| 6.849 | Class 20 | Nov. 20, 2012 |
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- Discussion of class as a whole & experimental split into video lectures + live classes

My perspective:

- you asked great questions, which shaped most of classes (lucky?)
- "true" interactivity mostly with few physical exercises (folding, etc.)
- old videos mostly good, >1x great (faster, potentially more focus)
- new videos add lots of complementary stuff
- most students seemed to follow schedule
- I probably needed more lead time

Questions for future:

- 5 hrs./week too much? doesn't scale
- more group activities?
- project-focused class?
- open-problem-focused class?
- in-person lectures better?
- or no fixed-time meetings?

- Equilateral vs. equiangular vs. obtuse <u>locked</u> 3D chains:

  1. universal joints:
     - <u>OPEN</u>: equilateral ⇒ not locked?
     - equiangular & obtuse don't seem too relevant here... not necc. preserved

  2. fixed-angle joints:
     - <u>OPEN</u>: equilateral, equiangular, & obtuse ⇒ not locked?
     - equilateral + equiangular: conjecture locked "crossed legs"
     - equiangular + obtuse: subdivided knitting needles lock
     - equilateral + obtuse: can simulate long bars ⇒ lock

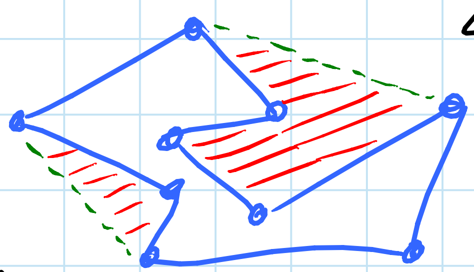
- Why model <u>ribosome</u> as a cone?
  - Because it's roughly a halfspace
    e.g. [Nissen, Hansen, Ban, Moore, Steitz – Science 2000]
    Nobel Prize in Chemistry, 2009

- HP protein folding NP-hardness:
  - 3D cube lattice [Berger & Leighton 1998] reduces from bin packing
  - 2D square lattice [Crescenzi, Goldman, Papadimitriou, Piccolboni, Yannakakis 1998] reduces from max.-degree-4 Hamiltonicity

- Flattening is strongly NP-hard [Demaine & Eisenstat – WADS 2011]

- Flips: (see 2010 L21 for more details)

Pocket of 2D polygon = region outside polygon & inside convex hull



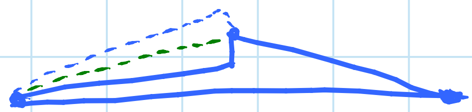
Pocket lid = convex-hull edge

Flip = reflect pocket through its lid
= rotate 180° through 3D around the lid
- avoids self-intersection (line of support)
- increases area

## "Erdős-Nagy" Theorem: [posed by Erdős 1935]

any polygon always convexifies after finite flips, no matter how flip sequence is chosen

- but can be arbitrarily many: [Joss & Shannon 1973]



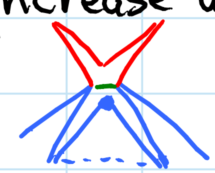
- $\boxed{\text{OPEN}}$: bound # flips in $n$ & $r$ = max. dist./min. dist
  - pseudopolynomial? [Overmars 1998]

## "Proofs" of Erdős-Nagy Theorem:

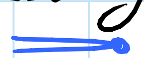
[Demaine, Gassend, O'Rourke, Toussaint 2007]

knowledge

- Nagy 1939 — flawed: "$P^0 \subseteq C^0 \subseteq P^1 \subseteq C^1 \subseteq \cdots$"
  (used to "prove" limit polygon convex)
- Reshetnyak 1957 — correct (though somewhat imprecise)
- Yusupov 1957 — flawed: "limit convex else flip" & more subtle error
- Bing & Kazarinoff 1959 — correct (though somewhat terse)
- Wegner 1993 — flawed: "move vertex ⇒ increase area by incident Δ"



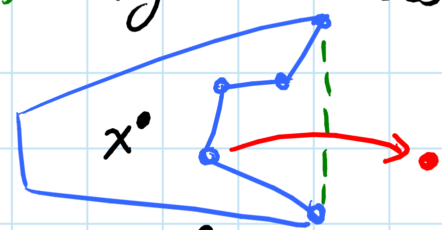
- all ↑ Grünbaum 1995 — omission: why limit polygon is convex
- all ↑ Toussaint 1999/2005 — flawed: "limit convex else flip"
- all ↑ Demaine, Gassend, O'Rourke, Toussaint 2008 — generalization to self-crossing assuming no "hairpins":



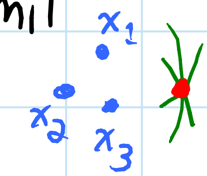
# Proof of "Erdős-Nagy" Theorem: [Bing & Kazarinoff 1959] & [DGOT 2006]

consider an infinite flip sequence

① distance from a vertex to fixed point $x$ inside the polygon (remains so) only increases
- pocket lid is Voronoi diagram of old & new




② each vertex approaches a unique limit
- apply ① to three noncollinear points $x_1, x_2, x_3$ inside the polygon




- distances from vertex ≤ perimeter of polygon/2

⇒ distances converge

⇒ vertex approaches intersection of 3 circles

③ turn angle at each vertex converges
- by ②, 3 vertices defining the angle converge
- by ①, vertices do not get closer to each other
- rest by continuity

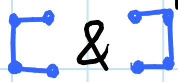
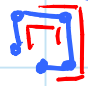
④ vertex moves infinitely ⇒ asymptotically flat
- each move negates sign of turn angle ⇒ → ∅

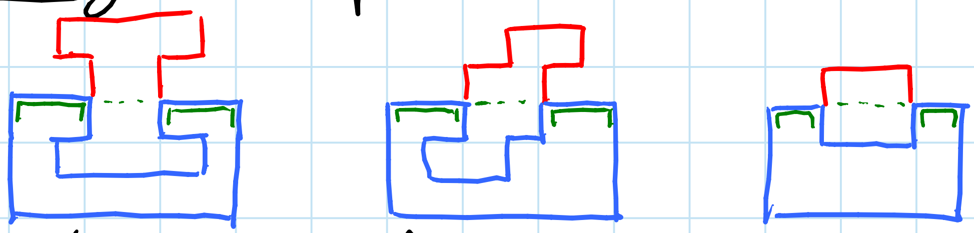
⑤ contradiction
- eventually asymptotically pointed vxs. stop moving

⇒ attain limit convex hull, but about to flip! □

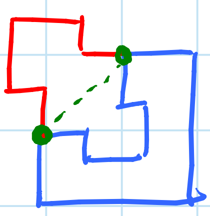
**Flipturn:** rotate pocket 180° in 2D around lid midpoint
- at most $n!$ configurations [Joss & Shannon 1973]
- always $O(n^2)$ flipturns [Aichholzer et al. 2002; Ahn et al. 2000 (diff. model)]
- sometimes $\Omega(n^2)$ flipturns [Biedl 2004]
- final polygon & location determined
- NP-hard to find *longest* flipturn sequence
- $\boxed{\text{OPEN}}$: finding *shortest* flipturn sequence?

[Aichholzer et al. 2002]

**Orthogonal polygons:** $< n$ flipturns
- count *brackets*:  (polygon interior on either side)
  - allow overlap  $\Rightarrow \le n$ brackets
- claim # brackets never decreases
  (13-case analysis)
- *orthogonal* flipturn kills two brackets:



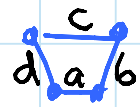
$\Rightarrow \le n/2$ orthogonal flipturns
- diagonal flipturn kills two vertices:



$\Rightarrow < n/2$ diagonal flipturns
$\Rightarrow < n$ total $\square$

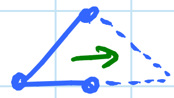
$\boxed{\text{OPEN}}$: $n - O(1)$ flipturns ever possible?
- best example requires $\frac{5}{6}n - O(1)$

**Deflation:** inverse of flip (avoiding crossings)
- conjectured finite [Wegner 1993]
- quadrilaterals with $a+c=b+d$ & $a \neq b \neq c \neq d \neq a$ always deflate infinitely




  [Fevens, Hernandez, Mesa, Morin, Soss, Toussaint 2001]
- that's all such quads [Ballinger, PhD 2003]
- no pentagon always deflates infinitely
  [Demaine, Demaine, Fevens, Mesa, Soss, Souvaine, Taslakian, Toussaint 2007]
- $\boxed{\text{OPEN}}$: $n \geq 6$ gon? (no flat vertices)

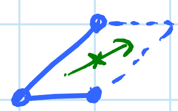
**Pop:** flip on 2 incident edges



[Millett 1994]
- can be forced to introduce crossings ⇒ allow
- possible to convexify any polygon in finitely many pops? [Ballinger & Thurston 2001]
  - NO: "alternating" polygons can't be [Dumitrescu & Hilscher 2009]

    vertices alternate between x & y axes

**Popturns:** flipturn on 2 incident edges



- equivalent to pops for equilateral polygons
- can convexify any polygon allowing self-intersection
- characterization of when possible without crossing

[Aloupis et al. 2007]

6.849 Geometric Folding Algorithms: Linkages, Origami, Polyhedra
Fall 2012