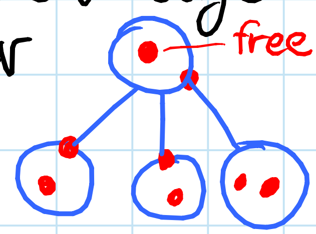
6.849	Class 11	Oct. 16, 2012

- Pebble algorithm: [Jacobs & Hendrickson 1997]

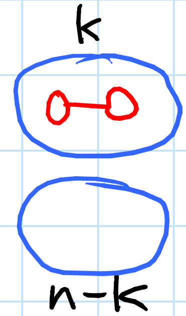
(1) test 2k property: every k vertices induce $\le 2k$ edges

- each vertex has 2 attached pebbles
- each pebble can cover 1 incident edge
  - free if not used to cover
- goal: cover every edge



Claim: 2k property $\Leftrightarrow$ pebble cover

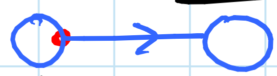
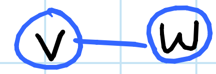
Proof:

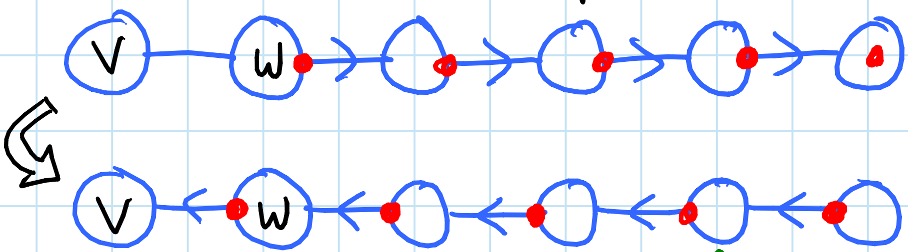


($\Leftarrow$) edges induced by k vertices must be covered by 2k pebbles of those vertices
$\Rightarrow$ $\le 2k$ induced edges

($\Rightarrow$) by correctness of algorithm below:
no pebble cover
$\Rightarrow$ algorithm below will fail
$\Rightarrow$ find vertex set violating 2k property □

Algorithm:

- add edges one at a time   INCREMENTAL
- view covered edge as directed from pebble 
- for each added edge vw: 
  - search for directed path from v or w to free pebble
  - if found: shift pebbles (reverse edges)



directed

  - else: nodes reachable from v & w violate 2k property

    Proof: no outgoing edges
    $\Rightarrow$ pebbles cover induced edges EXCEPT vw
    $\Rightarrow$ $>2k$ edges among k vertices $\square$

Running time: $O(V+E)$ per search
$\cdot\ O(V)$ searches
$= O(V^2 + \cancel{VE})$

just check whether $E > 2V$ at start ($\Rightarrow$ return NO)

② test 2k-3 property (Laman condition)

Claim: G has 2k-3 property
$\iff$ G+3e has 2k property
add 3 copies of e
for every edge e in G.

Proof: consider k vertices.
($\Rightarrow$) $\le 2k-3$ induced edges
if e among them:
G+3e induces $\le 2k$ edges
else: still $\le 2k-3 < 2k$ edges
($\Leftarrow$) if no induced edges: done
else: add 3 copies of induced edge
results in $\le 2k$ induced edges
remove 3 extra copies
$\Rightarrow \le 2k-3$ induced edges □

$O(V^3)$ algorithm: call previous on G+3e $\forall e$

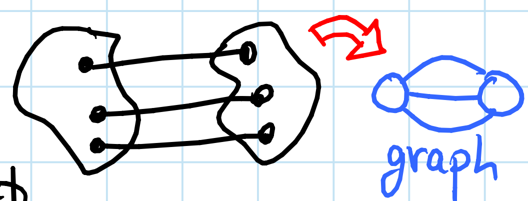
$O(V^2)$ algorithm: incremental as above
- for each added edge e:
  - add 4 copies of e as above
  - if succeed: remove 3 copies of e (freeing 3 pebbles)
  - if fail: remove all 4 copies of e
    mark edge as redundant
- gen. rigid $\iff$ 2n-3 nonredundant edges

Implementation [Audrey Lee]

Generalization to $a \cdot k - b$ property
[Lee & Streinu – Discr. Math. 2008]

Rigid component decomposition:
[above paper + Lee, Streinu, Theran – CCCG 2005]
roughly, component = what you can reach, including backward edges if reachable component on other side has no free pebbles

Body & bar frameworks:



generically rigid in d-D
$\Leftrightarrow$ graph has $ak-a$ property
where $a = \frac{d(d+1)}{2}$ $= 6$ in 3D
[Tay 1984 + Nash-Williams/Tutte (indep.)]

– can also support hinges (3D): equivalent to 5 bars

Angular rigidity: [Lee-St. John & Streinu – CCCG 2009]
– lines/planes & angles: angles min. gen. rigid in 3D
$\Leftrightarrow$ constraint graph is Laman
– bodies & angles: angles gen. rigid in 3D
$\Leftrightarrow$ constraint graph has $3k-3$ property

- 5-connected double bananas: [Mantler & Snoeyink - CCCG 2004]
  - in fact, any graph can be made 5-connected while preserving Laman & generic flexibility
    - just add spiders: