Homework Set #4 Solution

Problem #12

Classically:

$$p = \frac{h}{\lambda},\ \text{K.E.} = \frac{p^2}{2m_0} \quad \Rightarrow \quad \lambda = \frac{h}{\sqrt{2m_0\text{K.E.}}}.$$

Relativistically:

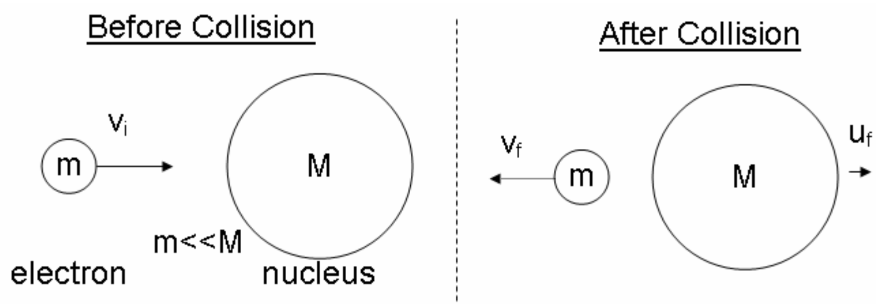
$$pc = \sqrt{\text{K.E.}^2 + 2\ \text{K.E.}\ m_0c^2},\ \lambda = \frac{hc}{pc} \quad \Rightarrow \quad \lambda = \frac{hc}{\sqrt{\text{K.E.}^2 + 2\ \text{K.E.}\ m_0c^2}}.$$

In the equations, p is the momentum, K.E. is the kinetic energy, c is the speed of light, m_0 is the rest mass of the electron.

K.E. (keV)	$\lambda_{\text{classical}}$ (nm)	$\lambda_{\text{relativistic}}$ (nm)
10	0.0123	0.0122
100	0.00388	0.00370
1000	0.0012	0.00087

Relativistic corrections are important above 100 keV.

Problem #13



a.) Elastic collision

Conservation of momentum

$$m(v_i) = M(u_f) - m(v_f)$$

Conservation of Energy

$$\tfrac{1}{2}m(v_i)^2 = \tfrac{1}{2}M(u_f)^2 + \tfrac{1}{2}m(v_f)^2$$

Combining these equations:

$$v_f = \frac{M-m}{M+m}v_i \qquad u_f = \frac{2m}{M+m}v_i \qquad \text{For M>>m, } v_f \approx v_i$$

b.) 100 KeV electron: m=9.1e-31 kg; resting gold nucleus: M=3.3e-25 kg

Energy lost for the electron is that imparted on the gold nucleus

$$Q_{lost} = \tfrac{1}{2} M (u_f)^2 = \tfrac{1}{2} M \left(\frac{2m}{M+m} v_i \right)^2 = \frac{4mM}{(M+m)^2} \left[\tfrac{1}{2} m (v_i)^2 \right] = \frac{4mM}{(M+m)^2} \left[100\ KeV \right]$$

$$Q_{lost} = 1.1\, eV$$

Problem #14

a) At 100Kev, λ = 3.7e-3 nm. Since $p = m\lambda / \sin\alpha = 0.54\, nm$,
$\alpha \approx \sin\alpha = m\lambda / p = 6(3.7\times10^{-3}\ nm) / 0.54\, nm$, or $\alpha = 4\times10^{-2}\ rad$.

b) For imaging $\alpha \sim 10^{-2} - 10^{-3}$, so $4\times10^{-2}\, rad$ is reasonable for obtaining diffraction patterns, which are less critical than imaging.

c) $\sin\alpha = m\lambda / p \quad \Rightarrow \quad 0.01 = 6(400\ nm)/p \quad \Rightarrow \quad p = 2.4\times10^{5}\, nm = 0.24\ mm$
Pretty big!

d) $p = m\lambda / \sin\alpha$. Let $\sin\alpha = 1$, then $p = m\lambda$. $m = p/\lambda = 2.4\times10^{5}\, nm / 4\times10^{2}\, nm = 600$.
Really high!

e) $\sin\alpha = m\lambda / p \quad \Rightarrow \quad \alpha = \sin^{-1}(m\lambda / p) = \sin^{-1}(0.285m)$
For $m = 1;\ \alpha = 17^{o}$.
For $m = 2;\ \alpha = 35^{o}$.
For $m = 3;\ \alpha = 59^{o}$.
For $m = 4;\ \alpha =$ cut off.
So, 3rd order is the highest.

Problem #15

$p_{min} = C_s^{1/4} \lambda^{3/4}$ where $C_s \approx f$.

a) @ $E = 100\, keV$, $\lambda = 3.7\times10^{-3}\, nm$. For $C_s = 1 mm$, $p_{min} = 0.47\, nm$

b) @ $E = 1 keV$, $\lambda = 3.88\times10^{-2}\, nm$. For $C_s = 10\, mm$, $p_{min} = 4.9\, nm$

c) To keep the focal length the same, the current in the magnetic lens must be adjusted such that the trajectories of electrons are identical for the two beam energies. To achieve

this, the ratio of the axial velocity to the radial velocity v_r / v_z should be independent of the beam energy. Hence, $v_r = \sqrt{E} h_r(z)$ and $v_z = \sqrt{E} h_z(z)$.
On the other hand, the B field is a linear function of the current I, or
$B_z = I\ g_z(z),\ B_r = I\ g_r(z)$, where $g_z(z)$ and $g_r(z)$ are functions of *z*. So, from the lecture notes we have

$$r \frac{d\theta}{dt} \propto \int v_z B_r dt = \int B_r dz$$

$$\begin{aligned} v_r &= \int a_r dt \\ &\propto \int F_r dt \\ &\propto \int \left(\int B_r dz \right) B_z dt \\ &= \int \left(\int B_r dz \right) \frac{B_z}{v_z} dz \\ &\propto \frac{I^2}{\sqrt{E}} \end{aligned}$$

Finally,

$$\frac{v_r}{v_z} \propto \frac{I^2}{\sqrt{E}} \frac{1}{\sqrt{E}} = \frac{I^2}{E}$$

Therefore, if energy E is increased by a factor of 100, the current I must be increased by a factor of 10.

Problem #16

The oxide is a uniform shade because it is amorphous, and thus no coherent Bragg diffraction occurs. The polysilicon is polycrystalline. If a grain is oriented properly, Bragg diffraction will enter the aperture and we will see a bright grain. Otherwise, the diffracted beam will miss the aperture and the grain will appear dark. Silicon is crystalline (single crystal) and scatters electrons coherently. Because TEM samples are typically very thin, they bend easily. In certain regions of the bend the Bragg condition is met such that the aperture is missed and the Si in those regions appears dark. Oxide does not exhibit these bend contours because it is amorphous.