MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF COMPUTER SCIENCE AND ELECTRICAL ENGINEERING

6.801/6.866 MACHINE VISION

Problem 1: You can approach the following problems geometrically or algebraically (whichever is easier);

(a) What is the image of a line in space?

(b) What is the image of a circular disc that lies in a plane that is parallel to the image plane?

(c) How is the shape of the image of a pattern that lies in a plane parallel to the image plane related to the shape of that pattern (think paper cutout, maybe)?

(d) What geometric shapes can the image of a sphere take on? (Careful: this is different from the image of a circular disc).

Notes: Assume perspective projection. Please explain your reasoning or show your calculations.

Problem 2: Consider the image of a polyhedral object (e.g. a dodecahedron) illuminated by distant light sources. Describe the resulting brightness pattern in the image plane. Note: Assume that the object is not only far from the light sources, but also far from the camera (i.e. a telephoto lens is used).

Think about the following: How does the brightness vary from place to place in the image? Is it constant? Does it have discontinuities? Does it vary linearly with position in the image? Are there areas of uniform properties? If so, what shapes are those image areas? (Note: we are not looking for a closed-form solution or mathematical formula).

Problem 3: Here we explore a simple method for estimating image motion. Suppose that at time $t = 0$ image brightness is $E_0(x, y)$, and that the image moves to the right (i.e. $+x$ direction) with velocity u.

(a) First, to get started, consider a simple situation where brightness ramps up linearly with position, $E_0(x, y) = mx + c$ say at time $t = 0$, with $m > 0$. If the brightness pattern moves to the right, does the brightness at a particular point (i.e. fixed x) go up or down? If the brightness pattern moves to the right by δx, then by how much does the brightness at a particular point in the image change? (This exercise helps get the signs of the derivatives right).

(b) If the pattern moves to the right with velocity u, then what is $E(x, y, t)$ in terms of $E_0(x, y)$?

(c) Now consider a simple image where, at time $t = 0$, brightness varies as

$$E_0(x, y) = 6 + 5\sin(x) - \sin(5x)$$

(Note that in this simple case brightness does not depend on y). Write an expression for $E(x, y, t)$ assuming the pattern moves to the right with velocity u. Find the derivatives with respect to x, y, and t.

(d) Estimate the velocity u from the partial derivatives E_x, E_y and E_t at a particular point (x, y) in the image.

(e) For $t = 0$, how well would you expect the method to work near $x = 0$. How about near $x = \pi/2$? (Consider the effect of small changes in brightness measurement. Think of "noise gain").

Problem 4: Here we explore using weighted averages of uncertain estimates derived from images in order to get more accurate results.

(a) If we have estimates of E_x and E_t for many points in the image, then we can average the corresponding local estimates of image motion u — obtained using the method of 3(d) — by integrating over an image area and dividing by the area. But as (hopefully) shown in 3(e), contribitions from some image regions may be less valuable than contributions from others. Suppose that we "weight" the estimate of u from image position (x, y) by multiplying by some suitable weight factor $w(x, y)$ (which is as yet to be determined). Give a formula for the "weighted average" velocity u. Note that you can no longer simply divide by the area of image region integrated over — there is now a need to compensate for the spatially varying weight factor $w(x, y)$.

(b) Suppose now that the method is to be used on more complex images. The brightness no longer varies according to some simple analytical expression, but as some arbitrary function $E_0(x, y)$ of image position. Find the value of u that minimizes:

$$\iint_R (E_t + uE_x)^2 \, dx \, dy$$

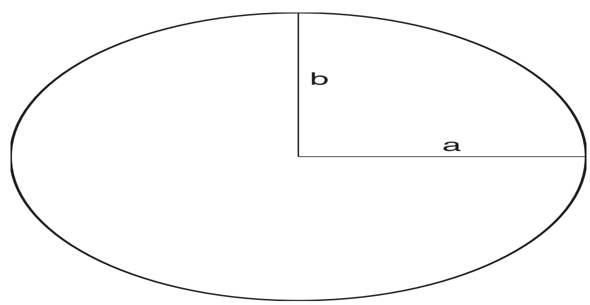
where the integration is carried out over the image region R of interest. (Assume that you can interchange the order of differentiation and integration).

(c) Interpret the formula for u that you obtained in part (b) in terms of the weighting scheme of part (a). What is the weight factor $w(x, y)$ here? Do the weights seem rasonable in the sense that more reliable information is weighted more heavily?

Problem 5: Here we use images to estimate object poses. Consider a system for estimating the oriention of a planar surface based on an image of a circle on that surface. If we view the plane “straight on”, the circle will be imaged as a circle. We get an ellipse in the image if the plane is tilted so that its normal vector is not perpendicular to the image plane. We want to determine how much the plane is tilted from the eccentricity of the ellipse.

To simplify matters, we here consider *orthographic* instead of *perspective* projection. This is corresponds to the limit of looking at the objects from further and further away, but with correspondingly higher and higher magnification.

Imagine parallel rays perpendicular to the image plane arriving from a very distant light source. A circular disk is suspended above the image plane, blocking some of the incident light. The result is an elliptical shadow in the image. The angle between the image plane and the plane containing the circular disk is θ.



(a) Find the length of the minor axis b in terms of the angle θ and the length of the major axis a.

(b) The eccentricity e of an ellipse can be defined in terms of the ratio of the semi-major axes using the formula $e^2 = 1 - (b/a)^2$. Express the eccentricity of the ellipse in the image in terms of the angle θ.

(c) Suppose we have an image processing method that can estimate the eccentricity of an elliptical image pattern. For what angles will the estimation of θ based on the eccentricity be particularly difficult in the presence of noise?