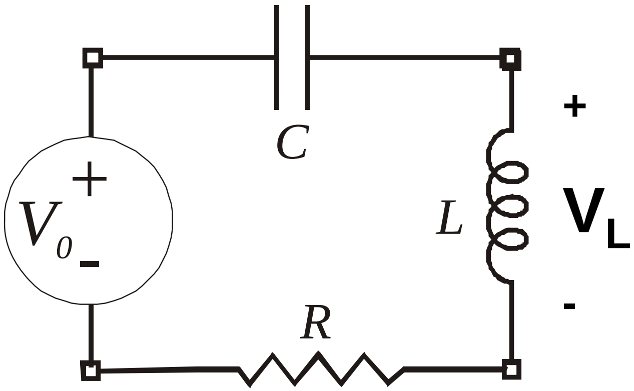
Example #1

- Solve for V_L



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Nodal analysis

- Element law approach becomes tedious for circuits with multiple loops
- Nodal analysis is a KCL-based approach

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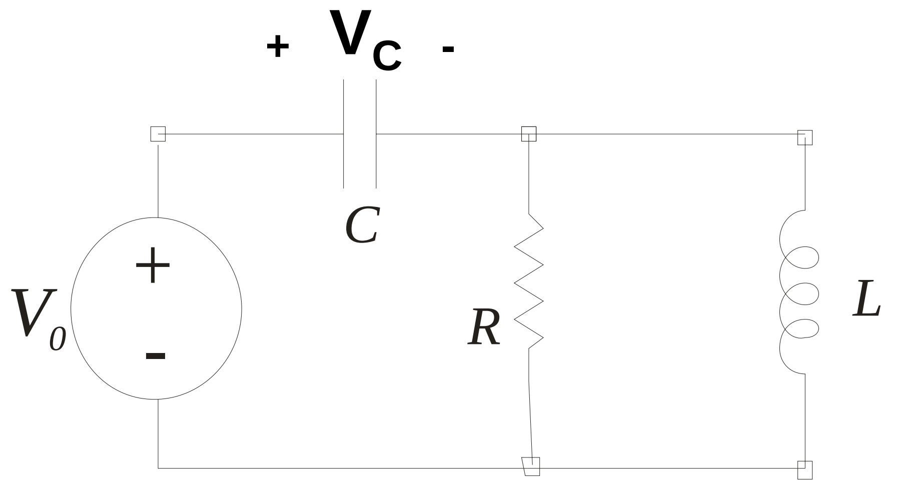
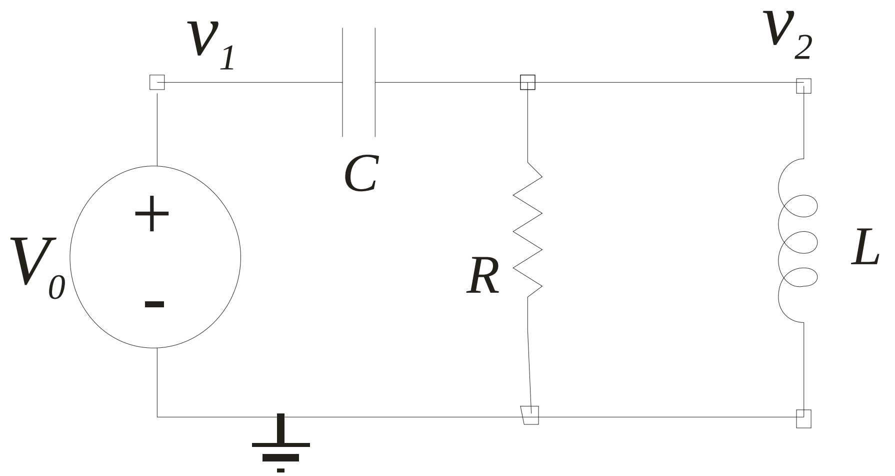
Nodal analysis

1. Figure out what are you trying to determine
2. Replace elements with complex impedances
3. Assign node voltages & ground node
4. Write KCL at each node
5. Solve for node voltages
6. Use node voltages to find what you care about





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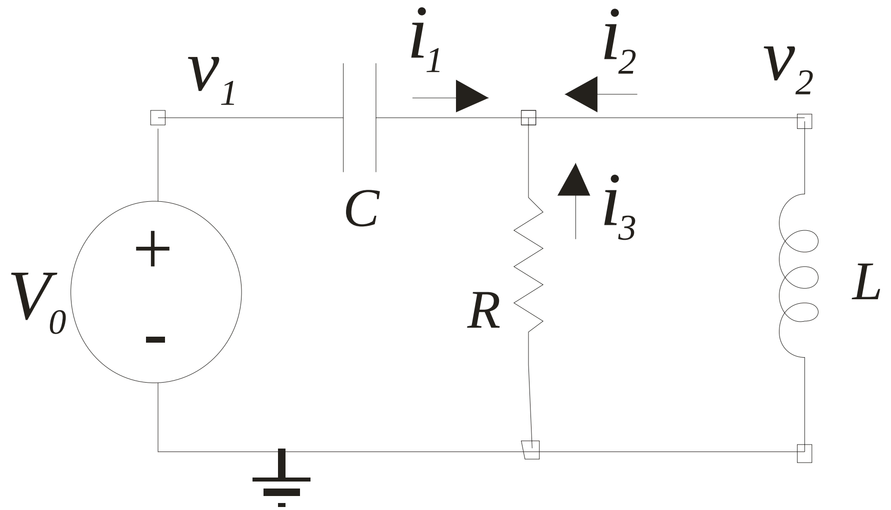

Node 1: $v_1 = V_0$

Node 2: $i_1 + i_2 + i_3 = 0$

$$\frac{v_1 - v_2}{Z_C} + \frac{0 - v_2}{Z_L} + \frac{0 - v_2}{Z_R} = 0$$

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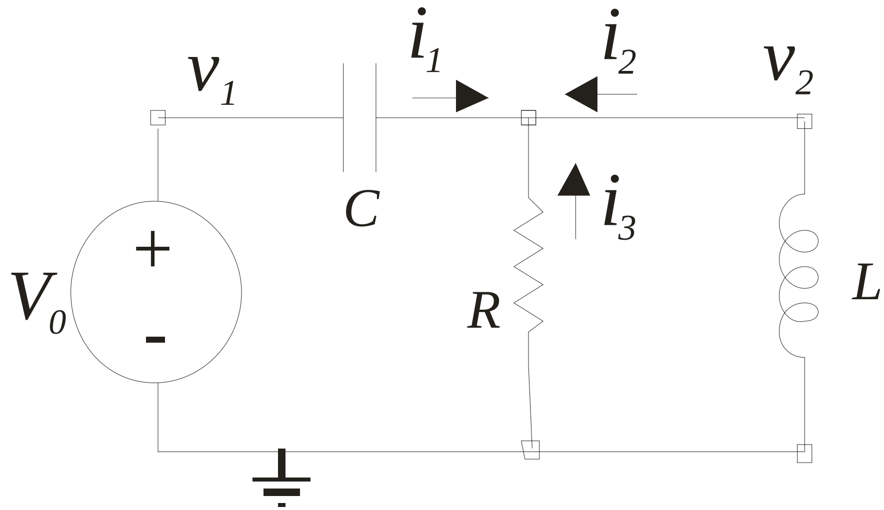

$$v_2\left(\frac{1}{Z_C}+\frac{1}{Z_L}+\frac{1}{Z_R}\right)=\frac{V_0}{Z_C}$$

$$v_2\left(Z_LZ_R+Z_CZ_R+Z_LZ_C\right)=V_0Z_LZ_R$$

$$v_2=V_0\frac{Z_LZ_R}{Z_LZ_R+Z_CZ_R+Z_LZ_C}$$

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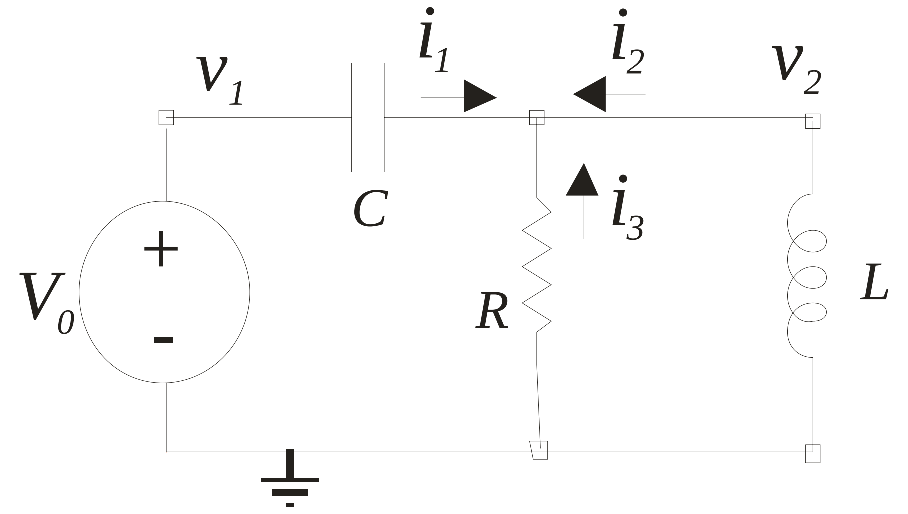

$$v_2 = V_0 \frac{LRs}{LRs + {}^{1}\!/_{Cs} R + Ls\, {}^{1}\!/_{Cs}}$$

$$v_2 = V_0 \frac{LRCs^2}{LRCs^2 + Ls + R}$$

$$V_C = v_1 - v_2 = V_0 - V_0 \frac{LRCs^2}{LRCs^2 + Ls + R}$$

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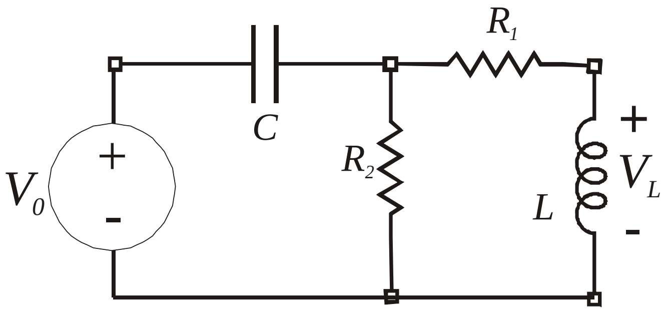
Example #2



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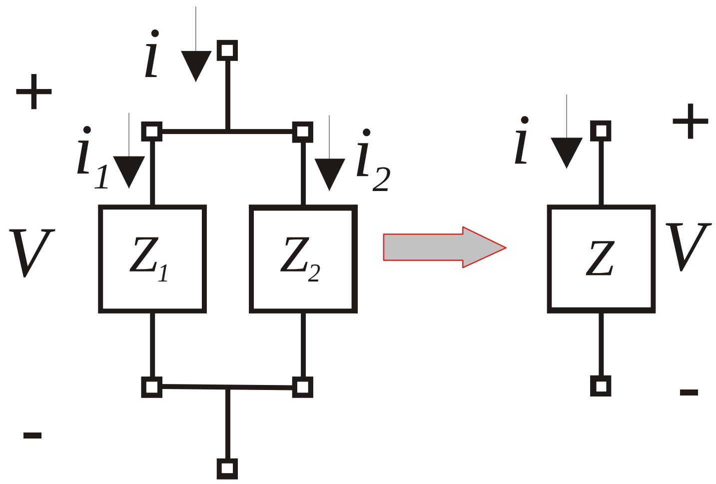
Intuitive methods

- Instead of "solving" the circuit using equations, use series/parallel tricks to analyze the circuit by inspection
- Current divider & impedances in parallel
  - Both elements have SAME voltage
  - Terminals connected together

$$i_1 = i\frac{Z_2}{Z_1 + Z_2}$$

$$i_2 = i\frac{Z_1}{Z_1 + Z_2}$$



$$V = i_1 Z_1 = i\frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = Z_1 // Z_2$$

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

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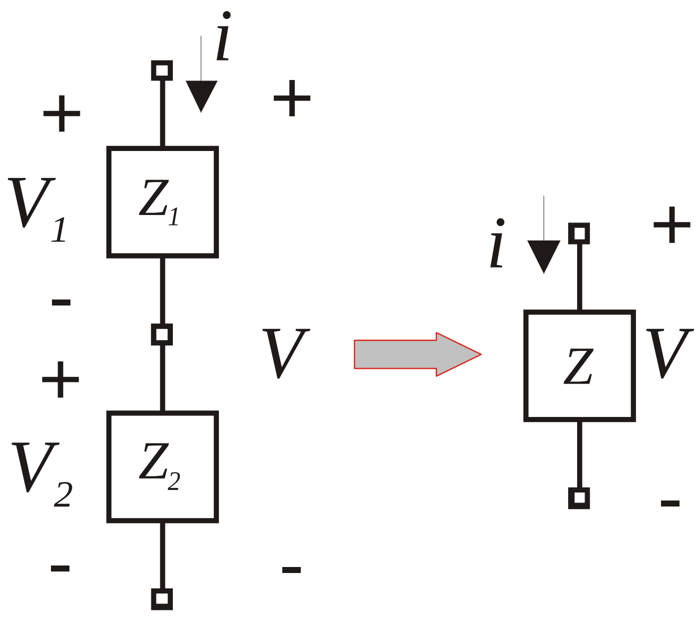
Intuitive methods

> **Voltage divider & impedances in series**

- **Both elements have SAME current**

$$V_1 = V\frac{Z_1}{Z_1 + Z_2}$$

$$V_2 = V\frac{Z_2}{Z_1 + Z_2}$$



$$i_1 = \frac{V_1}{Z_1} = \frac{V}{Z_1 + Z_2} = i$$

$$Z = Z_1 + Z_2$$

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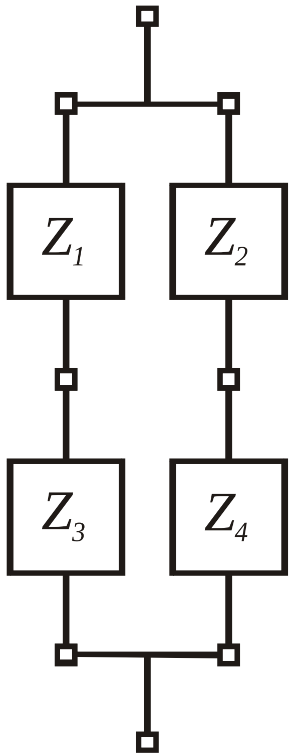
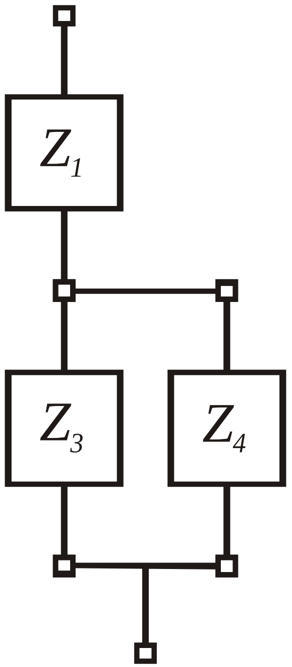
Intuitive methods

- Examples of elements NOT in series OR parallel



Z_1 and Z_3 in series
Z_2 and Z_4 in series
Z_1 and Z_2 NOT in parallel
Z_3 and Z_4 NOT in parallel



Z_3 and Z_4 in parallel
Z_1 and Z_3 NOT in series

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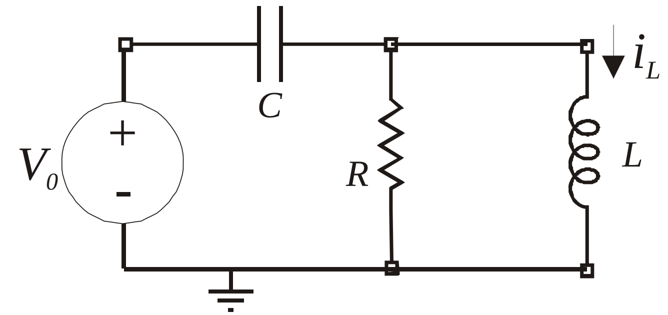
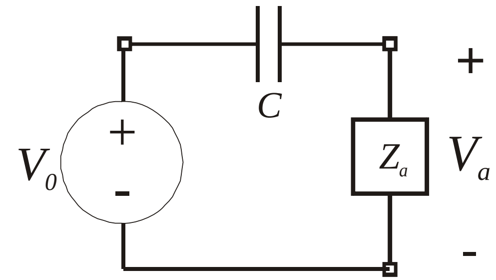
Intuitive methods

> **Let's use this approach to solve a circuit**



1. **Figure out what are you trying to determine**
2. **Replace elements with complex impedances**
3. **Collapse circuit in terms of series/parallel relations till circuit is trivial**
4. **Re-expand to find signal of interest**



$$Z_a = Z_R \,//\, Z_L$$

$$V_a = V_0 \frac{Z_a}{Z_a + Z_C}$$

$$i_L = \frac{V_a}{Z_L}$$

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Intuitive methods

> Let's use this approach to solve a circuit

1. Figure out what are you trying to determine
2. Replace elements with complex impedances
3. Collapse circuit in terms of series/parallel relations till circuit is trivial
4. Re-expand to find signal of interest

$$i_L = V_0 \frac{Z_a}{Z_a + Z_C} \frac{1}{Z_L} = V_0 \frac{Z_R // Z_L}{Z_R // Z_L + Z_C} \frac{1}{Z_L}$$

$$= V_0 \frac{\dfrac{Z_R Z_L}{Z_R + Z_L}}{\dfrac{Z_R Z_L}{Z_R + Z_L} + Z_C} \frac{1}{Z_L}$$

$$= V_0 \frac{Z_R Z_L}{Z_R Z_L + \left(Z_R + Z_L\right) Z_C} \frac{1}{Z_L}$$

$$= V_0 \frac{RLs}{RLs + \left(R + Ls\right) \frac{1}{Cs}} \frac{1}{Ls}$$

$$i_L = V_0 \frac{RCs}{RLCs^2 + Ls + R}$$

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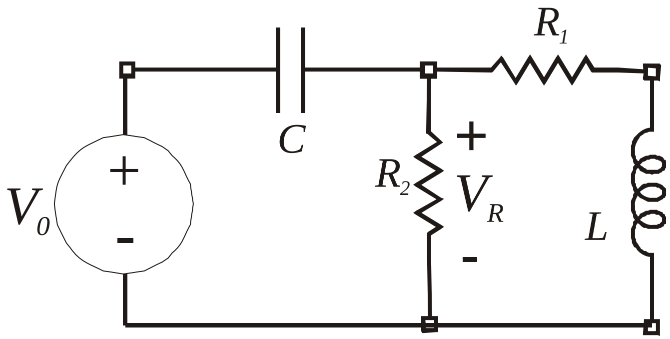
Example #3



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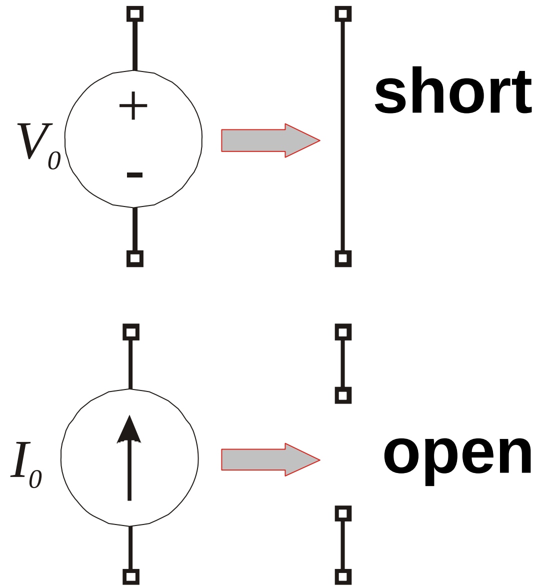
Superposition

> **These equivalent circuits are linear and obey the principles of superposition**
- **This can be useful**

> **For circuits with multiple sources,**
- **Turn off all independent sources except one**
- **Solve circuit**
- **Repeat for all sources, then add responses**

> **Turning off a voltage source gives a short circuit**

> **Turning off a current source gives an open circuit**



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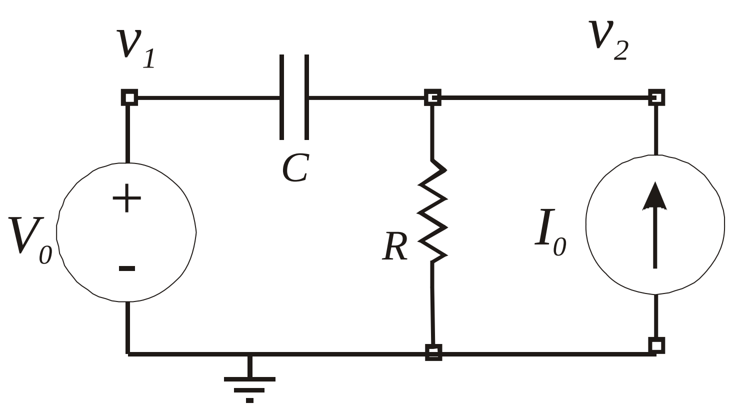
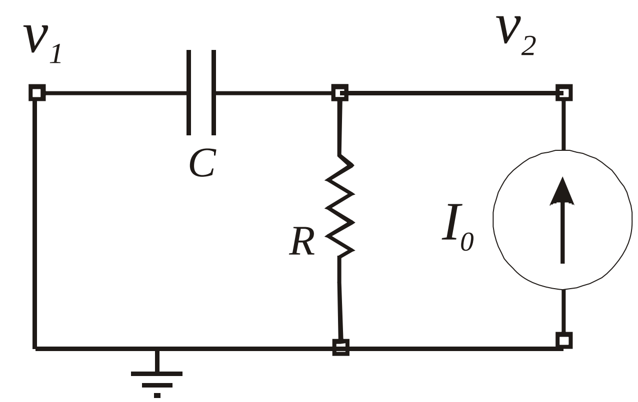
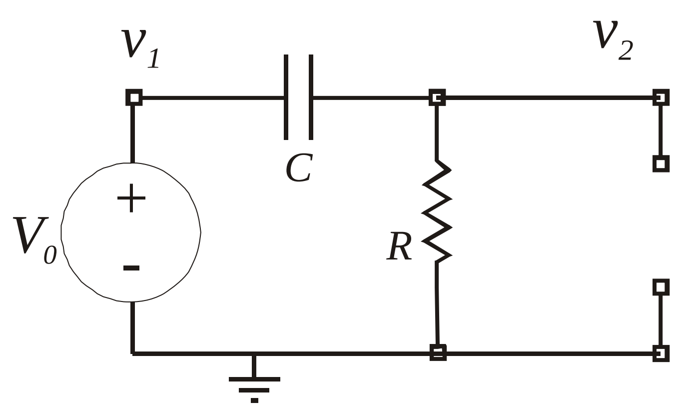
  - Solve circuit
  - Repeat for all sources, then add responses



Find v_2



$$v_2 = I_0 Z_R // Z_C$$



$$v_2 = V_0 \frac{Z_R}{Z_R + Z_C}$$

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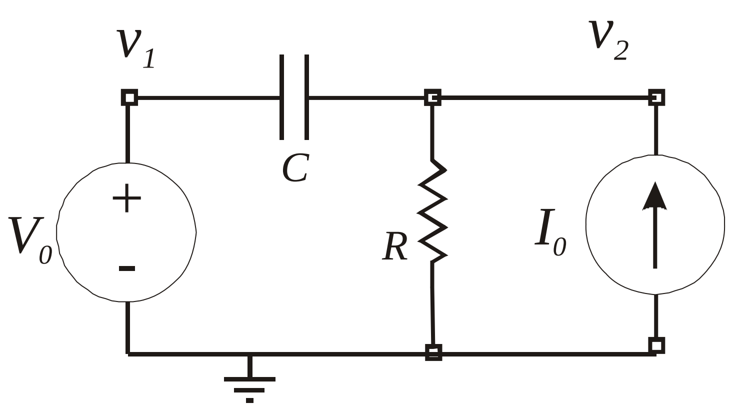
Superposition

> **For circuits with multiple sources,**
> - **Turn off all independent sources except one**
> - **Solve circuit**
> - **Repeat for all sources, then add responses**



Find v_2

$$v_2 = I_0 Z_R // Z_C + V_0 \frac{Z_R}{Z_R + Z_C}$$

$$v_2 = I_0 \frac{Z_R Z_C}{Z_R + Z_C} + V_0 \frac{Z_R}{Z_R + Z_C}$$

$$v_2 = \frac{I_0 R \, {}^{1}\!/_{Cs} + V_0 R}{R + {}^{1}\!/_{Cs}}$$

$$\boxed{v_2 = \frac{I_0 R + V_0 RCs}{RCs + 1}}$$

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Conclusions

- There are many ways to analyze equivalent circuits
- Use the simplest method at hand
- Element laws & connection laws are OK for simple ckts
- Nodal analysis works for most any circuit, but will be tedious for complicated circuits
- Try to use intuitive approaches whenever possible

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