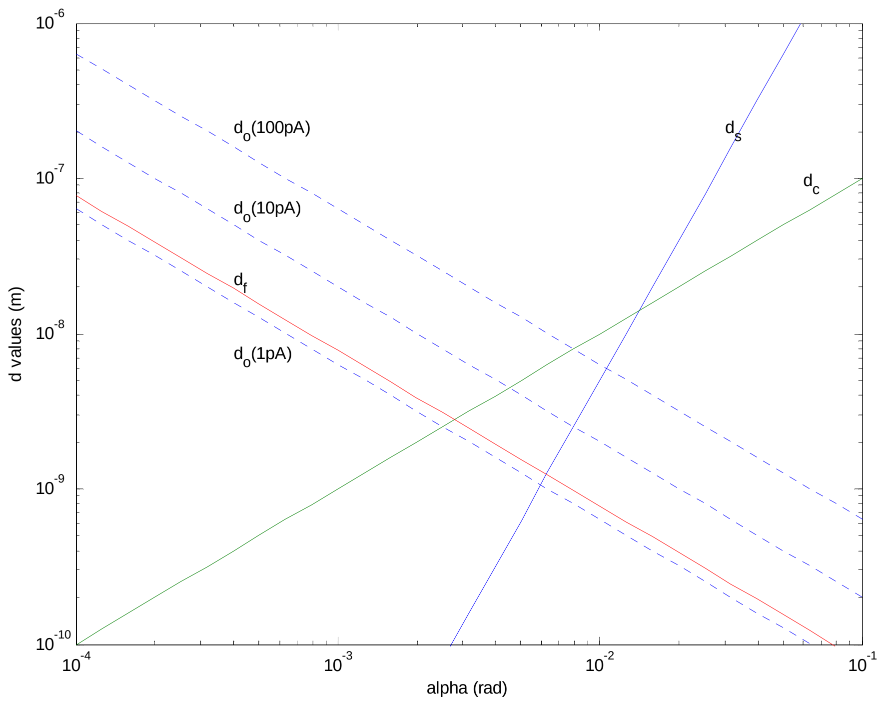
Homework #5 Solution

Problem #18

Maximum meaningful magnification $M = \dfrac{10cm/1000}{50nm} = 2000.$

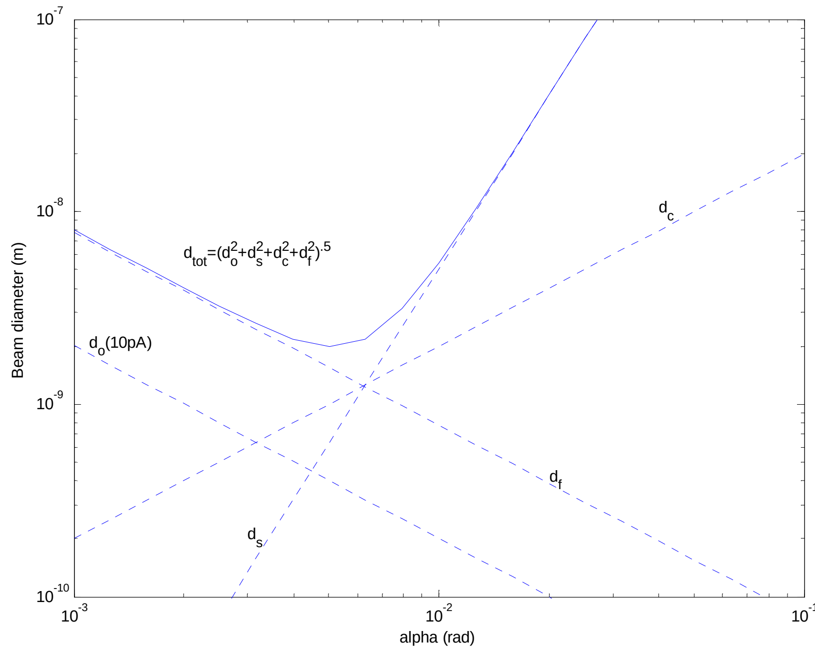
Problem #9

$$d_o = \left(\frac{4i}{\pi^2 B}\right)^{1/2} \alpha^{-1},\ d_s = \frac{1}{2} C_s \alpha^3,\ d_c = C_c \alpha \left(\frac{\Delta E}{E}\right),\ d_f = \frac{\lambda}{\alpha},\ \text{and}\ \lambda = \frac{h}{\sqrt{2mE}} = 7.76\,pm.$$



Problem #20

The formulas are still the same.



$d_{min} \approx 2nm, \alpha \approx 5\times10^{-3} rad$.

Problem #21

Without loss of generality, we can assume

$$h(x) = \frac{1}{\sigma_1\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma_1^2}}$$

$$g(x) = \frac{1}{\sigma_2\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma_2^2}}$$

Then,

$$h \otimes g = \int_{-\infty}^{\infty} h(y-x)g(x)dx$$

$$= \frac{1}{2\pi\sigma_1\sigma_2} \int_{-\infty}^{\infty} e^{\frac{-(y-x)^2}{2\sigma_1^2}} e^{\frac{-x^2}{2\sigma_2^2}} dx$$

$$= \frac{1}{2\pi\sigma_1\sigma_2} e^{\frac{-y^2}{2\sigma_1^2}} \int_{-\infty}^{\infty} e^{-\left[x^2\left(\frac{1}{2\sigma_1^2}+\frac{1}{2\sigma_2^2}\right)-\frac{2yx}{2\sigma_1^2}\right]} dx$$

$$= \frac{1}{2\pi\sigma_1\sigma_2} e^{\frac{-y^2}{2\sigma_1^2}} e^{\frac{y^2\sigma_2^2}{2\sigma_1^2(\sigma_1^2+\sigma_2^2)}} \int_{-\infty}^{\infty} e^{-\left[x^2\left(\frac{1}{2\sigma_1^2}+\frac{1}{2\sigma_2^2}\right)-\frac{2yx}{2\sigma_1^2}\right]} dx$$

$$= \frac{1}{2\pi\sigma_1\sigma_2} e^{\frac{-y^2}{2\sigma_1^2}} e^{\frac{y^2\sigma_2^2}{2\sigma_1^2(\sigma_1^2+\sigma_2^2)}} \int_{-\infty}^{\infty} e^{-\left(\frac{\sigma_1^2+\sigma_2^2}{2\sigma_1^2\sigma_2^2}\right)\left(x-\frac{y^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}\right)^2} dx$$

$$= \frac{1}{2\pi\sigma_1\sigma_2} e^{\frac{-y^2}{2\sigma_1^2}} e^{\frac{y^2\sigma_2^2}{2\sigma_1^2(\sigma_1^2+\sigma_2^2)}} \int_{-\infty}^{\infty} e^{-\left(\frac{\sigma_1^2+\sigma_2^2}{2\sigma_1^2\sigma_2^2}\right)\left(x-\frac{y^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}\right)^2} dx$$

$$= \frac{1}{2\pi\sigma_1\sigma_2} e^{\frac{-y^2}{2(\sigma_1^2+\sigma_2^2)}} \left(\frac{2\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}\right)^{\frac{1}{2}} \int_{-\infty}^{\infty} e^{-\tau^2} d\tau$$

$$= \frac{1}{(\sigma_1^2+\sigma_2^2)\sqrt{2\pi}} e^{\frac{-y^2}{2(\sigma_1^2+\sigma_2^2)}}$$

Of course, you can use moment generating function (one kind of Fourier transform) to prove it. The second method is a lot easier.

Problem #22

a) $t_p = \frac{1/30\,\mathrm{s}}{1000^2} = 33.3ns$

b) For low-energy electrons, $v = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{2\times 10eV \times 1.6\times 10^{-19}\ J/eV}{9.1\times 10^{-31}kg}} = 1.9\times 10^6\ m/s$.

Then, $t_t = \frac{d}{v} = 16ns$.

c) Let $t_t = t_p$, $v = \frac{d}{t_t} = \frac{3cm}{33.3ns} = 9\times 10^5\ m/s$. This corresponds to $E = \frac{1}{2}mv^2 = 2.3eV$.