HW #7 Solutions

#26

Let's split interval Δt into N bins where N is very large so that each bin can have at most one electron. Then the probability that an electron is collected by a bin is $p = \overline{n}/N$. The standard deviation is

$$\begin{aligned}
\sigma^2 &= \sum_{all-bins} \sigma_{bin}^2 \\
&= N\sigma_{bin}^2 \\
&= N\left(E\left[n_{bin}{}^2\right] - E\left[\overline{n}_{bin}\right]^2\right) \\
&= N\left[p - p^2\right] \\
&= Np(1-p) \\
&\approx Np \\
&= N\frac{\overline{n}}{N} \\
&= \overline{n}
\end{aligned}$$

#27

For $I = 10^{-11} A$, $\tau = 1\mu s$, $\overline{n} = (I\tau/e) = (10^{-11}\, C/s \times 10^{-6}\, s)/(1.6\times 10^{-19} C) = 62.5$ electrons.
Although 62 is not large, it is only the number of electrons over one time interval, and we are interested in the distribution of the electrons over many time intervals. Over large number of time intervals, the Poisson distribution can be approximated by a Gaussian distribution:

$$p(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n-\overline{n})^2}{2\sigma^2}}$$

$$P(n \le \overline{n} - 10) = \frac{1}{\sqrt{2\pi 62.5}} \int_0^{52.5} e^{-\frac{(n-62.5)^2}{2\times 62.5}} = .103$$

#28

a) For G=5, N=500, T=100sec

From Lecture 5: $d_{min} = 0.7C_s{}^{1/4}B^{-3/8}i_{beam}{}^{3/8}$.

From Lecture 7: $i_{beam} = (G^2K^2eN^2/T)\times F$, where $F = \left(\delta_B + \delta_A/2\right)/\delta_A^2$.

Given: $\overline{n}_{SW} = 0.02n_i = (\delta_A + \delta_B)n_i \Rightarrow \delta_A + \delta_B = 0.02$

From the graph, the black level is $\overline{n}_{SB} = \delta_B\overline{n}_i = 20$, counting from the bottom of the hole.

We can also see from the graph that $\delta_A\overline{n}_i = 5$.

Therefore, $(\delta_A / \delta_B) = 20/5 = 4 \Rightarrow \delta_B = 4\delta_A$

Substituting: $\delta_A + 4\delta_A = 0.02 \Rightarrow \delta_A = 4\times10^{-3}, \delta_B = 1.6\times10^{-2}, F = 1125$
For pixel error <=1%, 5<K<6.

With K=6, $i_{beam} = (5^2 \times 6^2 \times 1.6\times10^{-19} \times 500^2 / 100)\times 1125 = 4.05\times10^{-10}\,A$.
Now solving for the minimum bean diameter: $d_{min} = 0.7 C_s^{\ 1/4} B^{-3/8} i_{beam}^{\ \ 3/8} = 2.73nm$
The actual number is smaller than this because K<6.

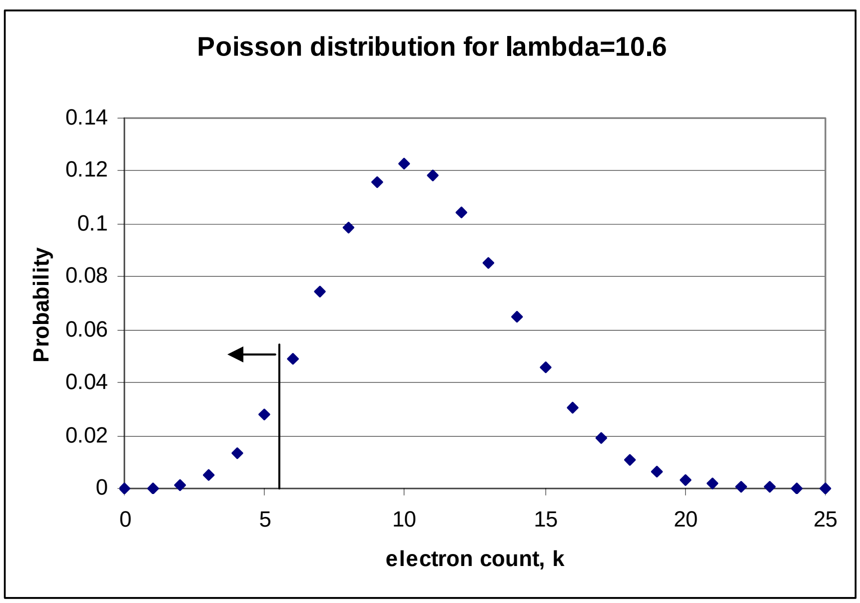
b) How could spatial resolution be improved?
- reduce beam current
- increase source brightness
To keep the same signal-to-noise ratio while reducing the beam current, we have to increase integration time.

#29
We assume a Poissonian distribution with average counts, λ=10.6

The probability that there are exactly k occurrences (counted electrons) is given by

$$f(k;\lambda) = \frac{e^{-\lambda}\lambda^k}{k!},$$

The distribution is plotted below for k=0,1,2…..25



Summing the probabilities from k= 0 to 5, we find the probability of getting the wrong pixel color to be 4.8%.