

```

MODULE ConcurrentTransactions [
  V, % Value
  S, % State of database
  T % Transaction ID
] EXPORT Begin, Do, Commit =

CONST s0:S := init()    % initial state

TYPE
  A = S -> [v,s]        % Action
  E = [a: A, v: V]      % Event
  H = SEQ E              % History
  Y = T -> H            % histories of transactions

  TS = SET T            % Transaction Set
  XC = (T, T)-> Bool    % eXternal Consistency
  TO = SEQ T            % Total Order on T's

VAR
  y := Y{}              % current transaction histories
  committed : TS{}      % committed transactions
  active      : TS{}     % active transactions (uncommitted)
  xc := XC{* -> false} % current required XC

```

```
FUNC Valid(y,to) -> Bool = RET Apply(+ : (to * y),s0)
```

```
FUNC Apply(h: H, s: S) =  
  IF h={} => RET True  
  [*] VAR [a,v] := h.head |  
    VAR [v',s'] := a(s) |  
    IF ~(v'=v) => RET False  
    [*] RET Apply(h.tail, s')  
FI
```

```
FUNC Consistent(to, xc) -> Bool =  
  ALL t1, t2: T |  
    xc.closure(t1,t2) ==> ( TO{t1,t2} <=< to)
```

```
FUNC Serializable(ts: TS, xc: XC, y: Y) -> Bool =  
  RET (EXISTS to: TO |  
    t.set=ts /\ Consistent(to, xc) /\ Valid(y,to))
```

```
FUNC Invariant(com: TS, act: TS, xc, y) -> Bool =  
  Serializable(com, xc, y)
```

```

APROC Begin() -> T = <<
  VAR t: T | ~ t IN (active \ / committed) =>
    y(t) := {};
    active := active \ / {t};
    xc(t,t) := true;
    DO VAR t' :IN committed | ~xc.closure(t',t) =>
      xc(t',t):=true
    OD;
>>

```

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APROC Do(t: T, a: A) -> V = <<
  VAR v: V,
    y' := t{t -> t(y) + {E{a,v}}}|
  Invariant(commited, active, xc, y') =>
    y := y';
    RET v;
>>

```

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APROC Commit(t: T) = <<
  VAR committed' :TS := commited \ / {t},
    active' :TS := active - {t} |
  Invariant(commited', active', xc, y) =>
    commited := committed'
    active := active';
>>

```

```

% INVARIANT Invariant(commited, active, xc, y)

```

```

FUNC Invariant(com: TS, act: TS, xc0: XC, y0: Y) -> Bool =
VAR current := com + act |
  Serializable(com, xc0, y0) /\ CONSTRAINT

CONSTRAINT is one of AC, CC, EO, OD, OC1, OC2, NC

AC: (ALL ts: TS | (com <= ts <= current) ==>
  Serializable(ts, xc0, y0))

CC: Serializable(current, xc0, y0)

EO: (ALL t :IN act | EXISTS ts | com <= ts <= current /\
  Serializable(ts + {t}, xc0, y0))

OD: (ALL t :IN act | EXISTS ts | AtBegin(t) <= ts <= current /\
  Serializable(ts + {t}, xc0, y0))

OC1: (ALL t :IN act, h :IN Prefixes(y0(t)) | EXISTS to, h1, h2 |
  to.set = AtBegin(t) /\ Consistent(to, xc0) /\ Valid(y0,to)
  /\ IsInterleaving(h1, {t' | t' IN current-AtBegin(t)-{t} | y0(t')})
  /\ h2 <<= h1
  /\ h.last.a(Apply(+ : (to * y0) + h2 + h.reml, s0) = h.last.v))

OC2: (ALL t :IN act, h :IN Prefixes(y0(t)) | EXISTS to, h1, h2, h3 |
  to.set = AtBegin(t) /\ Consistent(to, xc0) /\ Valid(y0,to)
  /\ IsInterleaving(h1, {t' | t' IN current-AtBegin(t)-{t} | y0(t')})
  /\ h2 <<= h1
  /\ IsInterleaving(h3, {h2, h.reml})
  /\ h.last.a(Apply(+ : (to * y0) + h2 + h.reml, s0) = h.last.v))

NC: true

FUNC Prefixes(h: T) -> SET H = RET {h' | h' M= h /\ h' # {}}

FUNC AtBegin(t: T) -> TS = RET {t' | xc.closure(t',t)}

FUNC IsInterleaving(h: H, s: SET H) -> Bool =
  ... sequence h is interleaving of sequences from the set s ...

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```

TYPE Lk    = String
      Lks   = SET Lk
      A     = S -> [v: V, s: S]

CONST
  protect   : A -> Lks
  conflict  : (Lk, Lk) -> Bool

% I1: ALL a1, a2 | ~commute(a1,a2) ==>
      EXISTS l1 IN protect(a1), l2 IN protect(a2) |
      conflict(l1, l2)

FUNC commute(a1: A, a2: A) -> Bool =
  RET (ALL s0: S |
      a2(a1(s0).s).s = a1(a2(s0).s).s /\
      a1(s0).v = a1(a2(s0).s).v /\
      a2(s0).v = a2(a1(s0).s).v)

% I2: ALL t :IN active, e :IN y(t) | protect(e.a) <= locks(t)

% I3: ALL t1 :IN active, t2 :IN active | t1 # t2 ==>
%     ALL lk1 :IN locks(t1), lk2 :IN locks(t2) |
%     ~conflict(lk1,lk2)

```