6.781 Homework #2 Solution

Problem 3

a) $$s_o = \left(\frac{1}{f} - \frac{1}{s_i}\right)^{-1} = \left(\frac{1}{0.01cm} - \frac{1}{1cm}\right)^{-1} = (100-1)^{-1}\, cm = 0.0101cm = 101\mu m$$

$$M_1 = -\frac{s_i}{s_o} = -\frac{1cm}{0.0101cm} = -99$$

b) $$M_{FINAL} = M_1 \cdot M_2 = 1000 \Rightarrow M_2 = \frac{1000}{-99} = -10.1 - \frac{s_{i_2}}{s_{o_2}}$$

Assume that the second lens has the same focal length as the first one. Then:

$$\frac{1}{100\mu m} = \frac{1}{s_{o_2}} + \frac{1}{s_{i_2}} = \frac{1}{s_{o_2}} + \frac{1}{10.1 s_{o_2}} \Rightarrow s_{o_2} = 100\,\mu m\left[1 + \frac{1}{10.1}\right] = 110\mu m$$

$$s_{i_2} = 10.1 s_{o_2} = 1.11mm$$

Problem 4

A periodic function with period p can be expanded into Fourier series as follows:

$$f(x) = \frac{A_0}{2} + \sum_{m=1}^{\infty}\left[A_m \cos(mk) + B_m \sin(mk)\right]$$

$$A_m = \frac{2}{p}\int_{-p/2}^{p/2} T(x)\cos(kmx)dx$$

$$B_m = \frac{2}{p}\int_{-p/2}^{p/2} T(x)\sin(kmx)dx$$

$$\text{with } k = \frac{2\pi}{p}$$

a.

$$A_m = \frac{2}{p}\int_{-p/2}^{p/2} T(x)\cos(kmx)dx = \frac{2}{p}\int_{-p/a}^{p/a} T(x)\cos(kmx)dx$$

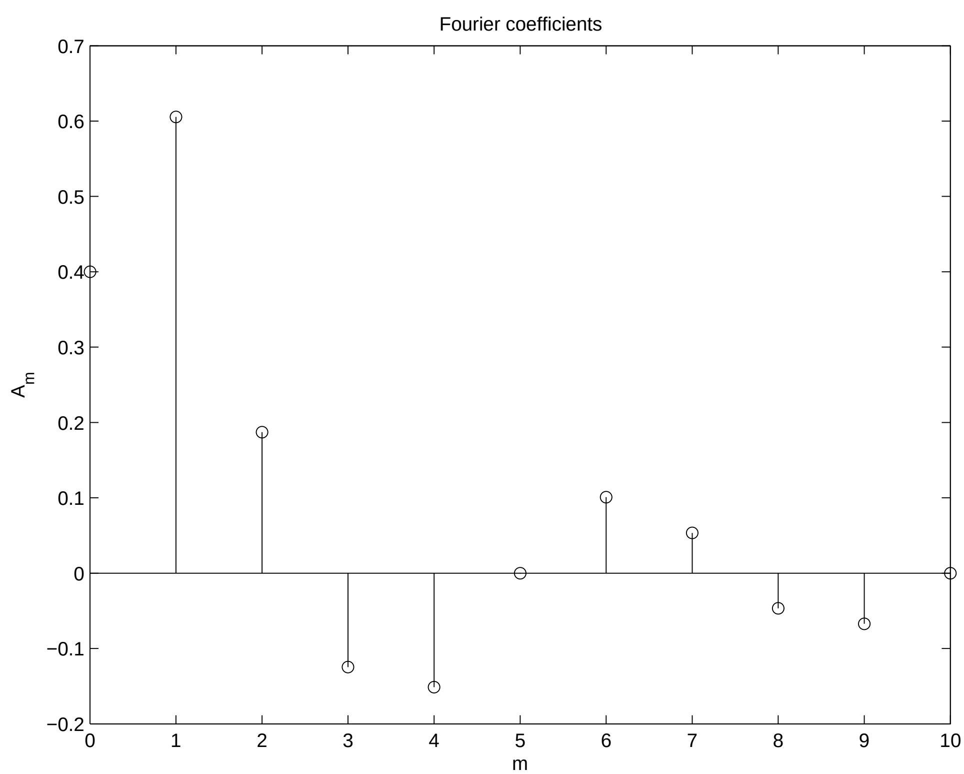
$$= \frac{2}{p}\frac{p}{2\pi m}\left(\sin\left(\frac{2\pi}{p}mx\right)\right)\Bigg|_{-p/a}^{p/a} = \frac{4}{a}\operatorname{sinc}\left(\frac{2\pi m}{a}\right)$$

$$B_m = \frac{2}{p}\int_{-p/2}^{p/2} T(x)\sin(kmx)dx = \frac{2}{p}\int_{-p/a}^{p/a} T(x)\sin(kmx)dx$$

$$= \frac{2}{p}\frac{p}{2\pi m}\left(-\cos\left(\frac{2\pi}{p}mx\right)\right)\Bigg|_{-p/a}^{p/a} = 0$$

b. $$T(x) = \frac{4}{a}\left[\frac{1}{2} + \operatorname{sinc}\left(\frac{2\pi}{a}\right)\cos(kx) + \operatorname{sinc}\left(\frac{4\pi}{a}\right)\cos(2kx)\operatorname{sinc}\left(\frac{6\pi}{a}\right)\cos(3kx)\right]$$

c. The Fourier coefficients can be in different forms depending on the base functions used. The base functions can be sinusoidal functions, or complex exponentials.



For the base functions we choose for this problem, $m \geq 0$.

Problem 5

$E(x) = E_o\left[1+\cos(k_o x)\right] = E_o\left[1+\frac{1}{2}e^{jk_o x}+\frac{1}{2}e^{-jk_o x}\right]$ where $k_o = \frac{2\pi}{p}$. A plane wave leaving the surface of the object splits into three components: one traveling perpendicular to the object's surface and two waves with lateral components $e^{\pm jkx}$. Therefore, a Fraunhofer diffraction pattern formed by these three 'beams' on a distant screen is simply three spots.

The Fourier transform of $E(x)$ is

$$\begin{aligned} F(k) &= \int_{-\infty}^{\infty} E(x)e^{jkx}dx \\ &= E\left[\int_{-\infty}^{\infty} e^{jkx}dx + \frac{1}{2}\int_{-\infty}^{\infty} e^{jk_o x}e^{jkx}dx + \frac{1}{2}\int_{-\infty}^{\infty} e^{-jk_o x}e^{jkx}dx\right] \\ &= E\left[2\pi\delta(k) + \pi\delta(k+k_o) + \pi\delta(k-k_o)\right] \end{aligned}$$

which is the same three spots as in the Fraunhofer diffraction pattern.

Problem 6

$\mathrm{NA} = \frac{1}{2f_\#} \Rightarrow p_{min} = \frac{\lambda}{2\mathrm{NA}} = f_\#\lambda$. For $\lambda = 500nm$, $p_{min} = 500nm \times 64 = 32\mu m$. Then 32mm film is needed to record 1000 slats, and 32cm is needed for 10000 slats.

Problem 7

$\mathrm{NA} \approx \sin\alpha \approx \frac{4m/2}{100km} = 2\times 10^{-5}$. For $\lambda = 500nm$, $p_{min} = \frac{\lambda}{2\mathrm{NA}} = 1.25cm$.

Problem 8

The diffraction pattern like this is produced by an orthogonal grid with different spatial periods and duty cycles along grid's axes. Both periods can be determined from the constructive interference conditions for the first diffraction order with $p = \frac{\lambda}{\sin\alpha}$. Then $p_1 \approx \frac{632.8nm}{0.1m/10m} = 63\mu m$, and $p_2 \approx \frac{632.8nm}{1m/10m} = 6.3\mu m.$

To determine the duty cycles, we need to plot out the intensity distribution. From the answer for problem 3 and the class handout on Fourier transforms, we realize that the distribution function is a sinc function with zero crossings at $m = 2, 4, 6, \ldots$. This suggests a 50% duty cycle in the x direction. The intensity in the y direction is a constant, which indicates that the duty cycle in that direction is very small, or it is periodic slits.