6.781 Homework #3 Solution

Problem 9

$$
\begin{aligned}
a &= E_0 E_{+1}^* + E_0^* E_{+1} \\
&= Ae^{j(\phi-\omega t)} \varepsilon e^{-j(k_x x-\omega t)} + Ae^{-j(\phi-\omega t)} \varepsilon e^{j(k_x x-\omega t)} \\
&= 2A\varepsilon \frac{e^{j(\phi-k_x x)} + e^{-j(\phi-k_x x)}}{2} \\
&= 2A\varepsilon \cos(\phi - k_x x)
\end{aligned}
$$

$$
\begin{aligned}
b &= E_0 E_{-1}^* + E_0^* E_{-1} \\
&= Ae^{j(\phi-\omega t)} \varepsilon e^{-j(-k_x x-\omega t)} + Ae^{-j(\phi-\omega t)} \varepsilon e^{j(-k_x x-\omega t)} \\
&= 2A\varepsilon \frac{e^{j(\phi+k_x x)} + e^{-j(\phi+k_x x)}}{2} \\
&= 2A\varepsilon \cos(\phi + k_x x)
\end{aligned}
$$

$$
\begin{aligned}
c &= E_{+1} E_{-1}^* + E_{+1}^* E_{-1} \\
&= \varepsilon e^{j(k_x x-\omega t)} \varepsilon e^{-j(-k_x x-\omega t)} + \varepsilon e^{-j(k_x x-\omega t)} \varepsilon e^{j(-k_x x-\omega t)} \\
&= 2\varepsilon^2 \frac{e^{2jk_x x} + e^{-2jk_x x}}{2} \\
&= 2\varepsilon^2 \cos(2k_x x)
\end{aligned}
$$

Problem 10

a) In the experimental setup, DIC is used for locating the bead, not for measuring the movement of the bead. The movement of the bead is detected by a scheme that is similar to ellipsometry. A polarized light beam is first split into two orthogonally polarized beams. These two beams are focused onto the bead, and they are largely overlapped. Because the two beams go through different parts of the bead, they are out of phase from each other. When they are recombined, they form an elliptically polarized light. The motion of the bead can be detected by measuring the change in the ellipticity of the polarization.

b) The answer to this question is it depends. The Voltage-vs-Time plot in Fig. 1 is quite noisy, and the noise can be from either the detection system, or the random motion of the bead. If the noise is from the detection system, it is a little farfetched to claim a resolution of 1 nm. The spectrum plot shows a unit of $nmHz^{-1/2}$ which indicates that the measurement is taken over a certain period of time. This time can be substantially longer than 10ms, the period of the 100Hz calibration signal, so the noise in the detection signal can easily be averaged out during the measurement to give the nice peak in the frequency plot. The question is asking if a 1nm motion can be detected. In this context, Brownian motion should not be considered as noise.

c) The sensitivity of the detection can be enhanced by using a shorter wavelength, because the same thickness difference will result in a larger phase shift of the two beams when a shorter wavelength is used. On the other hand, a shorter wavelength results in a larger shot noise. So, there is a balance to play with. Another concern in this particular application is that a shorter wavelength may destroy the sample.

d) There are many reasons for which SEM is not suitable for this task. First, the sample will not survive in the vacuum. Second, the sample is too thick for the electrons to go through. Third, even the sample is thin enough for the beam to go through, a polarized electron beam will no longer be polarized after passing through solid.

Problem 11

a) The scheme, point-spread-function engineering (PSFE), improves the resolution limit by reducing the effective size of the point-spread function (PSF) through exploring nonlinearities in the fluorescent processing. It requires fluorophores that behaves as 4-level system (states S_0, S_1 and their corresponding vibrational manifolds), and light with two different wavelengths, the shorter one excites the fluorophores and the longer one quenches it. A phase mask is used to null the center of the PSF for the longer wavelength, but preserves the wings. By doing so, the molecules at the tail of the shorter-wavelength PSF are quenched during the fluorescent process, and the shorter-wavelength PSF is effectively reduced. This mechanism works in all three dimensions of the space.

b) According to the article, the lateral resolution is improved by a factor of 2.

c) According to the article, the axial resolution is improved by a factor of 6.