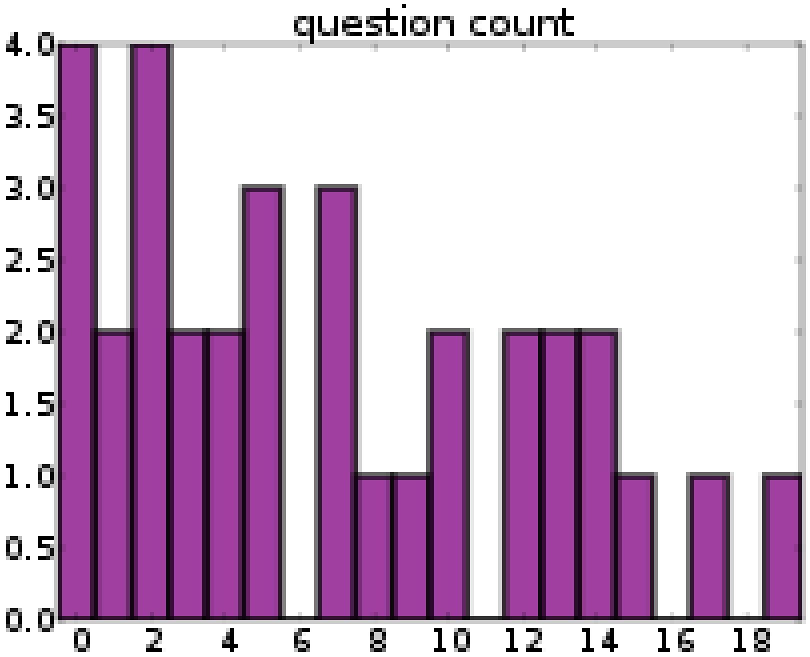
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3.0
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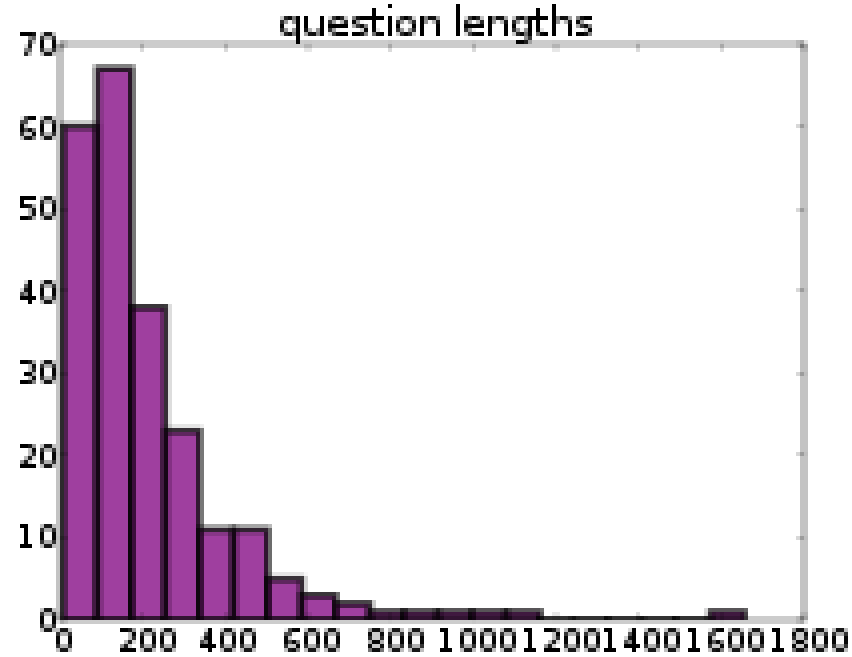
12
14
16
18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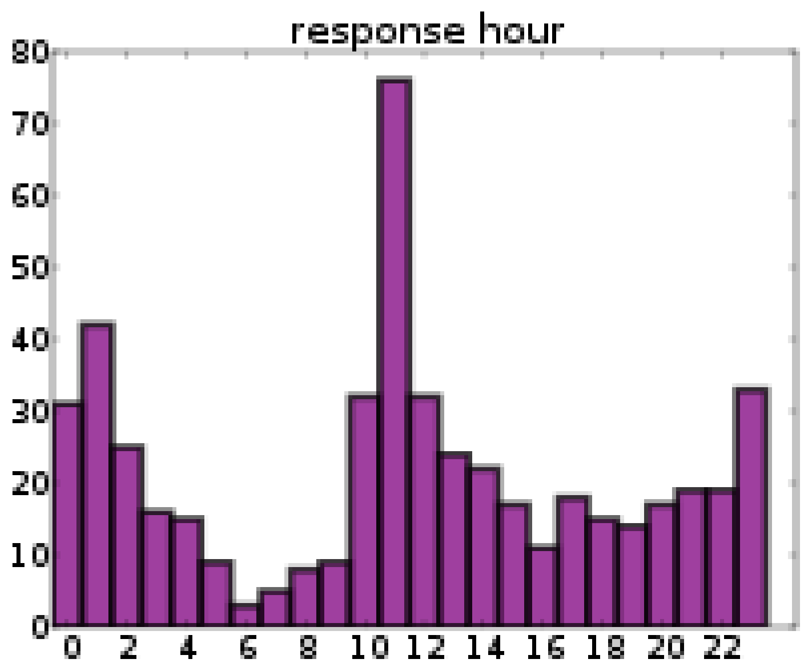
response hour
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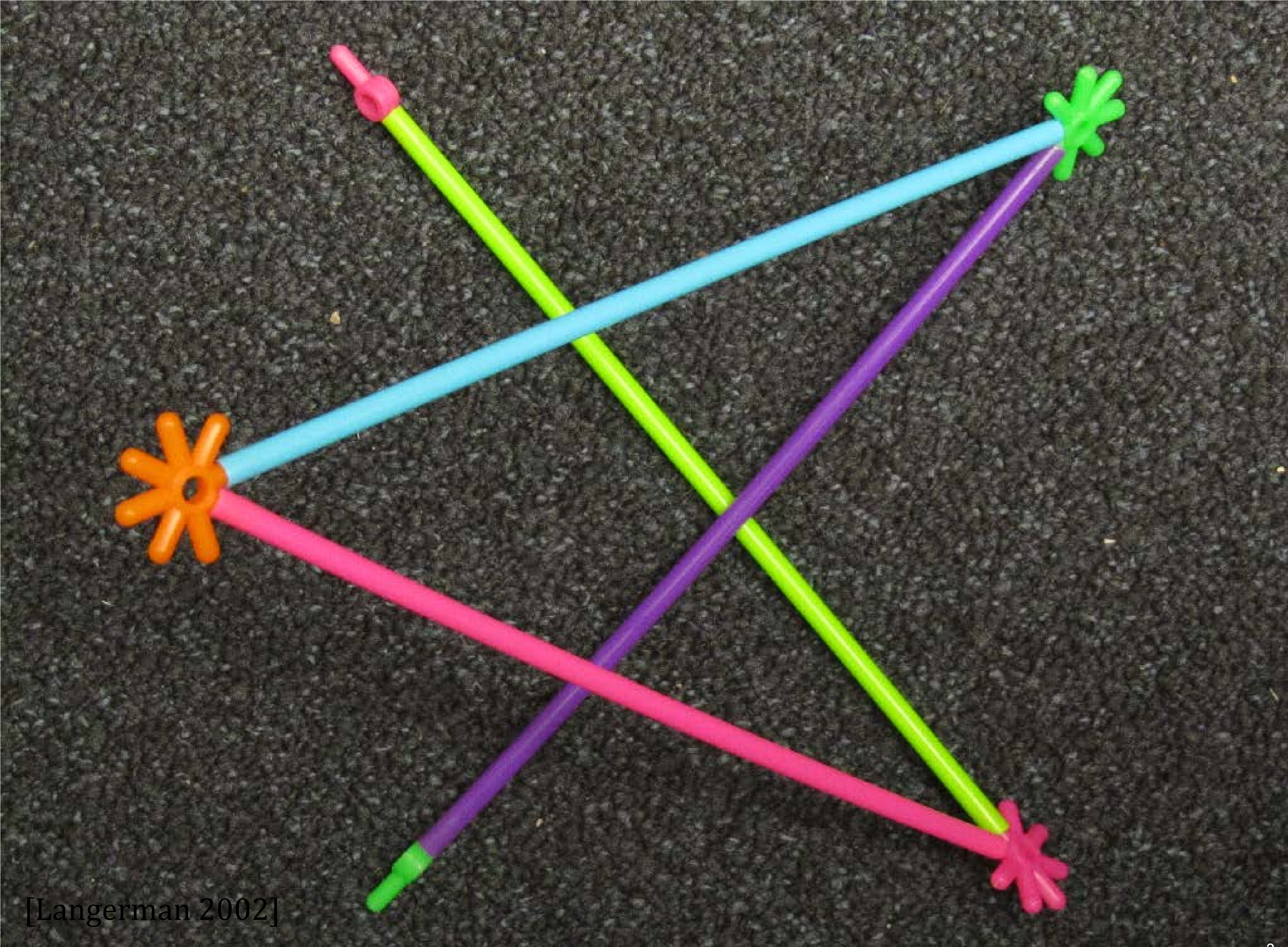
70
60
50
40
30
20
10
0
0
2
4
6
8
10
12
14
16
18
20
22

We've done several combinations of the constraints of equilateralness, equiangularity, and obtusehood for 3-D chains we want to know whether can be locked. What about the others?



[Langerman 2002]

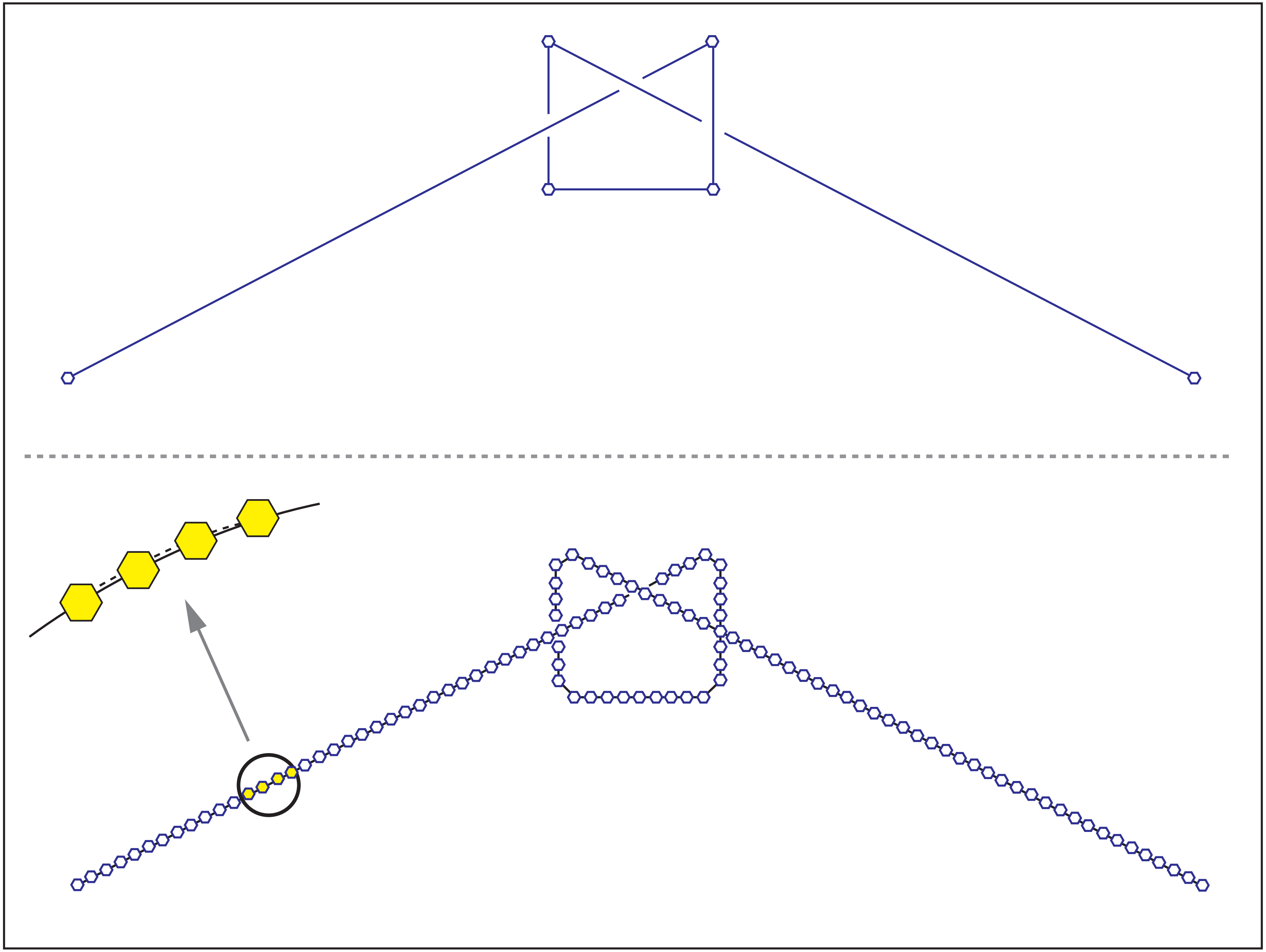


Image by MIT OpenCourseWare.

The geometric (cone) model for a ribosome seems too simple. Is it actually based on some verified model from biology?

Image of surface of polypeptide exit tunnel removed due to copyright restrictions.

[Nissen, Hansen, Ban, Moore, Steitz 2000]

[L21] In proving the NP-hardness of the 2D HP-model folding problem, what are the NP-hard problems used in various reductions?

Protein Folding in the Hydrophobic-Hydrophilic (HP) Model is NP-Complete

Bonnie Berger* Tom Leighton†

bin packing

Figures removed due to copyright restrictions.
Refer to: Fig. 4-5 from Berger, B., and T. Leighton. "Protein Folding in the Hydrophobic-hydrophilic(HP) is NP-complete." *Proceedings of the Second Annual International Conference on Computational Molecular Biology* (1998): 30-9.

On the Complexity of Protein Folding

Pierluigi Crescenzi, Deborah Goldman, Christos Papadimitriou
Antonio Piccolboni, Mihalis Yannakakis

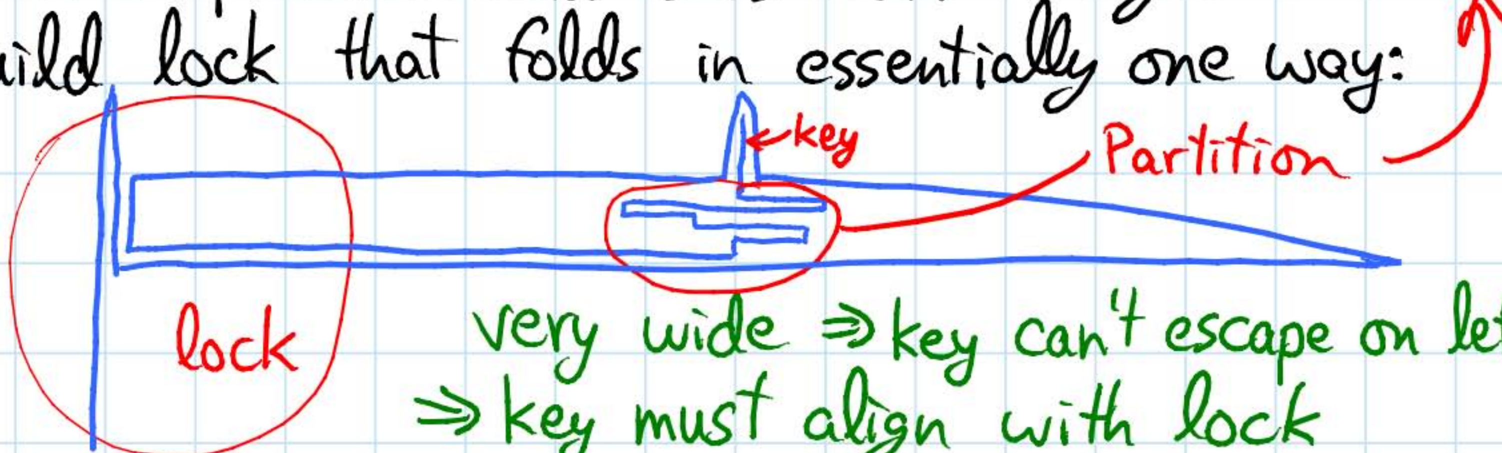
Hamiltonicity in max-degree-4 graphs

Any progress on any of the open problems?

Flattening: weakly NP-hard [Soss & Toussaint 2000]

- reduction from Partition: divide n integers into 2 equal sums
- horizontal bars for integers
- vertical bars in between, length $< \frac{1}{n}$

⇒ can flip horizontal bars left & right
- build lock that folds in essentially one way:



very wide ⇒ key can't escape on left
⇒ key must align with lock

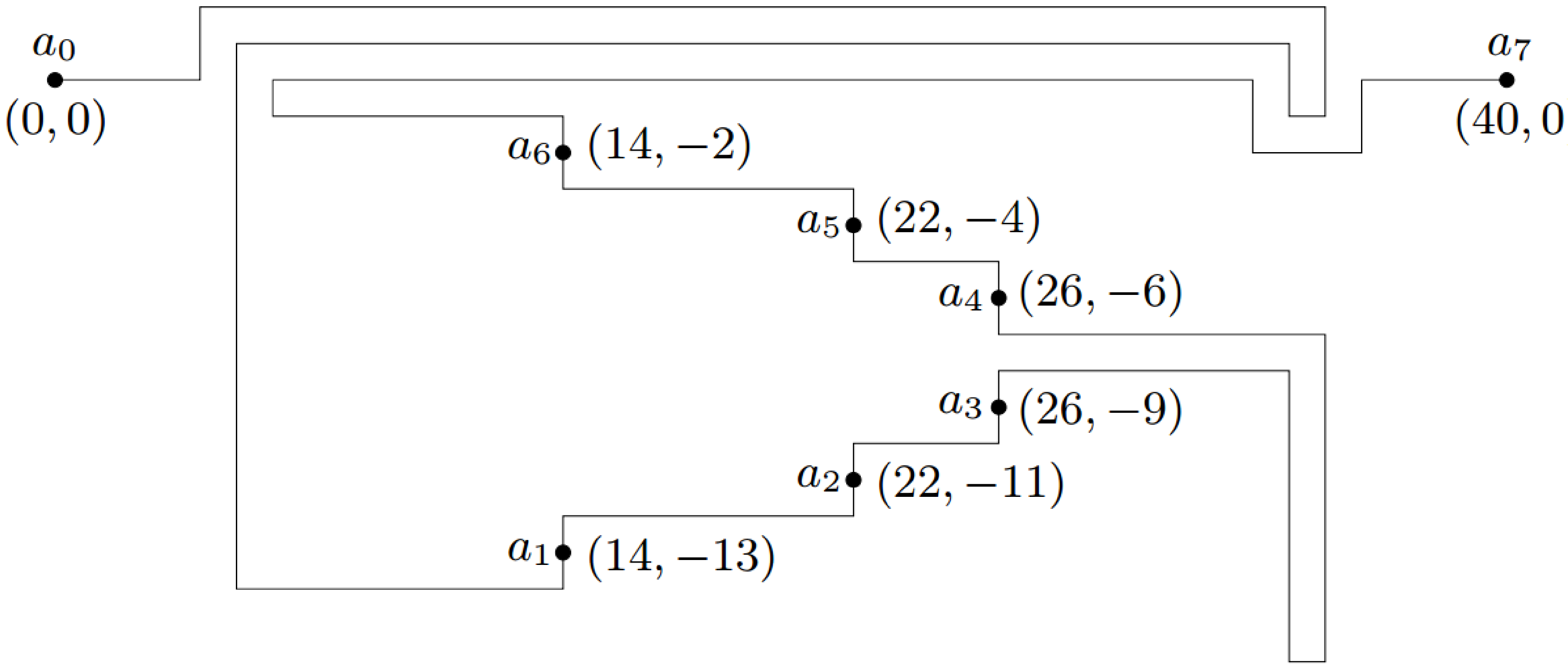
OPEN: pseudopolynomial-time algorithm?

Flattening Fixed-Angle Chains Is Strongly NP-Hard

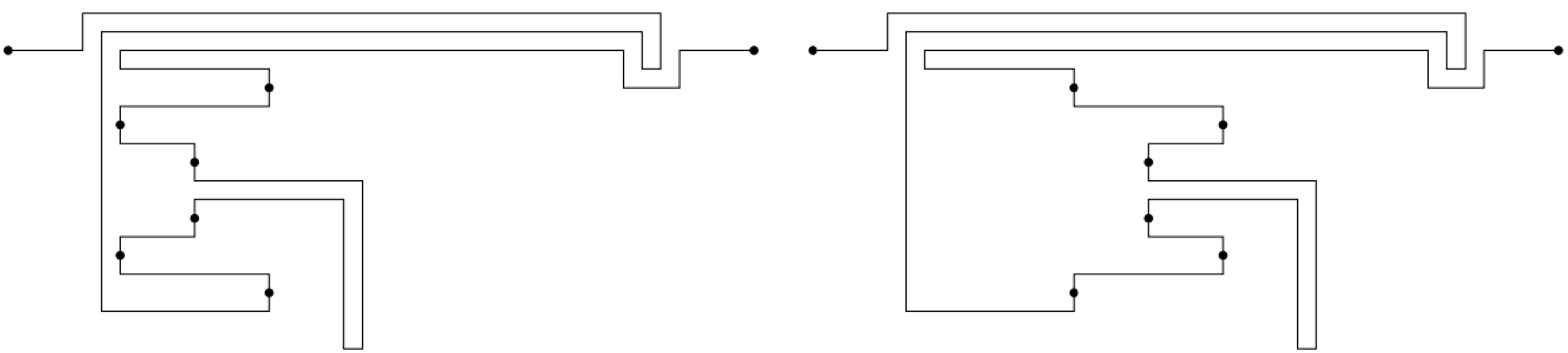
Erik D. Demaine* and Sarah Eisenstat*

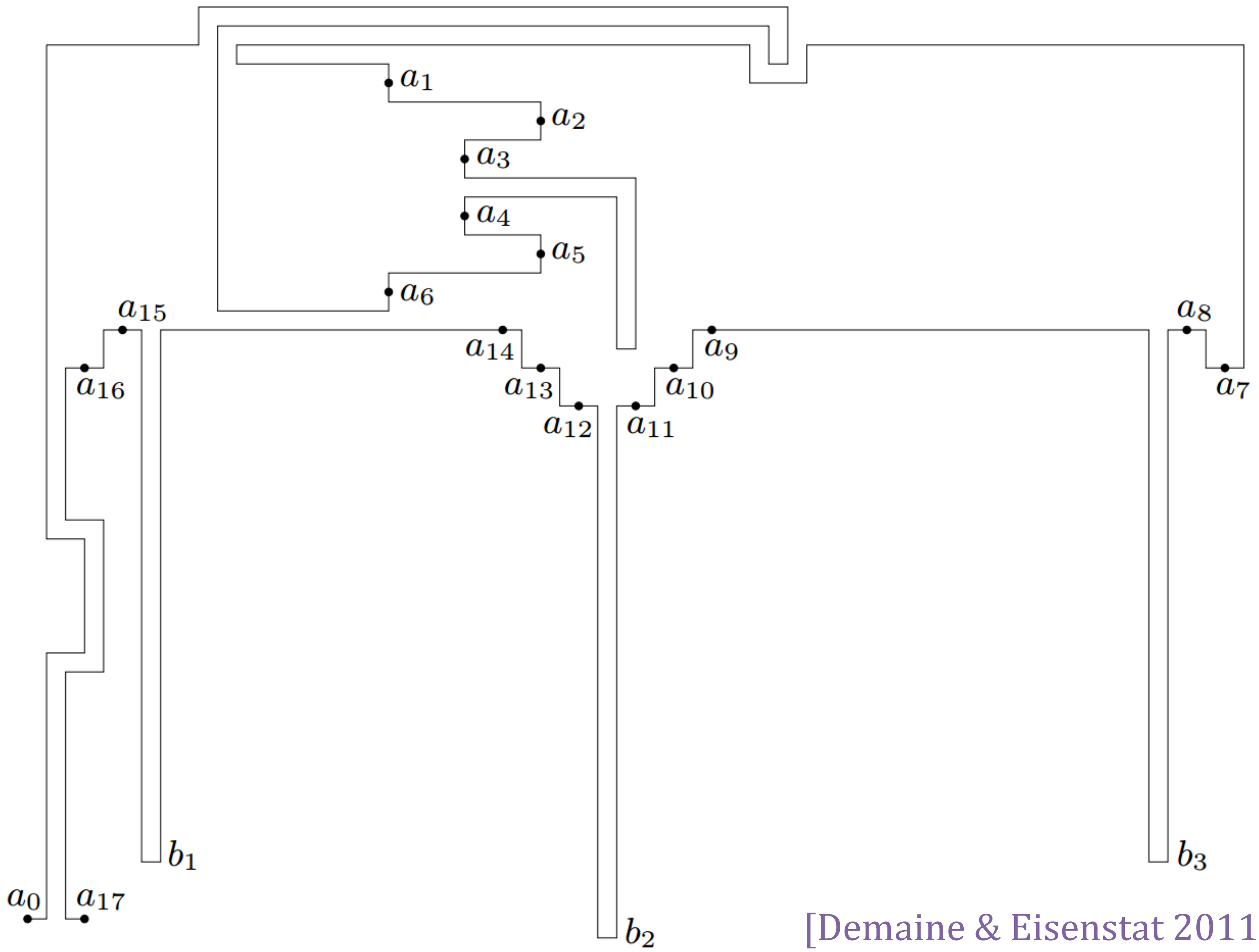
MIT Computer Science and Artificial Intelligence Laboratory,
32 Vassar St., Cambridge, MA 02139, USA

Problem	Linkage	Edge lengths	Angle range
Flattening	fixed-angle chain	equilateral	$[16.26°, 180°]$
Flattening	fixed-angle chain	$\Theta(1)$	$[60 - \varepsilon°, 180°]$
Flattening	fixed-angle caterpillar tree	equilateral	$\{90°, 180°\}$
Min flat span	fixed-angle chain	equilateral	$[16.26°, 180°]$
Max flat span	fixed-angle chain	equilateral	$[16.26°, 180°]$

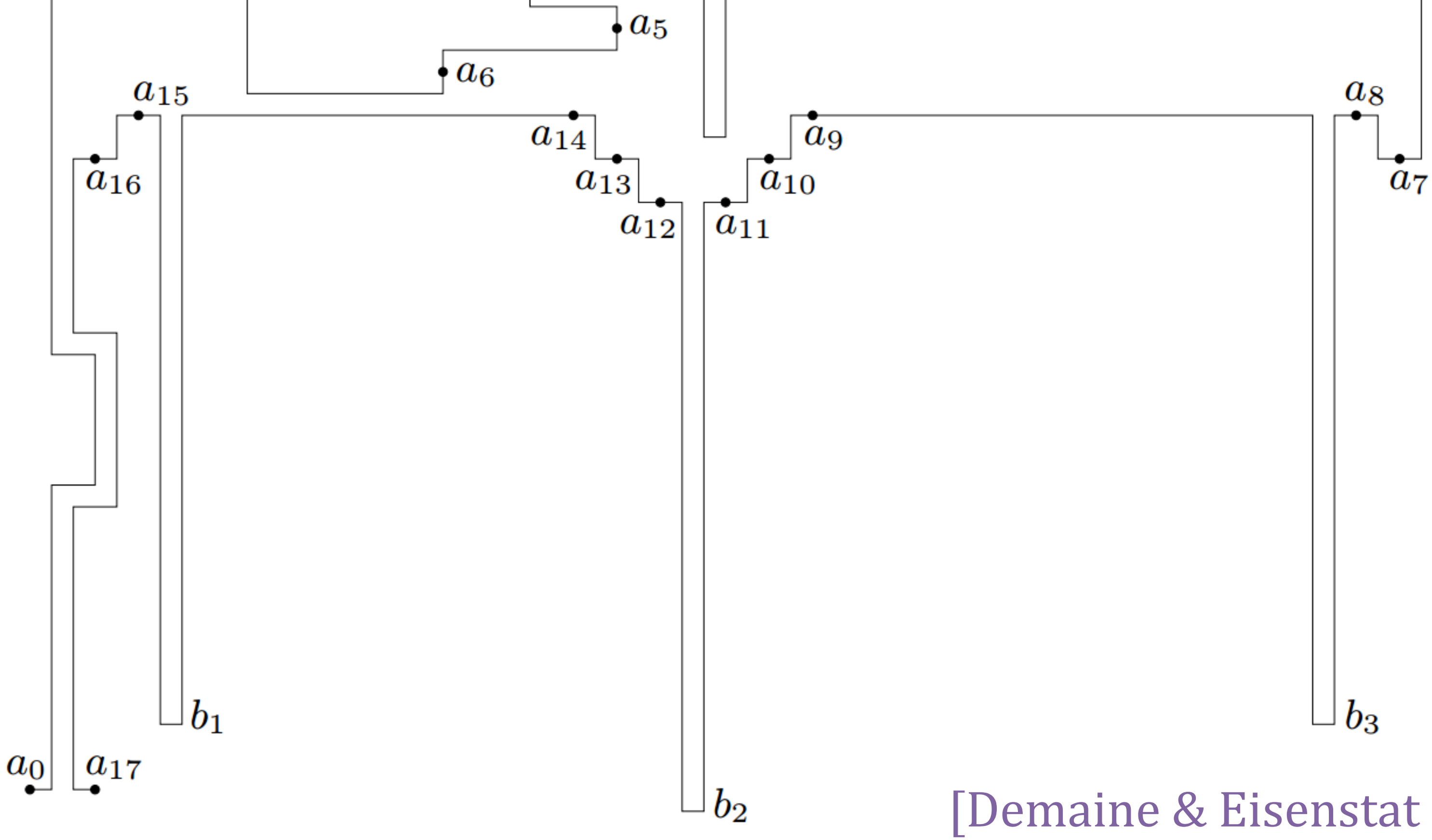


[Demaine & Eisenstat 2011]

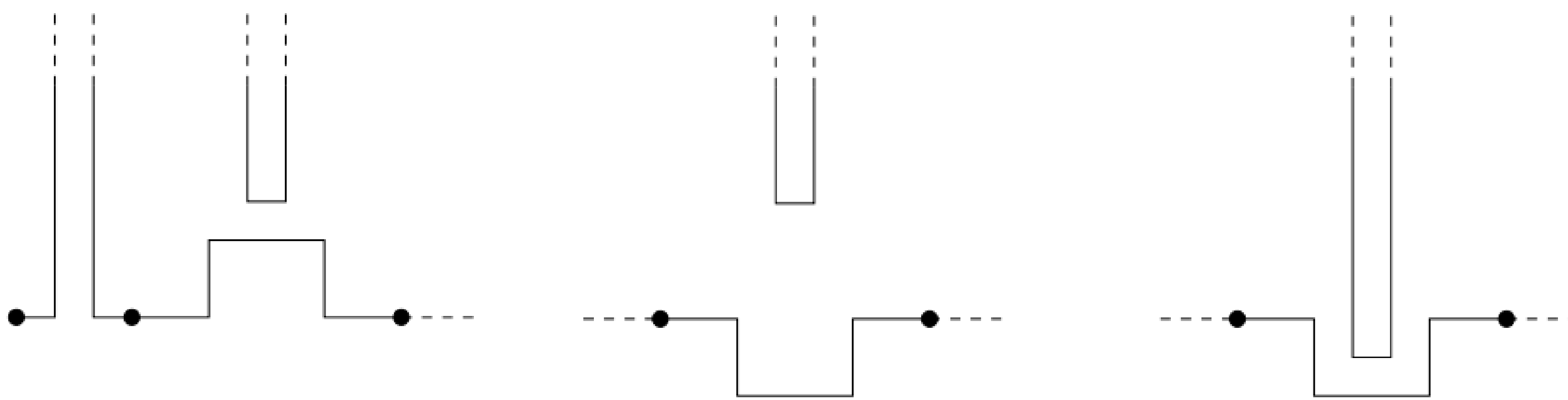


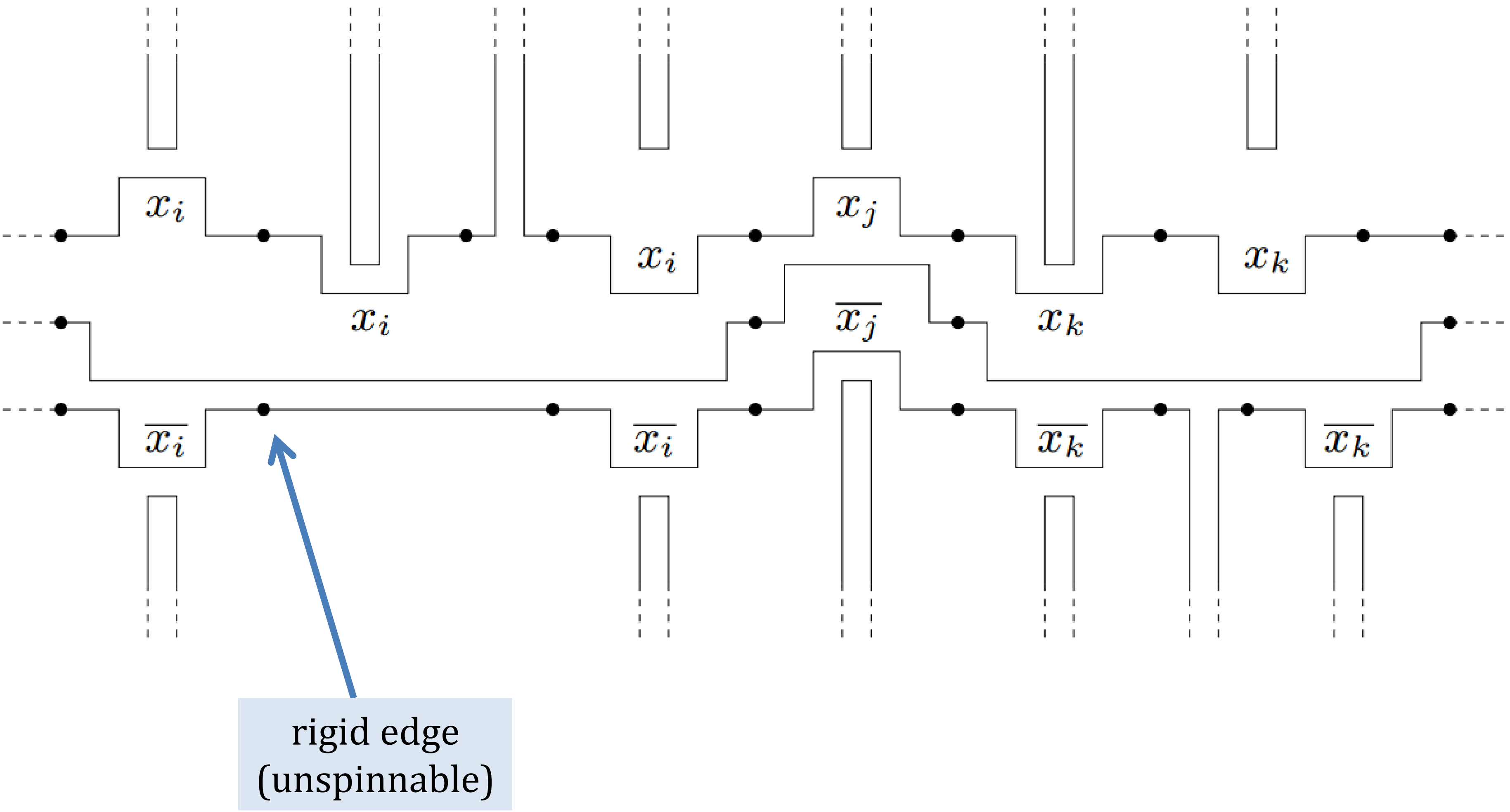


[Demaine & Eisenstat 2011]

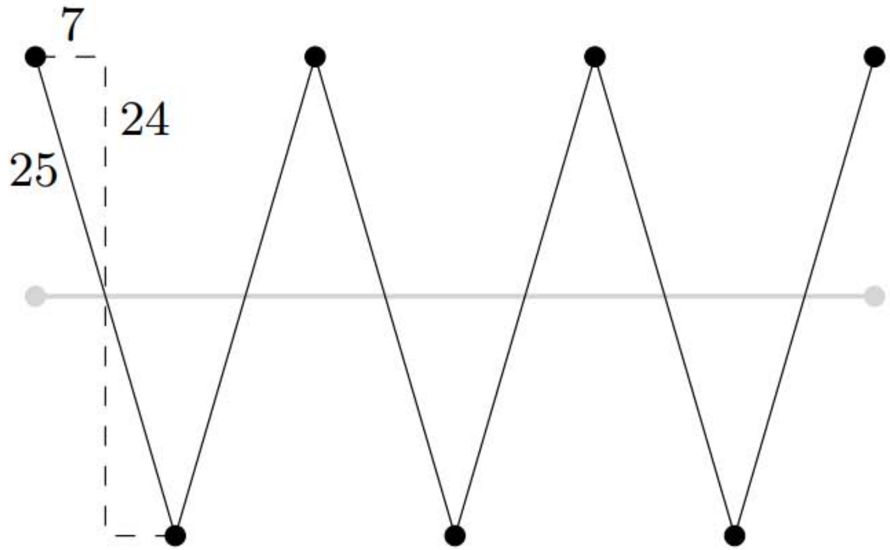


[Demaine & Eisenstat 2011]

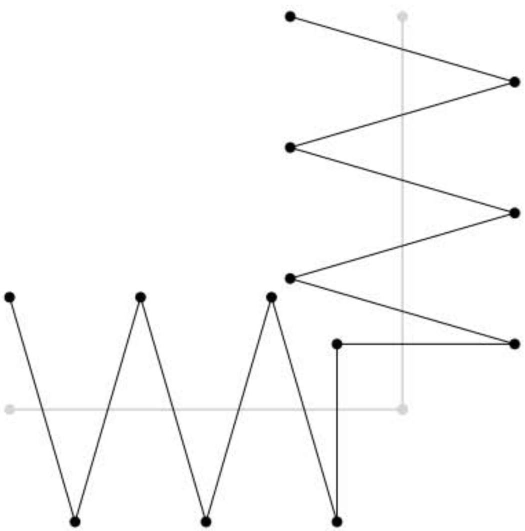




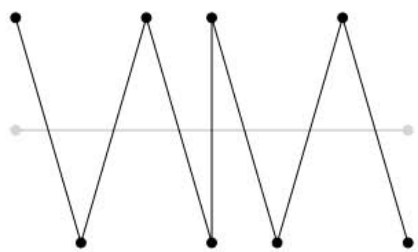
[Demaine & Eisenstat 2011]



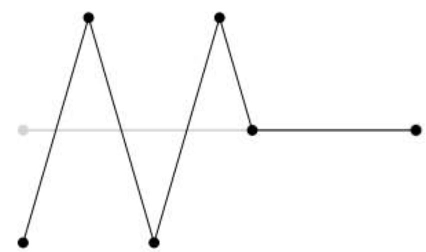
(a) Zig-zag gadget.



(b) Turn gadget.



(c) Switch gadget.



(d) Articulation gadget.

[Demaine & Eisenstat 2011]

Fig. 1a removed due to copyright restrictions.
Refer to: Demaine, E. D., B. Gassend, J. O'Rourke, et al. "All Polygons Flip Finitely… Right?" *Contemporary Mathematics* 453 (2008): 231–55.

Advanced Problems

3763. *Proposed by Paul Erdös, The University, Manchester, England.*

Given any simple polygon P which is not convex, draw the smallest convex polygon P' which contains P. This convex polygon P' will contain the area P and certain additional areas. Reflect each of these additional areas with respect to the corresponding added side, thus obtaining a new polygon P_1. If P_1 is not convex, repeat the process, obtaining a polygon P_2. Prove that after a finite number of such steps a polygon P_n will be obtained which will be convex.

Erdős 1935

Fig. 1a and 1b removed due to copyright restrictions.
Refer to: Demaine, E. D., B. Gassend, J. O'Rourke, et al. “All Polygons Flip Finitely… Right?” *Contemporary Mathematics* 453 (2008): 231–55.

Erdős 1935

Fig. 2 removed due to copyright restrictions.
Refer to: Demaine, E. D., B. Gassend, J. O'Rourke, et al. "All Polygons Flip Finitely… Right?" *Contemporary Mathematics* 453 (2008): 231–55.

de Sz. Nagy 1939

Fig. 6 removed due to copyright restrictions.
Refer to: Demaine, E. D., B. Gassend, J. O'Rourke, et al. “All Polygons Flip Finitely… Right?” *Contemporary Mathematics* 453 (2008): 231–55.

Joss & Shannon 1973

Table 1 removed due to copyright restrictions.
Refer to: Demaine, E. D., B. Gassend, J. O'Rourke, et al. “All Polygons Flip Finitely… Right?”
Contemporary Mathematics 453 (2008): 231–55.

Demaine, Gassend, O’Rourke, Toussaint 2008

Table 1 removed due to copyright restrictions.
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Demaine, Gassend, O’Rourke, Toussaint 2008

Доказательство этой теоремы, данное Б. Секефальви-Надем [см. B. Sz.-Nady, Amer. Math. Monthly **46**, 1939, стр. 176—177], неверно.

"The proof of this theorem, given by B. Sz. Nagy, is incorrect"

"Bing and Kazarinoff remark that Nagy's proof is invalid, but there is no basis for this claim."

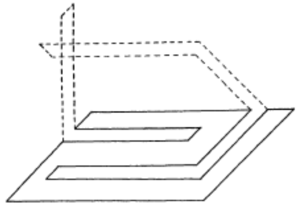
Demaine, Gassend, O'Rourke, Toussaint 2008

3763 [1935, 627]. *Proposed by Paul Erdös, The University, Manchester, England.*

Given any simple polygon P which is not convex, draw the smallest convex polygon P' which contains P. This convex polygon P' will contain the area P and certain additional areas. Reflect each of these additional areas with respect to the corresponding added side, thus obtaining a new polygon P_1. If P_1 is not convex, repeat the process, obtaining a polygon P_2. Prove that after a finite number of such steps a polygon P_n will be obtained which will be convex.

Solution by Béla de Sz. Nagy, Szeged, Hungary.

The process described in the above problem, *i.e.*, the reflection of *all* additional areas, does not always lead from a simple polygon to a simple one, as shown in the following example:



This means that the repeating of this process is not always possible.

In order to avoid this difficulty we modify the process in the following way. Instead of reflecting *all* additional areas mentioned in the problem we reflect *only one* of them, so obtaining obviously always a simple polygon again. We agree to define the process also for convex polygons as the process of leaving them invariant.

Let $A_1^0, A_2^0, \cdots, A_\sigma^0$ be the vertices of the given simple polygon P^0. Applying the process n times leads to a polygon P^n, the points A_ν^0 $(\nu=1, 2, \cdots, \sigma)$ being carried thereby into the points A_ν^n. Let us denote by C^n the least convex polygon containing P^n in its interior. Each polygon in the sequence P^0, C^0, P^1, C^1, P^2, C^2, $\cdots$ contains obviously the foregoing ones in its interior. The lengths of all polygons P^n being plainly the same, there is a circle containing all P^n's in its interior. This implies that the sequence of the points A_ν^n $(n=0, 1, 2, \cdots)$ has at least one point of accumulation.

It follows readily from the nature of the above process that if B is a point on, or inside of, P^m, then dist $(B, A_\nu^n) \leqq$ dist (B, A_ν^{n+1}) for $n \geqq m$. Especially we have: dist $(A_\nu^m, A_\nu^n) \leqq$ dist (A_ν^m, A_ν^{n+1}) for $n \geqq m$. From this it follows that the sequence of the points A_ν^n $(n=0, 1, 2, \cdots)$ may have only a single point of accumulation. It is thus convergent: $A_\nu^n \to A_\nu$ for $n \to \infty$.

The polygon $P=(\overline{A_1A_2}, \overline{A_2A_3}, \cdots, \overline{A_{\sigma-1}A_\sigma}, \overline{A_\sigma A_1})$, being the limit of the sequence P^n, is also the limit of the sequence C^n and is therefore convex.

Denote by $c_\nu(r)$ the interior of the circle of radius r drawn around A_ν as center.

Let A_μ be a convexity-point of P (i.e., such that $A_{\mu-1}$, A_μ, $A_{\mu+1}$ do not lie on the same straight line; A_σ being denoted also as A_0, A_1 as $A_{\sigma+1}$). We may find then obviously a straight line L and a positive number ρ such that $c_\mu(\rho)$ lies wholly on one side of L while all $c_\lambda(\rho)$ $(\lambda \neq \mu)$ lie on the other side. For $n \geqq n_0(\mu)$ we shall certainly have: $A_\nu^n \in c_\nu(\rho)$ for $\nu=1, 2, \cdots, \sigma$. L separates thus A_μ^n from the other points A_λ^n $(\lambda \neq \mu)$. Hence A_μ^n is a convexity-point of P^n. It must be therefore invariant: $A_\mu^{n+1}=A_\mu^n$. This implies that for $n \geqq n_0(\mu)$: $A_\mu n_0(\mu)=A_\mu^n$. So is $A_\mu^n=A_\mu$ for $n \geqq n_0(\mu)$.

Let now $A_{\mu_1}, A_{\mu_2}, \cdots, A_{\mu_s}$ be all the convexity-points of P. We have then $A_{\mu_r}^N=A_{\mu_r}$ $(r=1, 2, \cdots, s)$ for $N=\max(n_0(\mu_1), n_0(\mu_2), \cdots, n_0(\mu_s))$.

This involves that $C^N=P$ and therefore also that $P^n=P$ for $n \geqq N$. We thus obtain after a finite number of steps a convex polygon indeed.

de Sz. Nagy 1939

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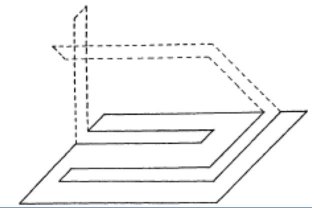

polygon containing P^n in its interior. Each polygon in the sequence P^0, C^0, P^1, C^1, P^2, C^2, $\cdots$ contains obviously the foregoing ones in its interior. The lengths

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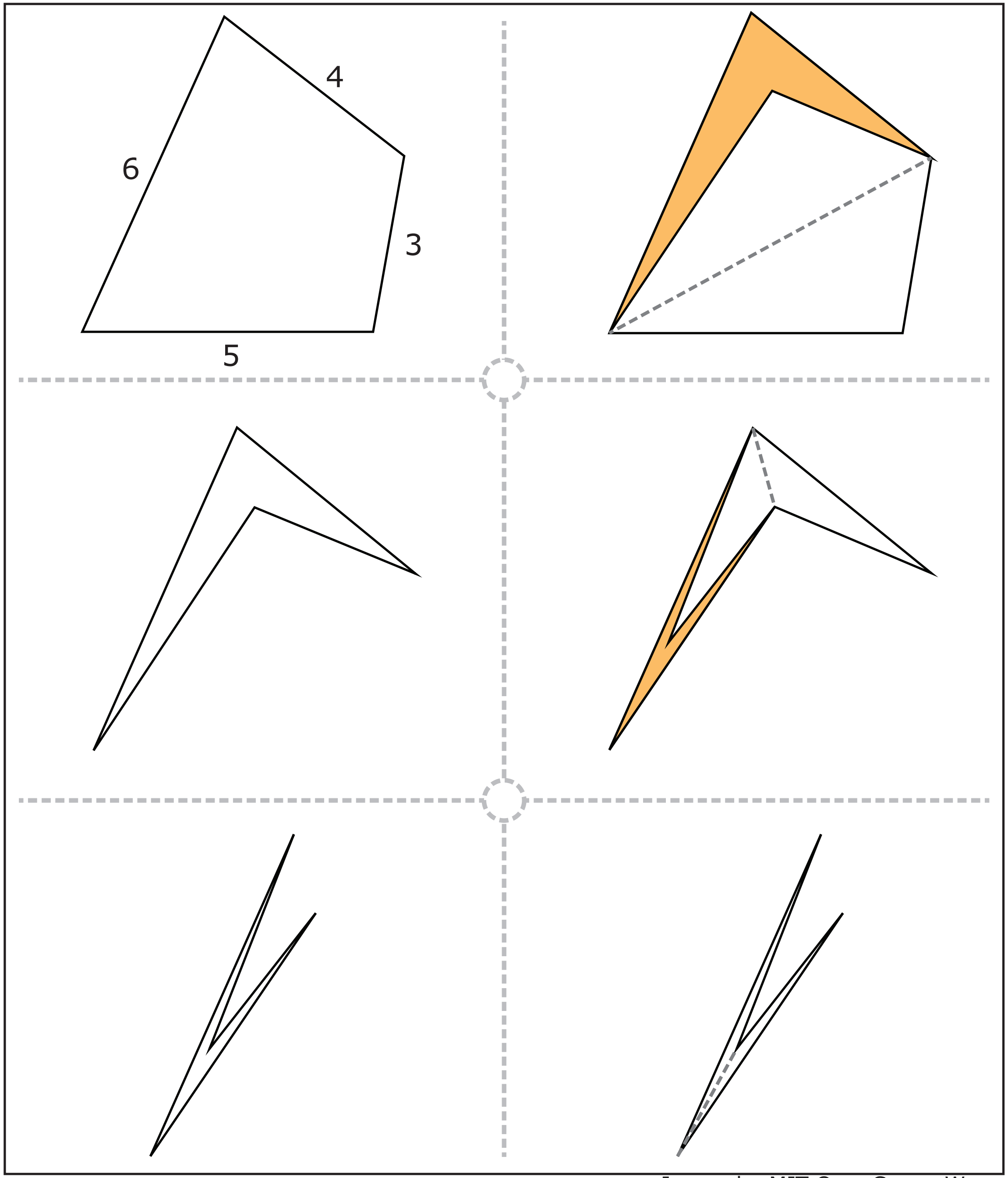
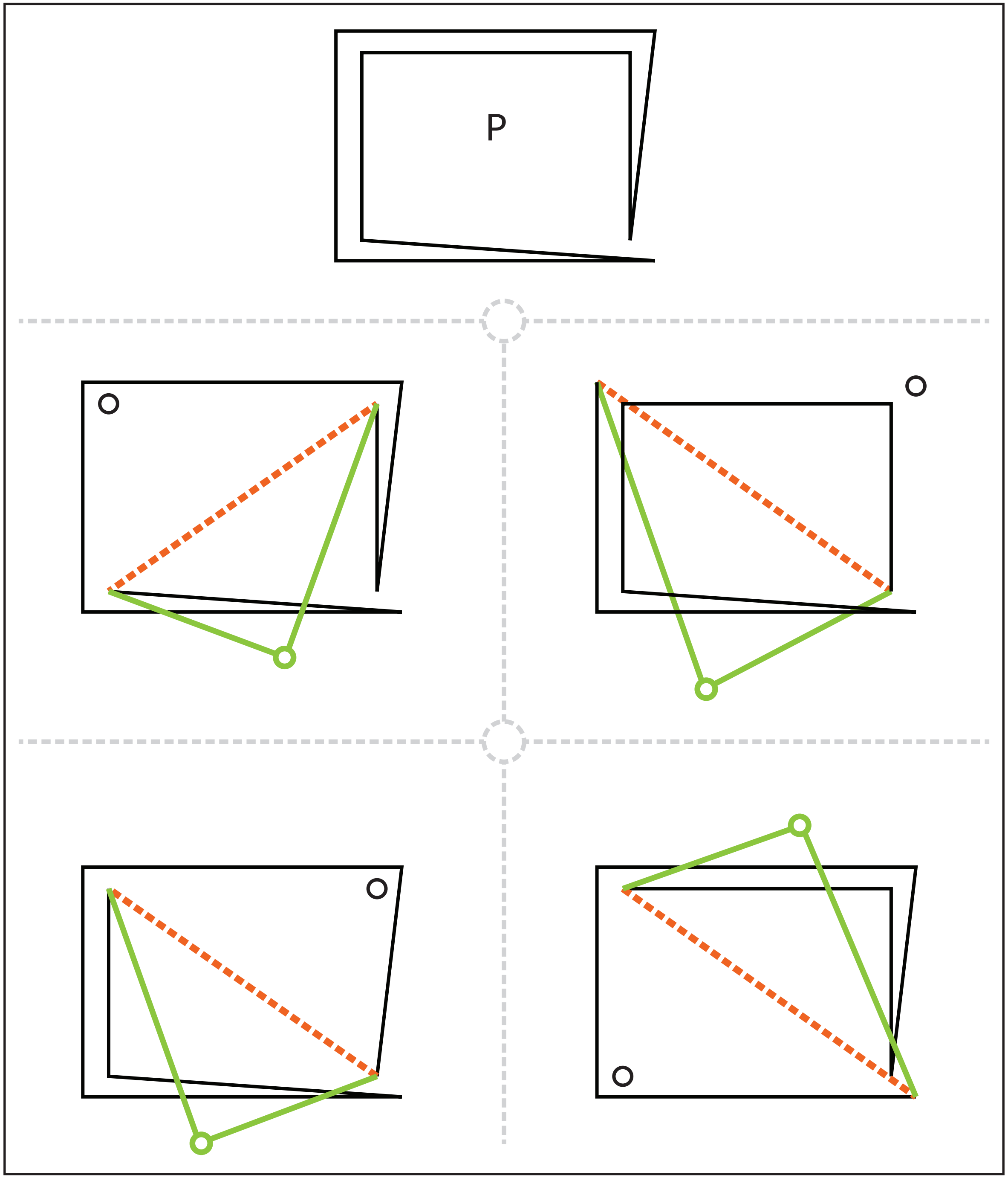


Image by MIT OpenCourseWare.

Fevens, Hernandez, Mesa, Morin, Soss, Toussaint 2001

Thurston 2001



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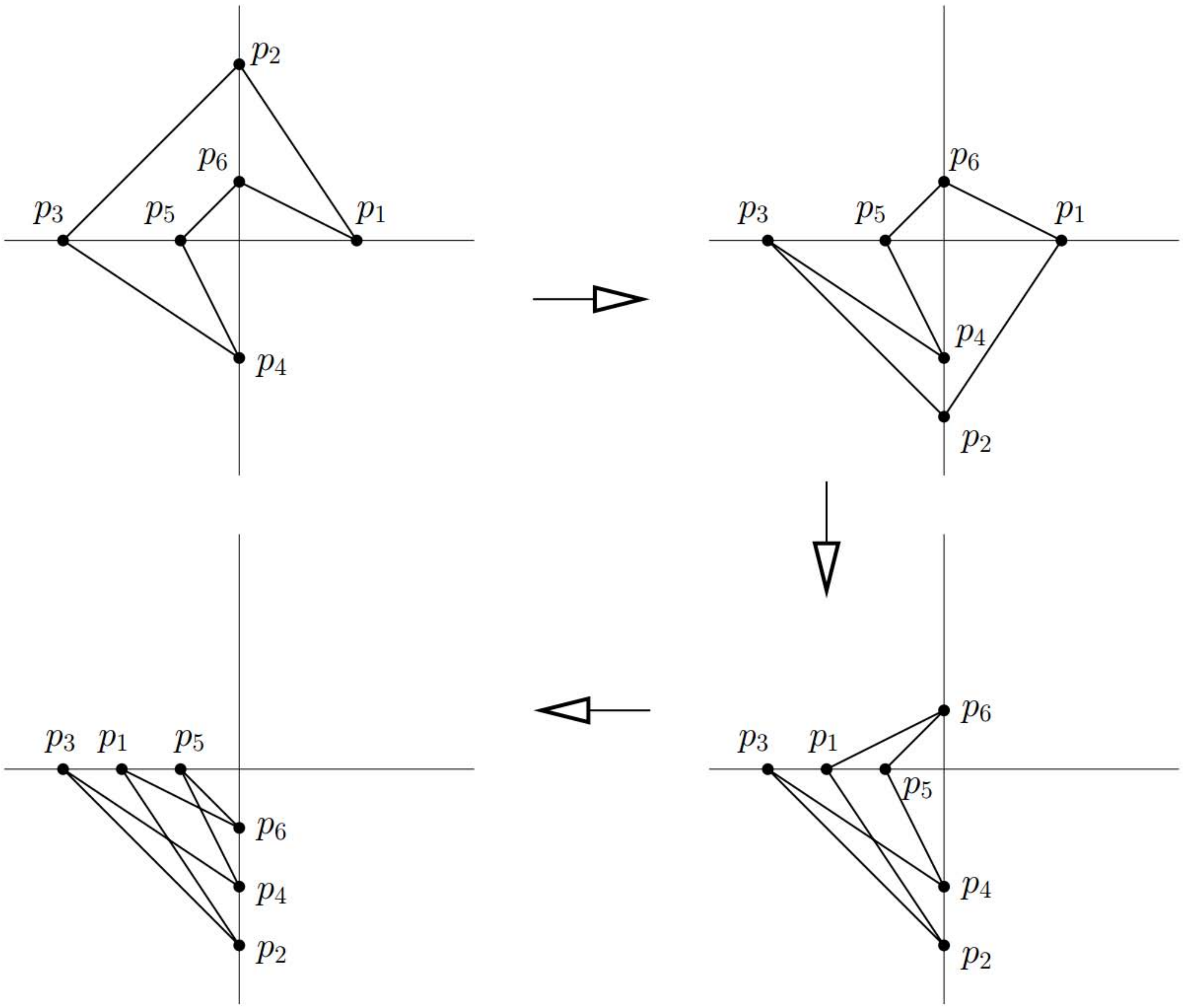

[Dumitrescu & Hilscher 2009]