

**Has the conjecture based
on “fractal paper” been
resolved?**

Construction of Common Unfolding of a Regular Tetrahedron and a Cube

Toshihiro Shirakawa*

Takashi Horiyama†

Ryuhei Uehara‡

Drawing of unfolding of a cube and a tetramonohedron removed due to copyright restrictions.
Refer to: Fig. 4 from Shirakawa, T., T. Horiyama, et al. "[Construct of Common Development of Regular Tetrahedron and Cube.](#)" *27th European Workshop on Computational Geometry* (2011): 47-50.

**Any new results in a net for
3 different boxes?**

Common Developments of Several Different Orthogonal Boxes

Zachary Abel*

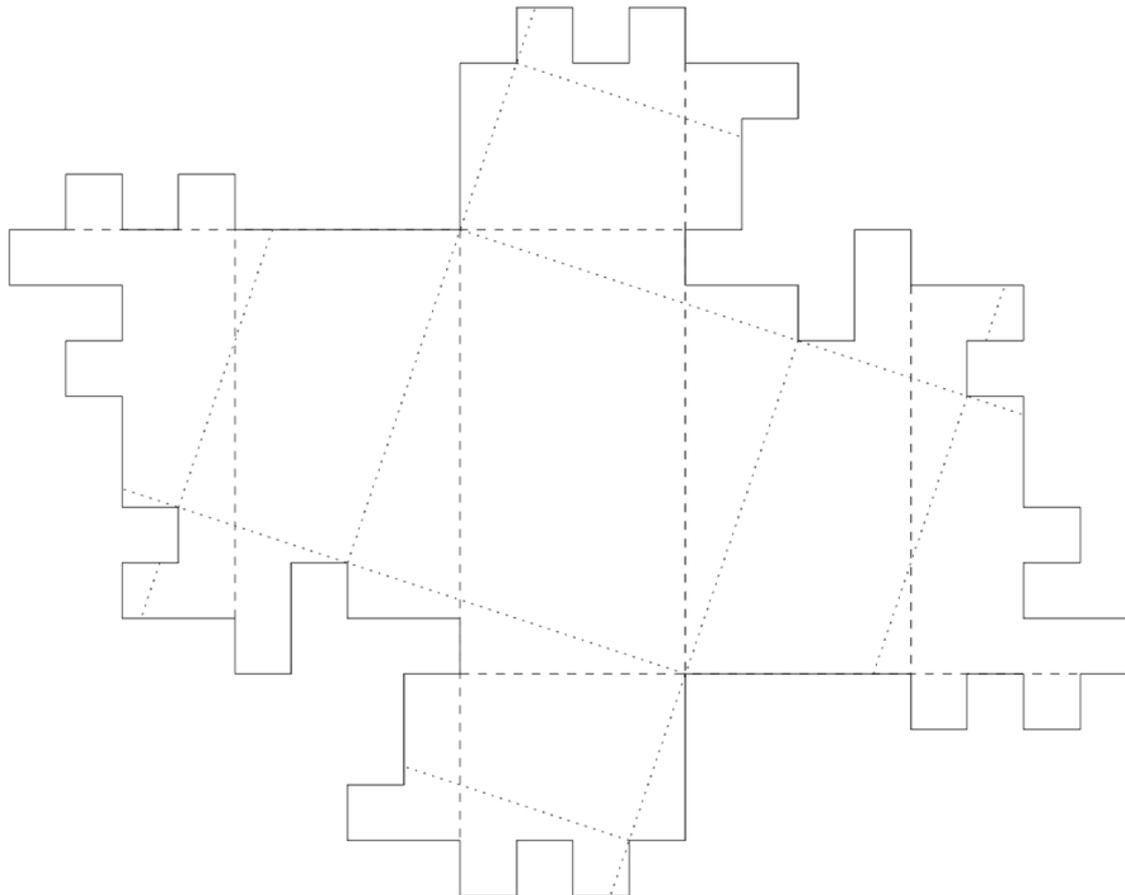
Erik Demaine[†]

Martin Demaine[‡]

Hiroaki Matsui[§]

Günter Rote[¶]

Ryuhei Uehara^{||}



Common unfolding of
 $4 \times 4 \times 8$ box and
 $\sqrt{10} \times 2\sqrt{10} \times 2\sqrt{10}$ box

[Abel, Demaine, Demaine,
 Matsui, Rote, Uehara 2011]

Courtesy of Zachary Abel, Erik Demaine, Martin Demaine, Hiroaki Matsui, Günter Rote, and Ryuhei Uehara. Used with permission.

Common Developments of Several Different Orthogonal Boxes

Zachary Abel*

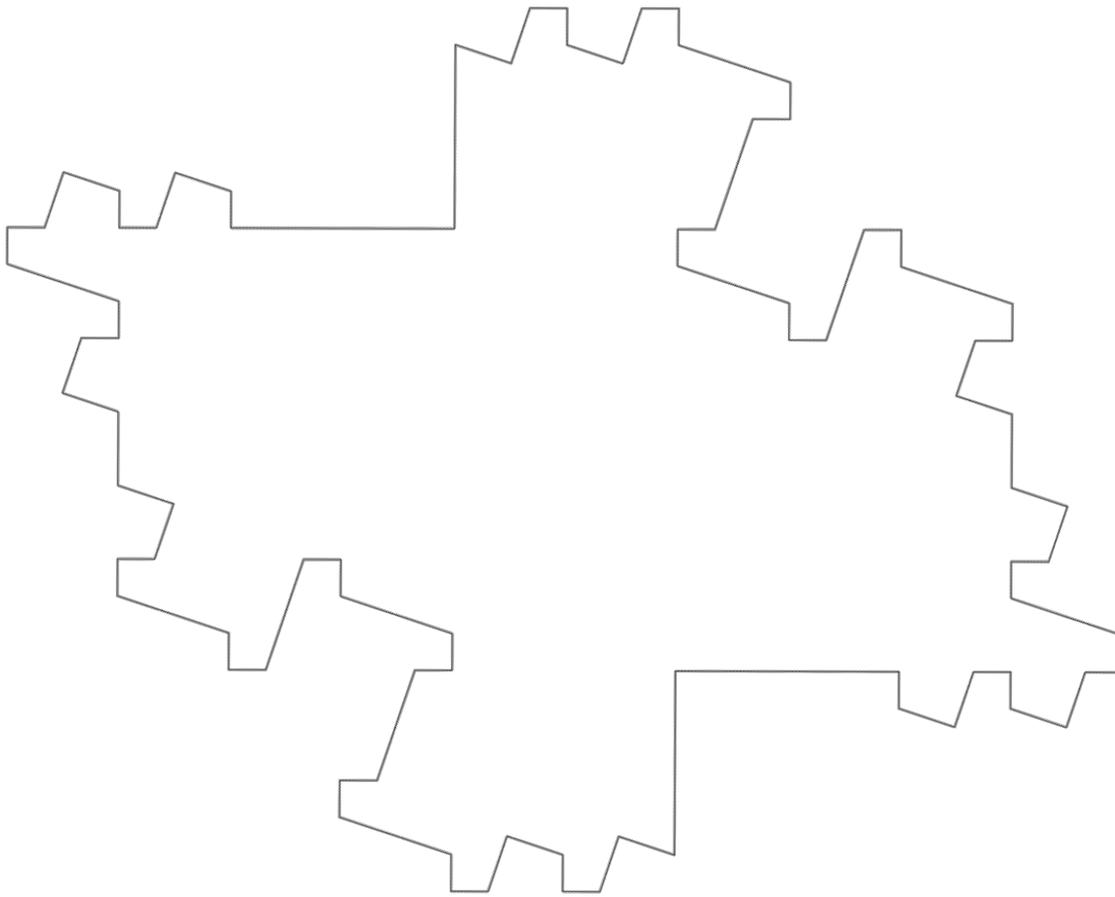
Erik Demaine[†]

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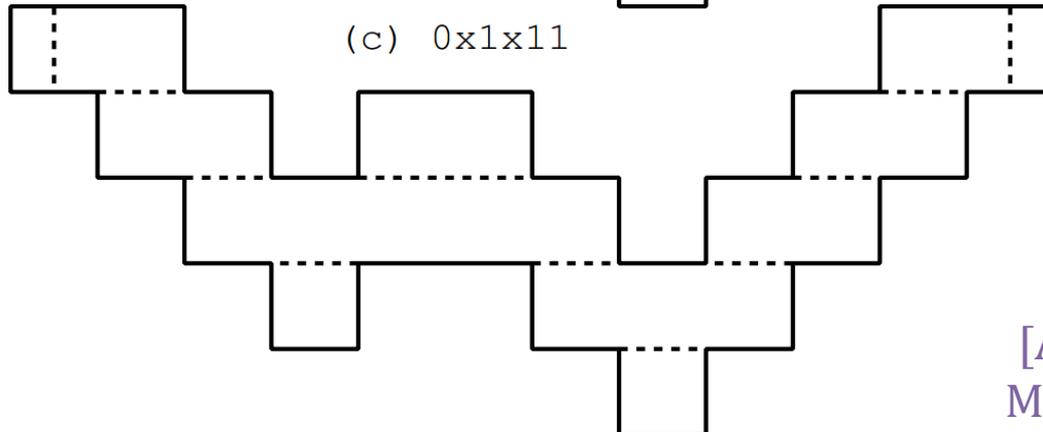
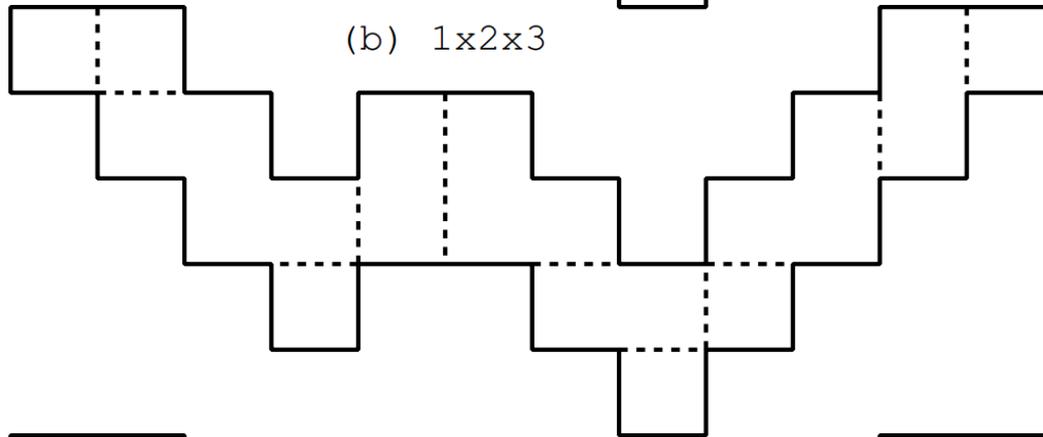
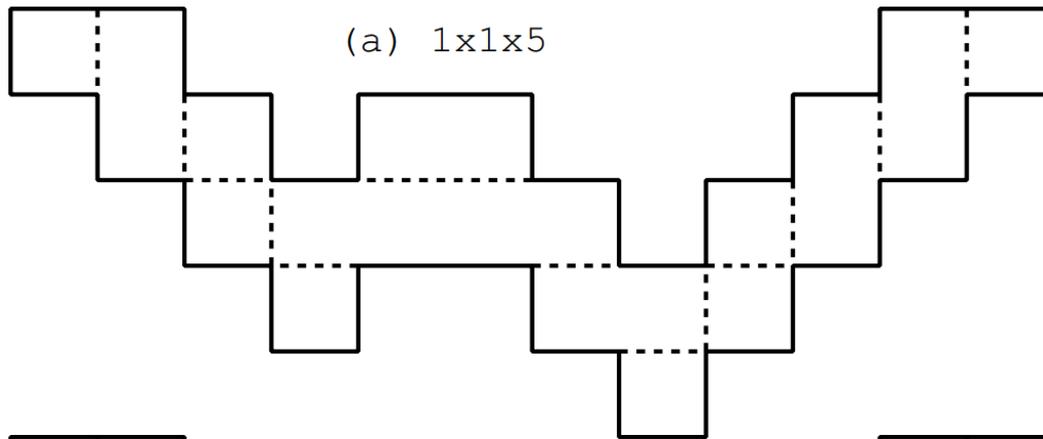
Ryuhei Uehara^{||}



Common unfolding of
 $4 \times 4 \times 8$ box and
 $\sqrt{10} \times 2\sqrt{10} \times 2\sqrt{10}$ box

[Abel, Demaine, Demaine,
Matsui, Rote, Uehara 2011]

Courtesy of Zachary Abel, Erik Demaine, Martin Demaine, Hiroaki Matsui, Günter Rote, and Ryuhei Uehara. Used with permission.



[Abel, Demaine, Demaine,
Matsui, Rote, Uehara 2011]

Courtesy of Zachary Abel, Erik Demaine, Martin Demaine, Hiroaki Matsui, Günter Rote, and Ryuhei Uehara. Used with permission.

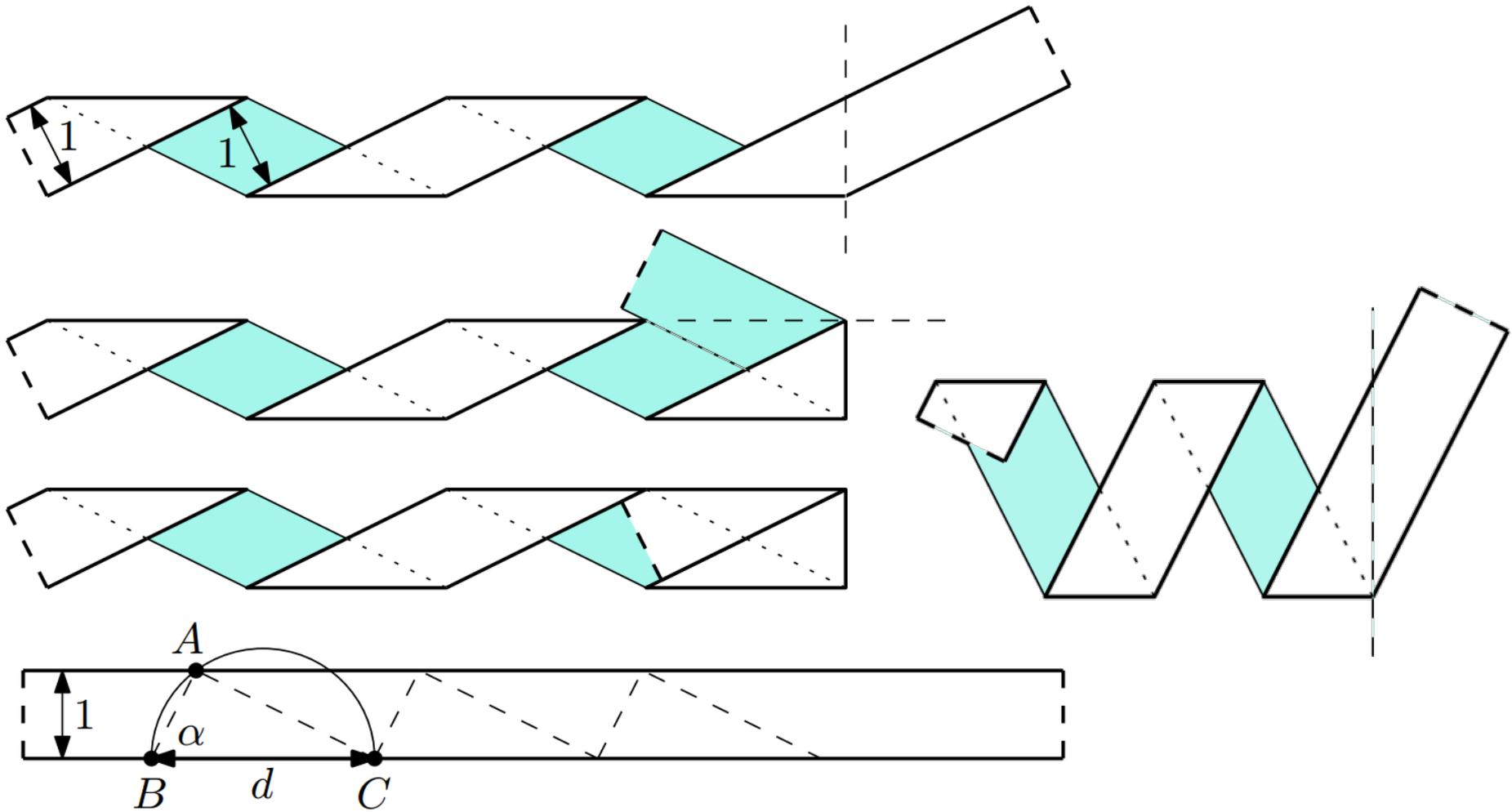
Input : None;

Output: Polygons that consist of 22 squares and fold to boxes of size $1 \times 1 \times 5$ and $1 \times 2 \times 3$;

[Abel, Demaine, Demaine, Matsui, Rote, Uehara 2011]

```
1 let  $L_1$  be a set of one unit square;
2 for  $i = 2, 3, 4, \dots, 22$  do
3    $L_i := \emptyset$ ;
4   for each common partial development  $P$  in
      $L_{i-1}$  do
5     for every polygon  $P^+$  of size  $i$  obtained by
       attaching a unit square to  $P$  do
6       check if  $P^+$  is a common partial
         development, and add it into  $L_i$  if it is a
           new one;
7     end
8   end
9 end
10 output  $L_{22}$ ;
```

i	1	2	3	4	5	6	7	8	9
L_i	1	1	2	5	12	35	108	368	1283
i -ominos	1	1	2	5	12	35	108	369	1285
i	10	11	12	13	14				
L_i	4600	16388	57439	193383	604269				
i -ominos	4655	17073	63600	238591	901971				
i	15	16	17	18					
L_i	1632811	3469043	5182945	4917908					
i -ominos	3426576	13079255	50107909	192622052					
i	19	20	21	22					
L_i	2776413	882062	133037	2263					



Courtesy of Zachary Abel, Erik Demaine, Martin Demaine, Hiroaki Matsui, Günter Rote, and Ryuhei Uehara. Used with permission.

[Abel, Demaine, Demaine, Matsui, Rote, Uehara 2011]

Common Developments of Three Different Orthogonal Boxes

Toshihiro Shirakawa

Ryuhei Uehara*

Abstract

We investigate common developments that can fold into plural incongruent orthogonal boxes. It was shown that there are infinitely many orthogonal polygons that fold into two incongruent orthogonal boxes in 2008. In 2011, it was shown that there exists an orthogonal polygon that folds into three boxes of size $1 \times 1 \times 5$, $1 \times 2 \times 3$, and $0 \times 1 \times 11$. It remained open whether there exists an orthogonal polygon that folds into three boxes of positive volume. We give an affirmative answer to this open problem: there exists an orthogonal polygon that folds into three boxes of size $7 \times 8 \times 56$, $7 \times 14 \times 38$, and $2 \times 13 \times 58$. The construction idea can be generalized, and hence there exists an infinite number of orthogonal polygons that fold into three incongruent orthogonal boxes.

1 Introduction

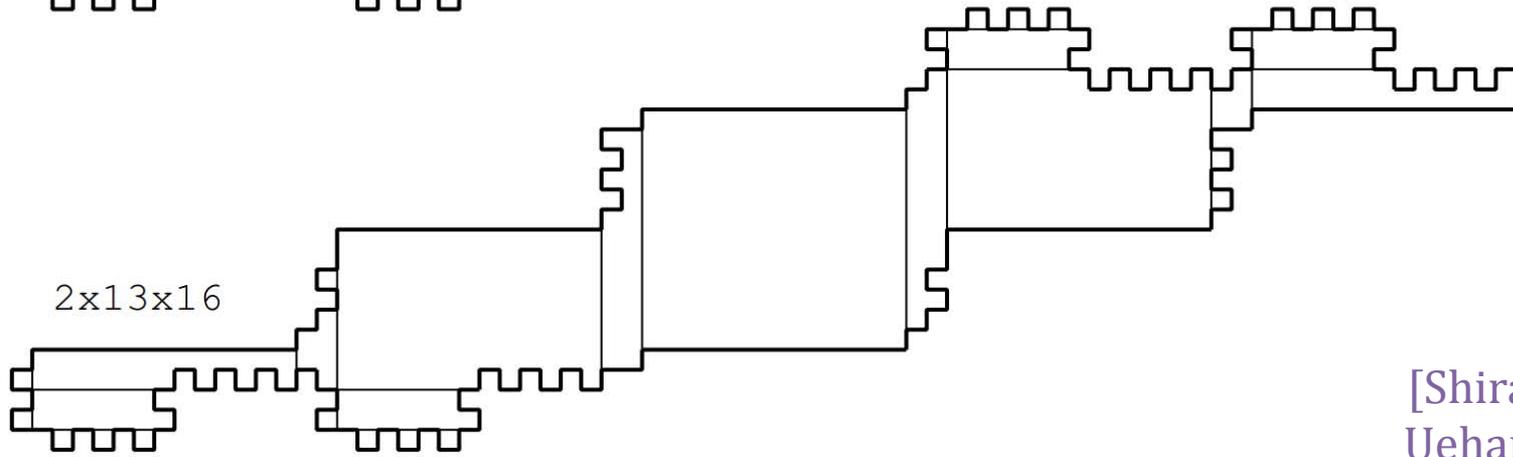
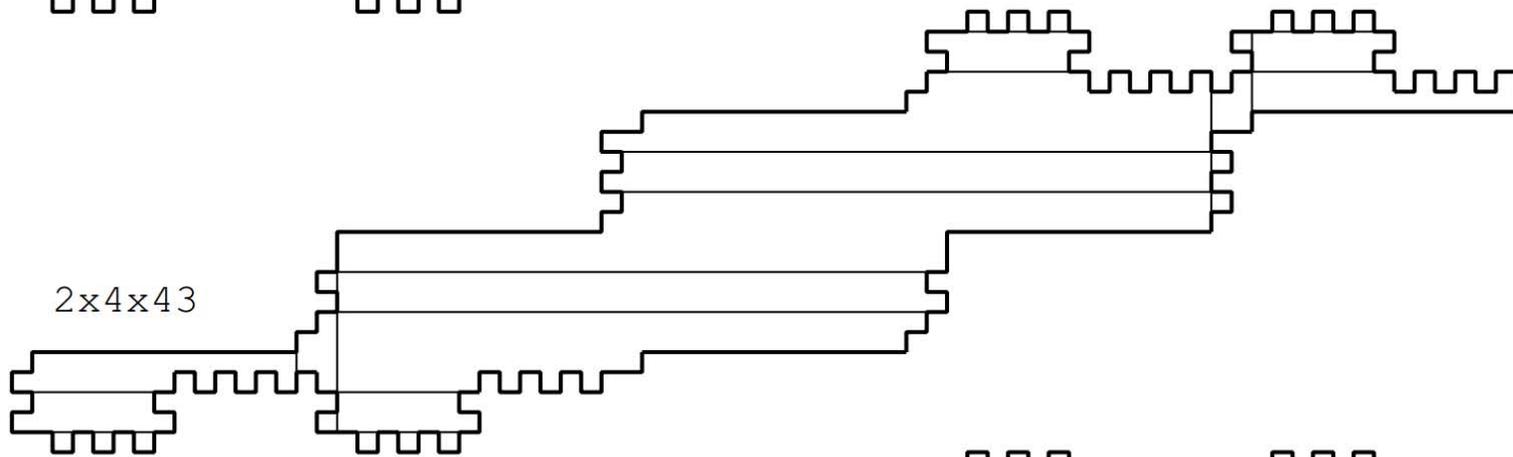
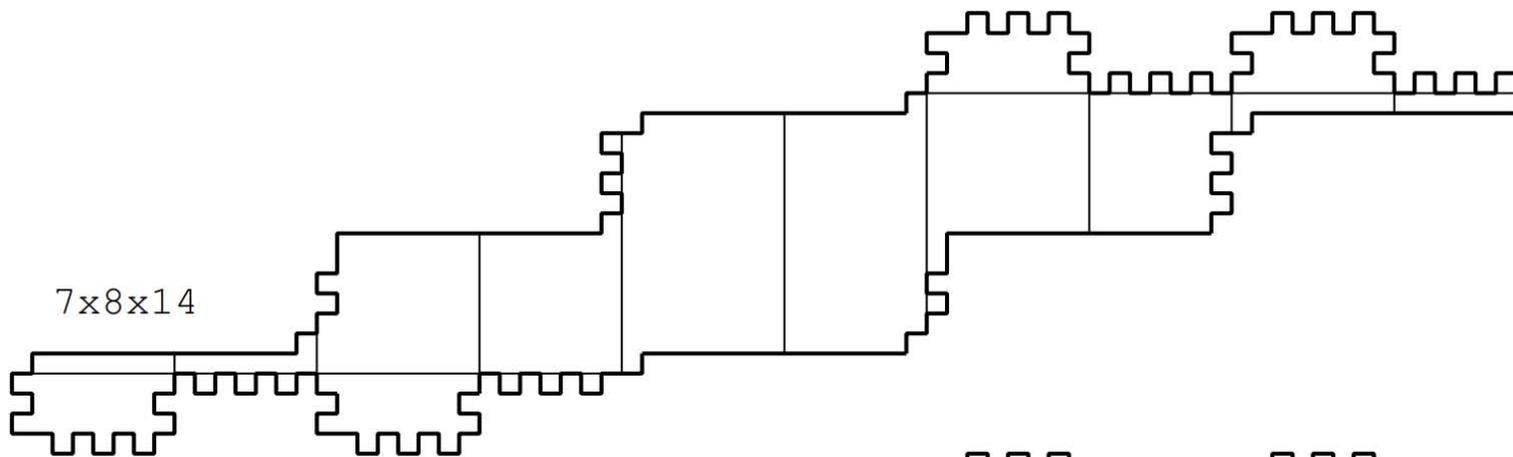
Since Lubiw and O'Rourke posed the problem in 1996



Figure 1: Cubigami.

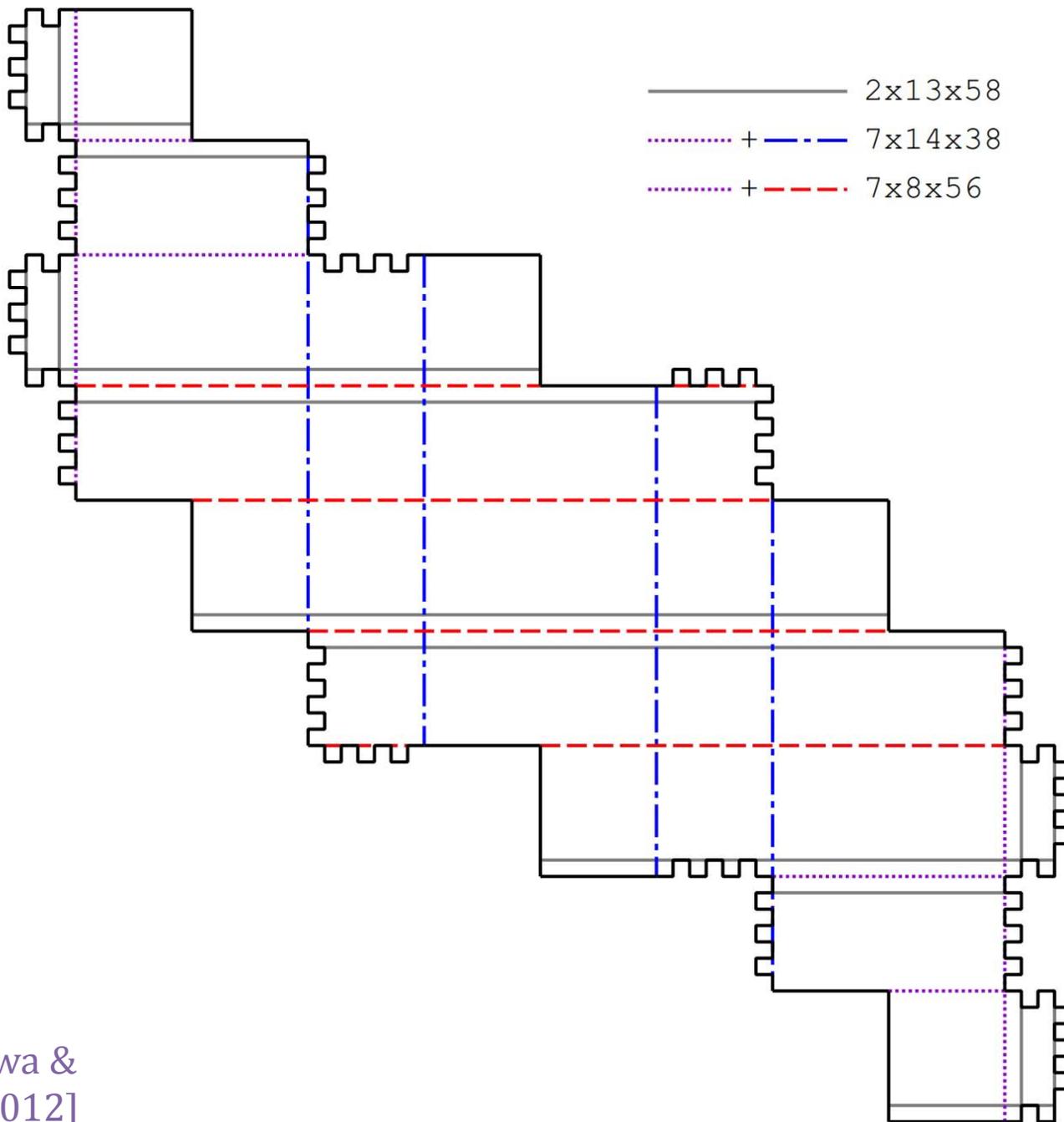
three incongruent orthogonal boxes of size $7 \times 8 \times 56$, $7 \times 14 \times 38$, and $2 \times 13 \times 58$ (Figure 2)¹.

The construction idea can be generalized. Therefore, we conclude that there exist infinitely many orthogonal polygons that can fold into three incongruent ortho-



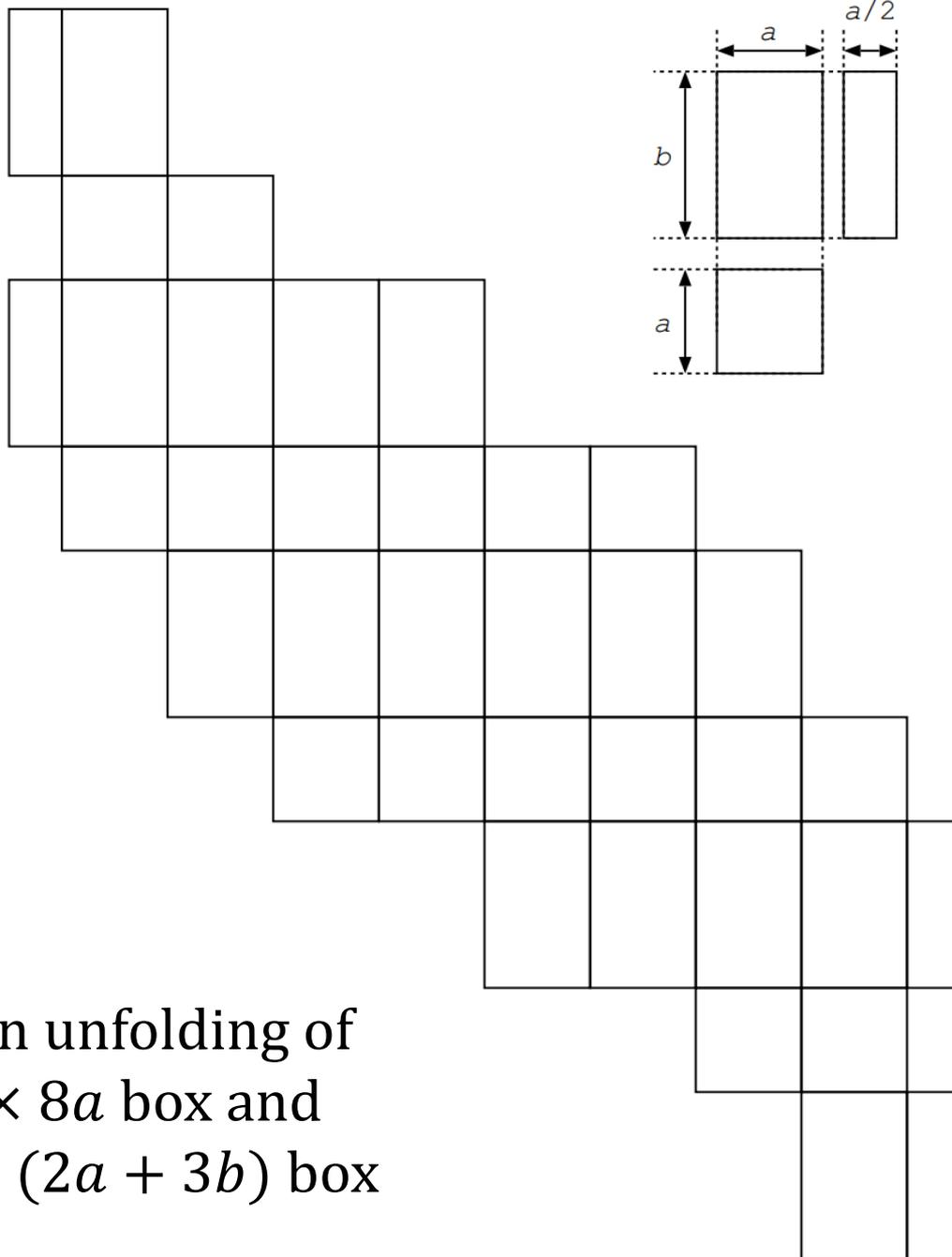
[Shirakawa & Uehara 2012]

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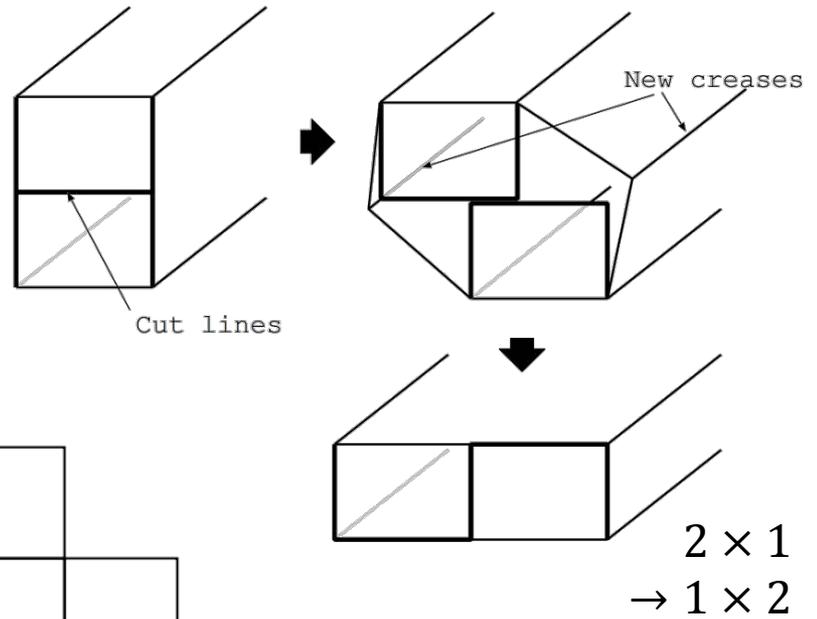
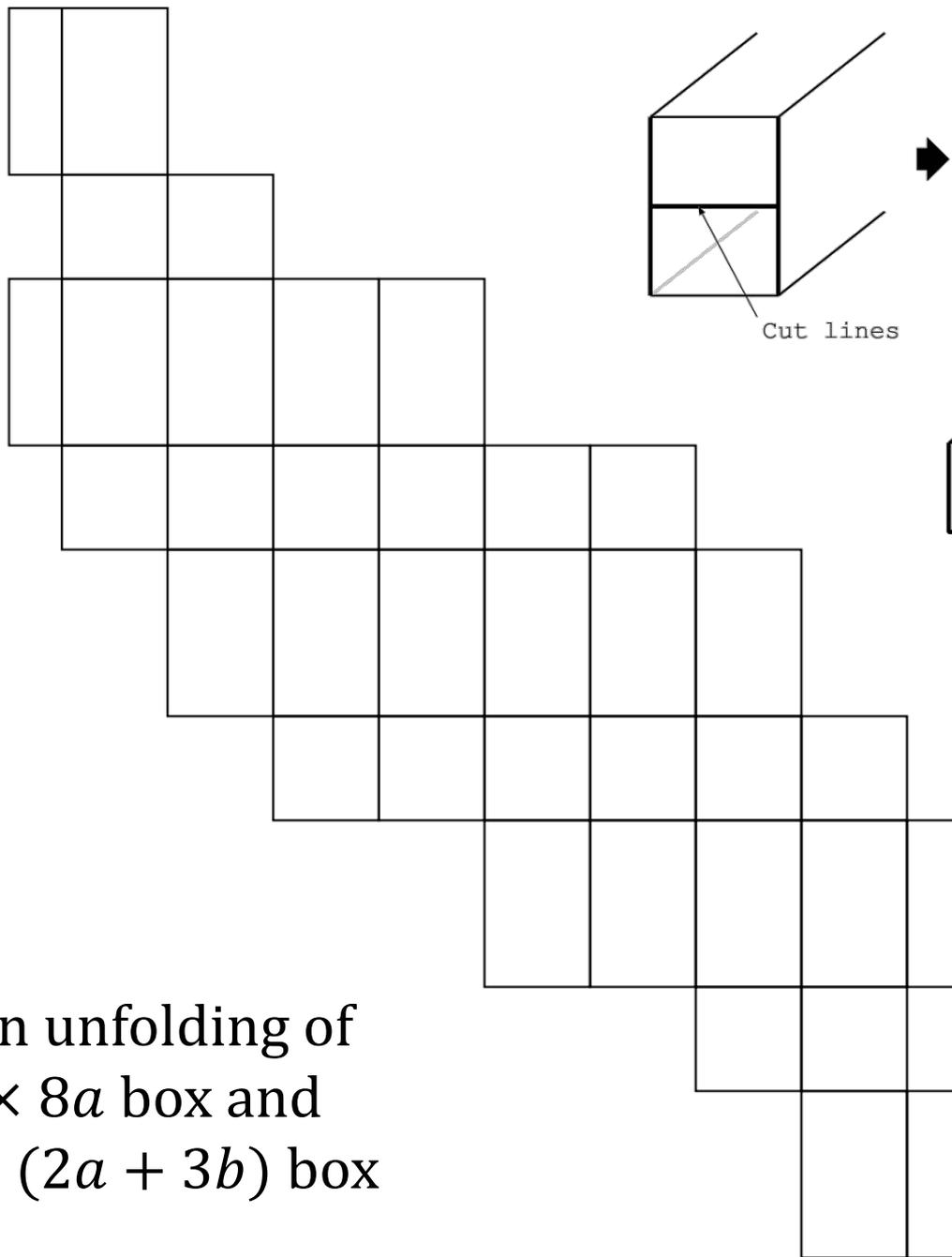
[Shirakawa & Uehara 2012]

Courtesy of Toshihiro Shirakawa and Ryuhei Uehara. Used with permission.



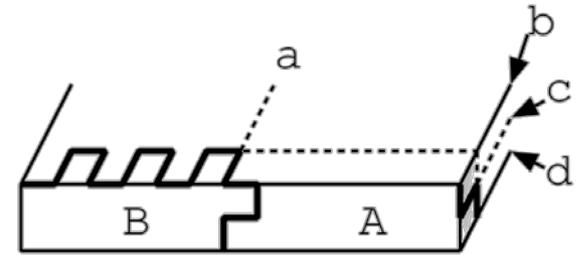
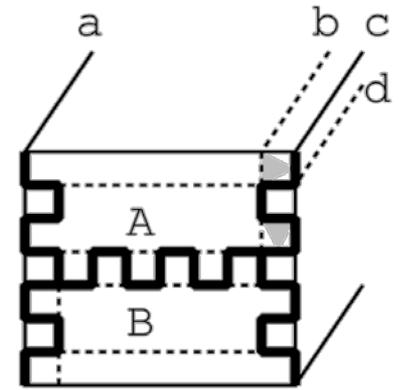
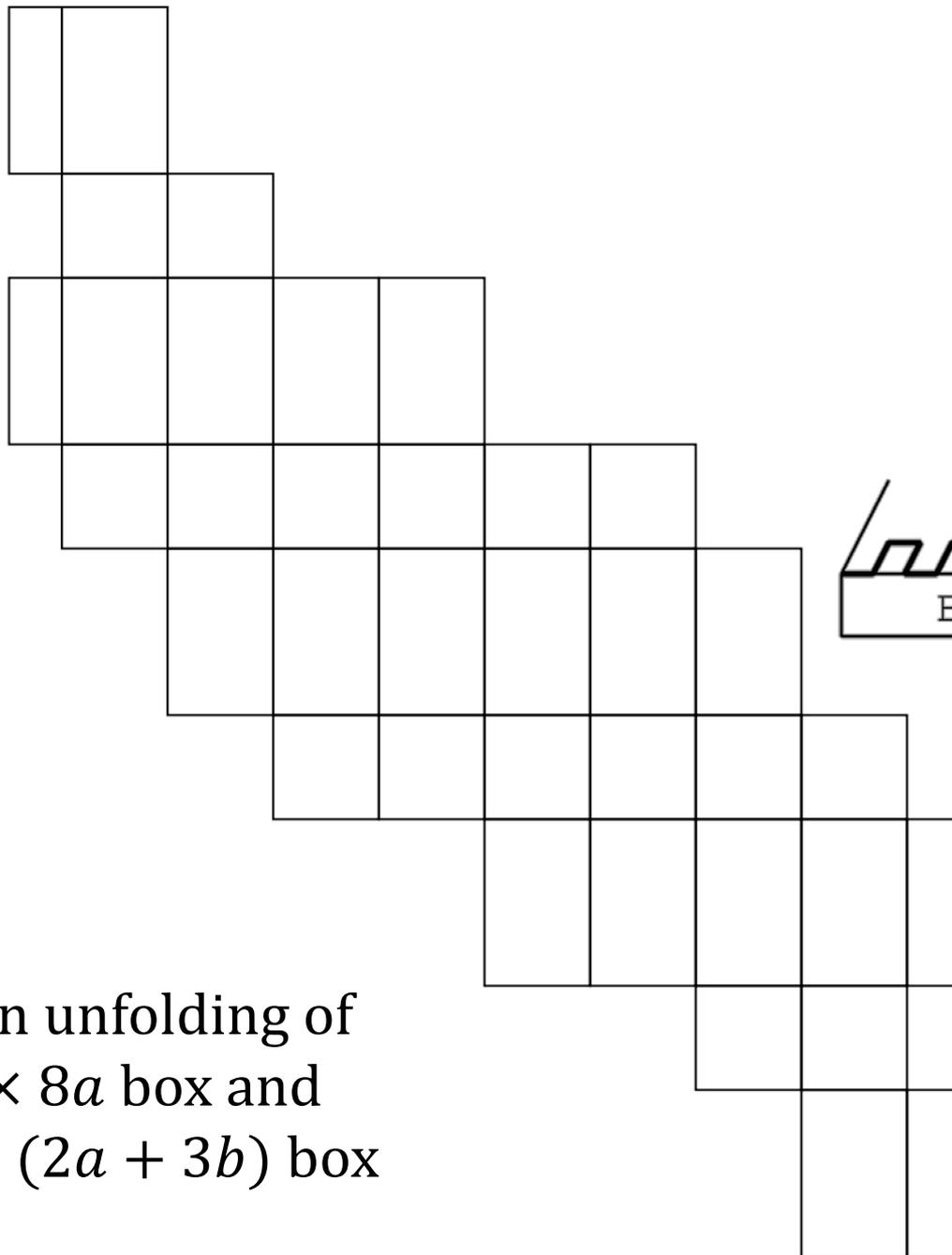
Common unfolding of
 $a \times b \times 8a$ box and
 $a \times 2a \times (2a + 3b)$ box

[Shirakawa &
 Uehara 2012]



Common unfolding of
 $a \times b \times 8a$ box and
 $a \times 2a \times (2a + 3b)$ box

[Shirakawa &
 Uehara 2012]

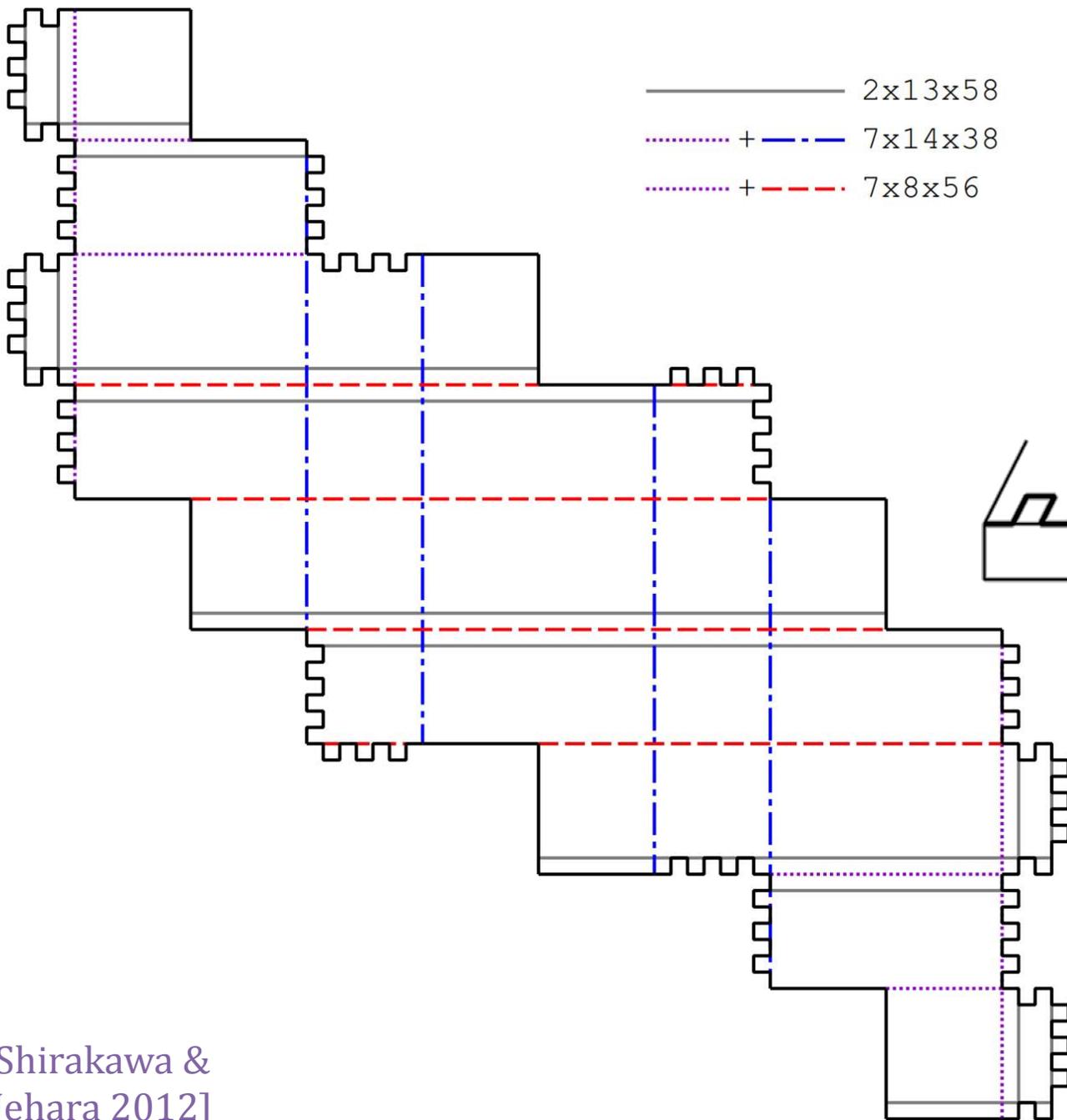


$$7 \times 8$$

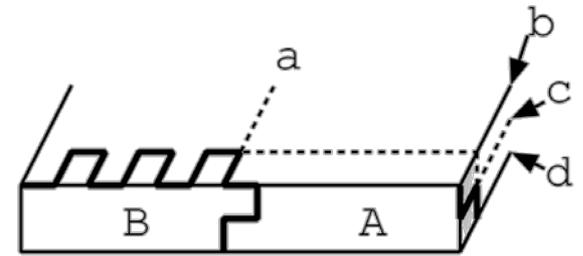
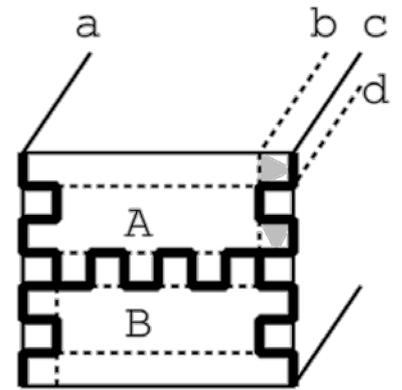
$$\rightarrow 2 \times 13$$

Common unfolding of
 $a \times b \times 8a$ box and
 $a \times 2a \times (2a + 3b)$ box

[Shirakawa &
 Uehara 2012]



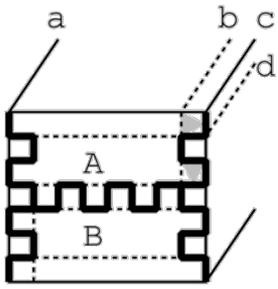
- $2 \times 13 \times 58$
- + + ———— $7 \times 14 \times 38$
- + + ———— $7 \times 8 \times 56$



$$7 \times 8$$

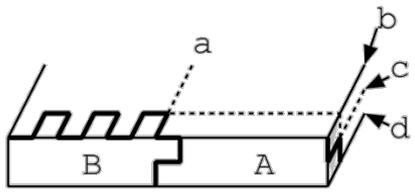
$$\rightarrow 2 \times 13$$

[Shirakawa & Uehara 2012]

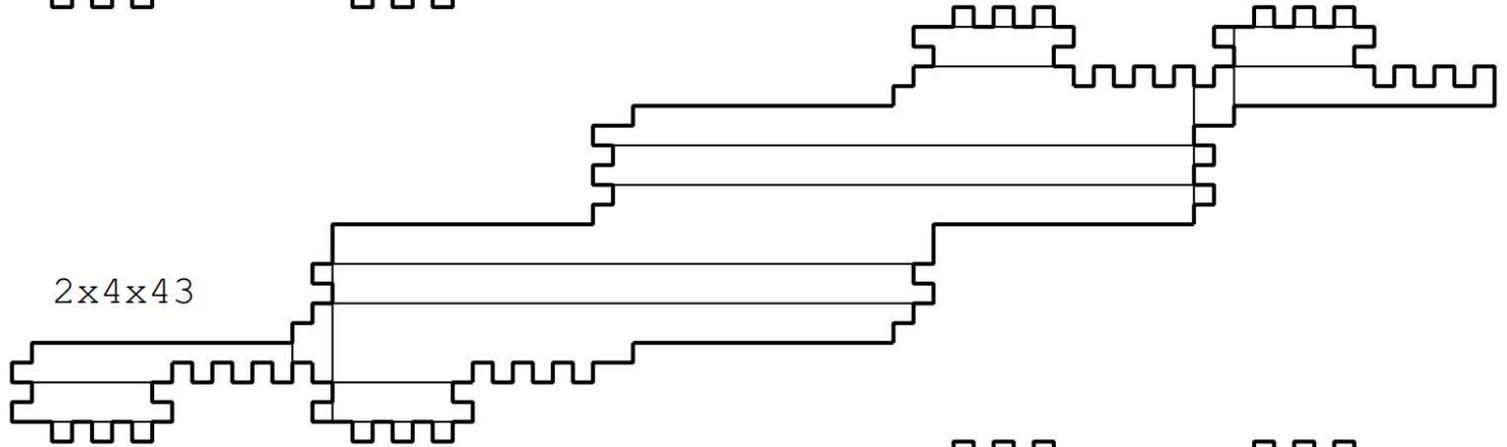
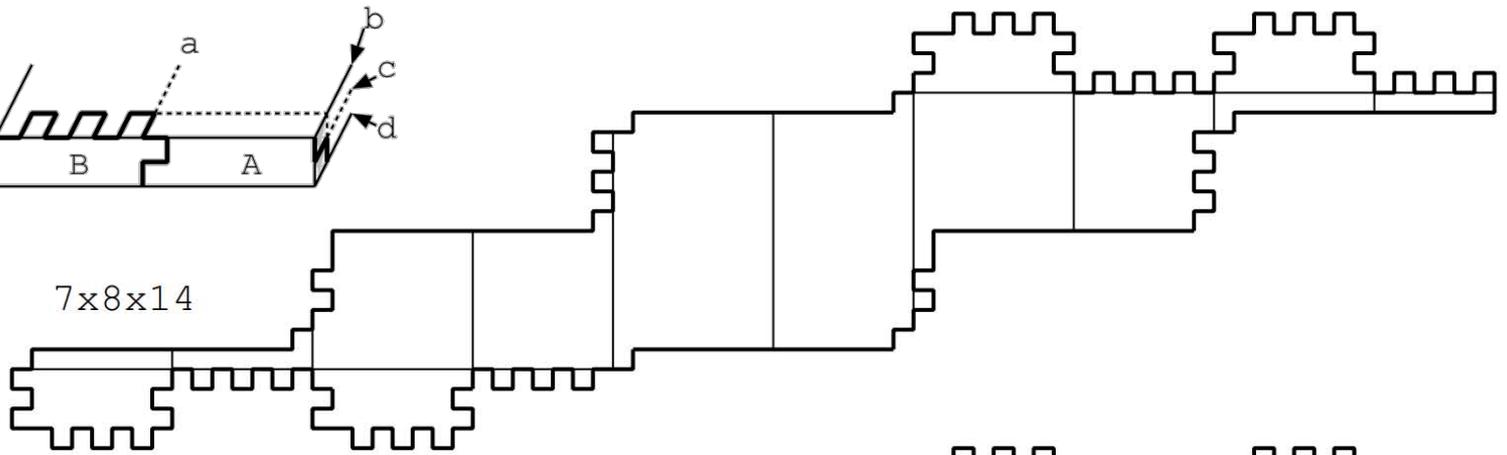


$$7 \times 8$$

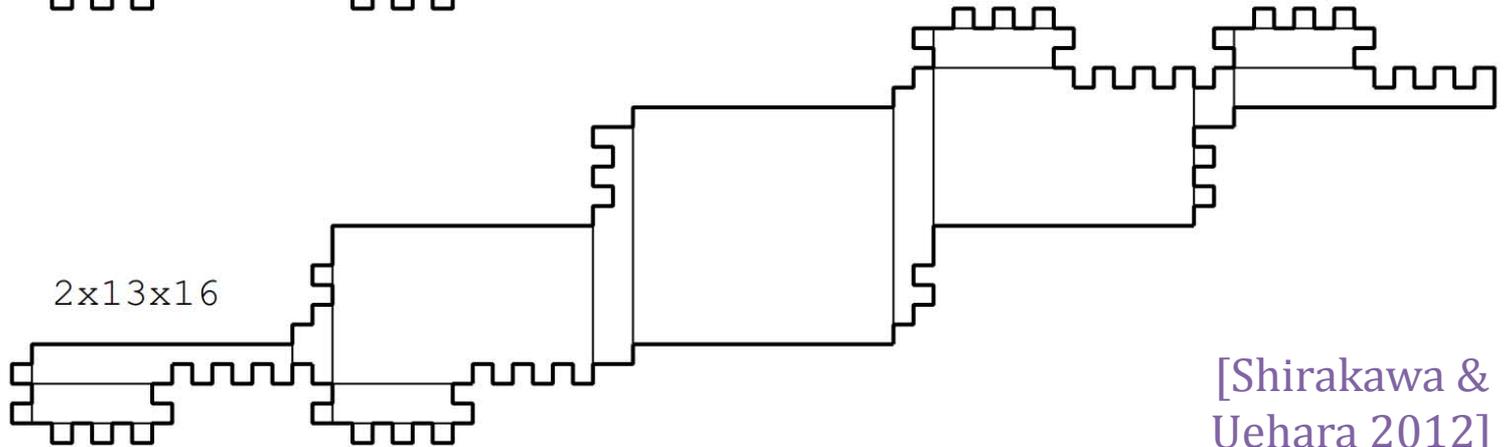
$$\rightarrow 2 \times 13$$



$$7 \times 8 \times 14$$

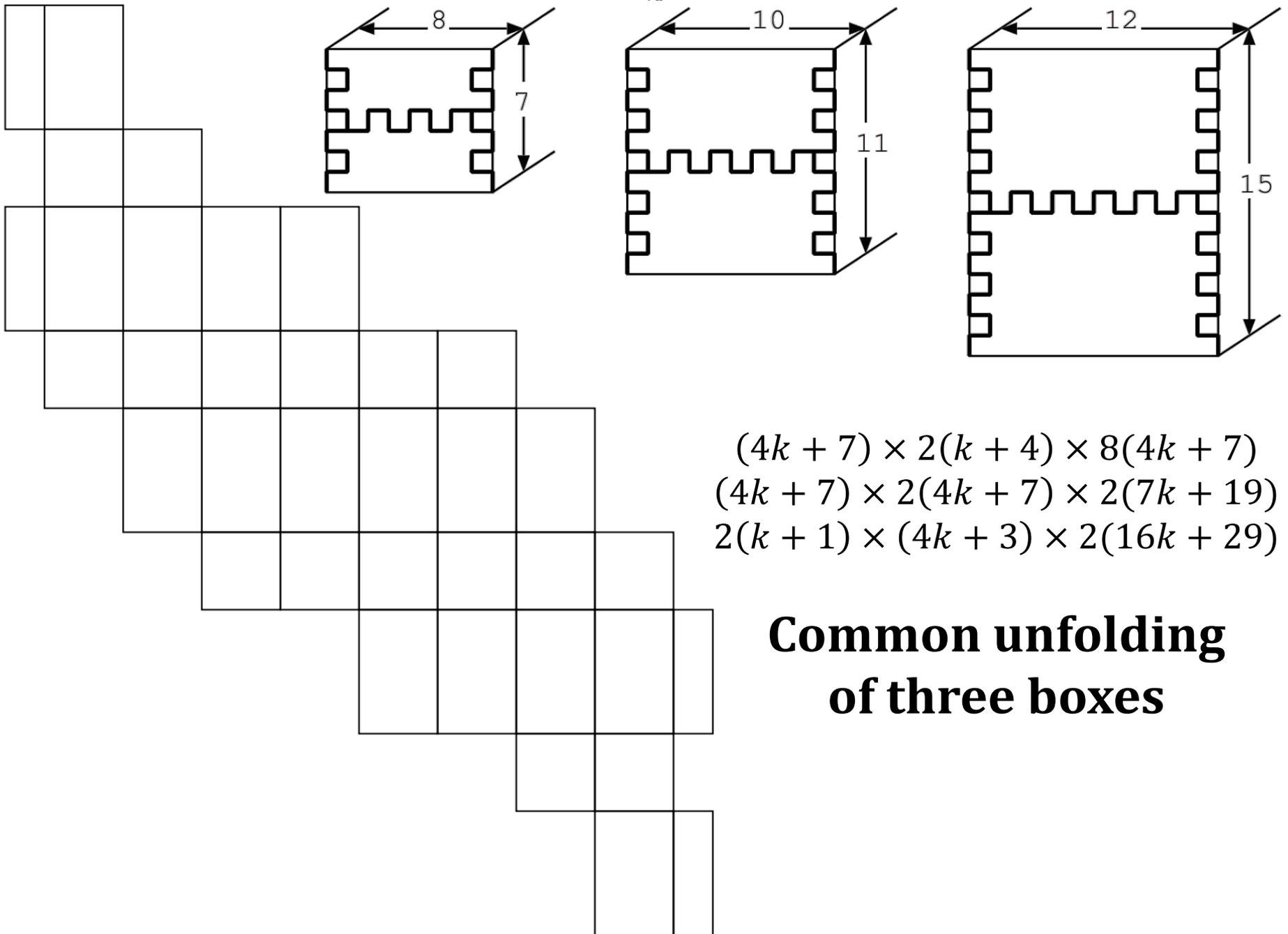


$$2 \times 4 \times 43$$



$$2 \times 13 \times 16$$

[Shirakawa & Uehara 2012]



$$\begin{aligned}
 &(4k + 7) \times 2(k + 4) \times 8(4k + 7) \\
 &(4k + 7) \times 2(4k + 7) \times 2(7k + 19) \\
 &2(k + 1) \times (4k + 3) \times 2(16k + 29)
 \end{aligned}$$

Common unfolding of three boxes

I'm kind of unsettled by the non-area-preserving unfolding. If it were a true limit then we'd be able to get arbitrarily close to the non-preserved area by unfolding into sufficiently many pieces. But this isn't the case: either we get the non-preserved area by unfolding into infinitely many pieces, or we get the original area, by unfolding into finitely many pieces.

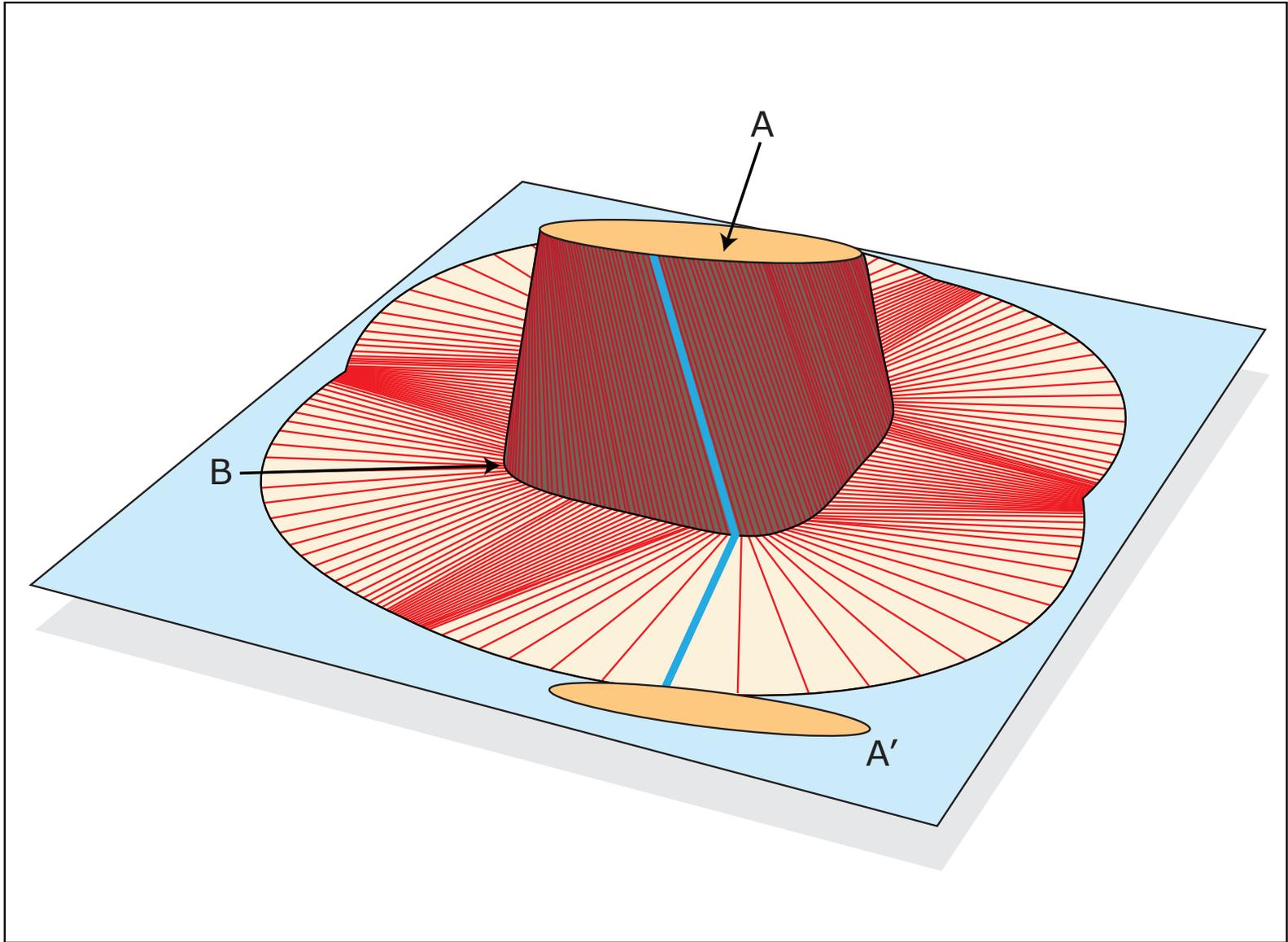


Image by MIT OpenCourseWare.

[Benbernou, Cahn, O'Rourke 2004]

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<http://vimeo.com/14648143>

“Ordis 2007”

Theo Jansen's *Strandbeests*

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<http://vimeo.com/14647032>

“Umerus 2009”

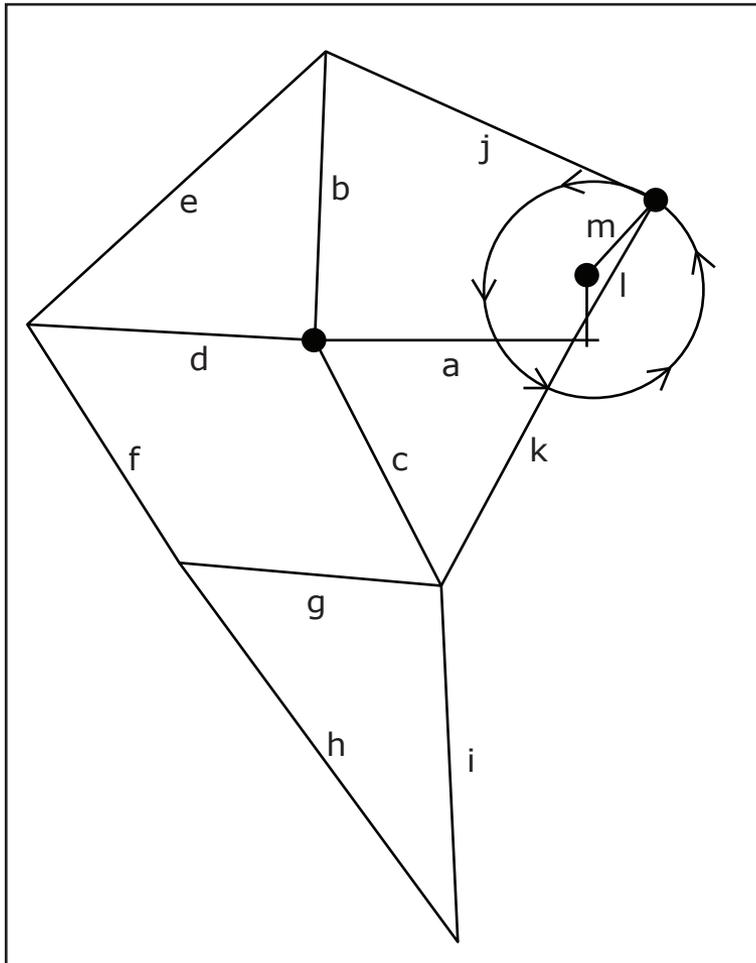


Image by MIT OpenCourseWare.
See also http://www.strandbeest.com/beests_leg.php/.

- $a = 38$
- $b = 41.5$
- $c = 39.3$
- $d = 40.1$
- $e = 55.8$
- $f = 39.4$
- $g = 36.7$
- $h = 65.7$
- $i = 49$
- $j = 50$
- $k = 61.9$
- $l = 7.8$
- $m = 15$

[Theo Jansen]

“Eleven holy numbers”

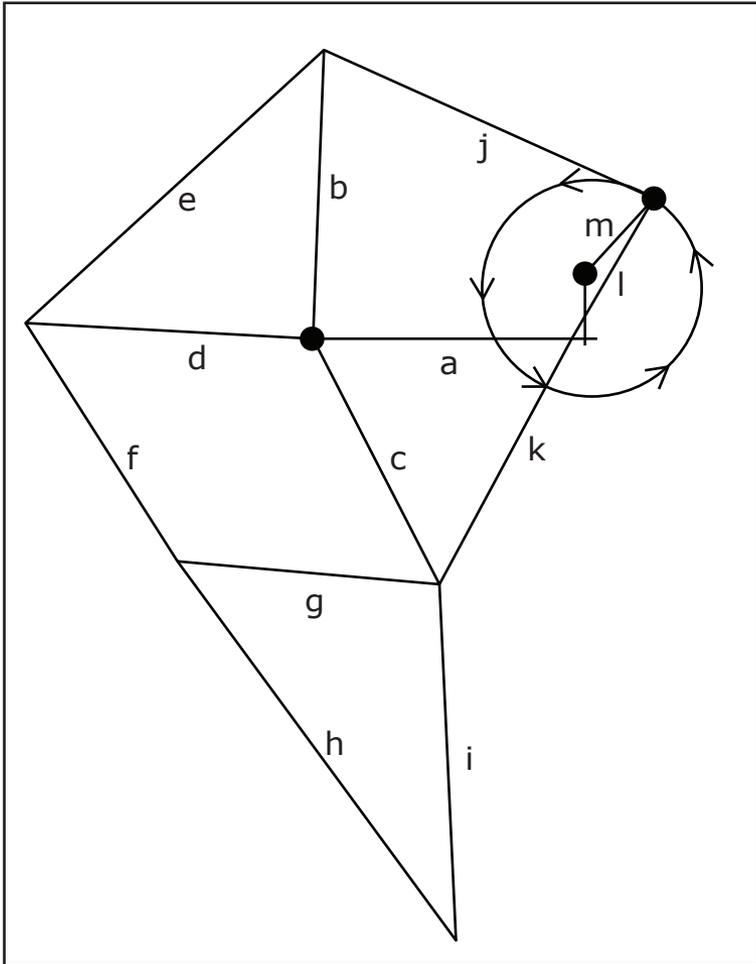
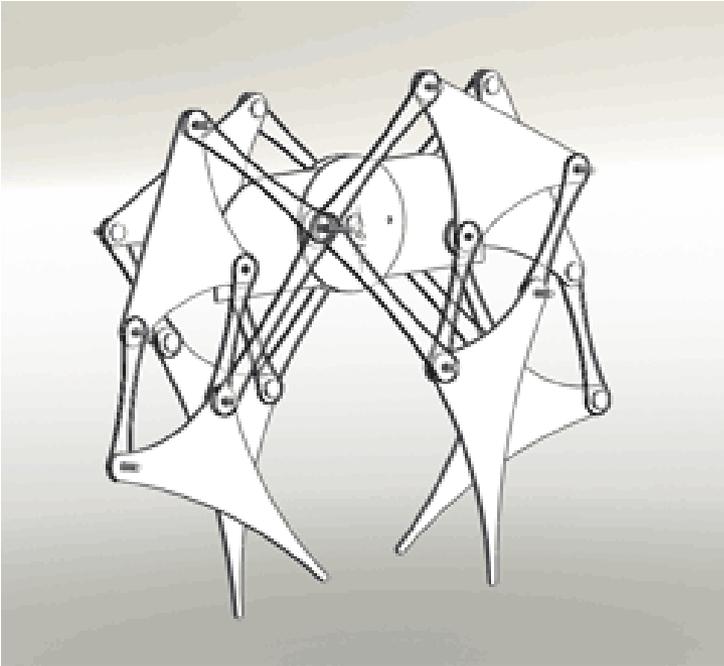


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Diagram removed due to copyright restrictions.

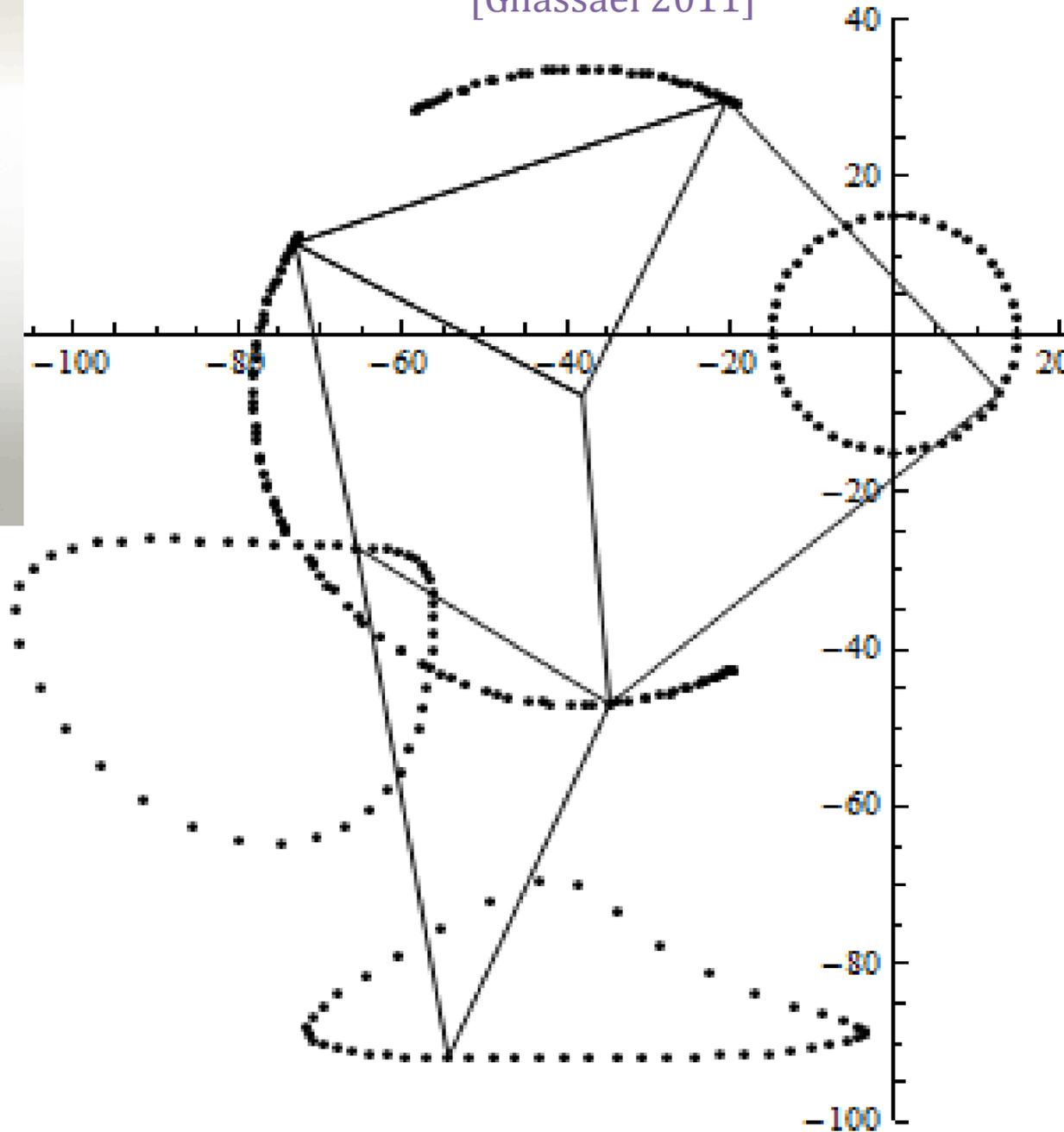
[Theo Jansen]



Courtesy of Jansen Walker by 4volt.com.
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[4volt.com]

[Ghassaei 2011]



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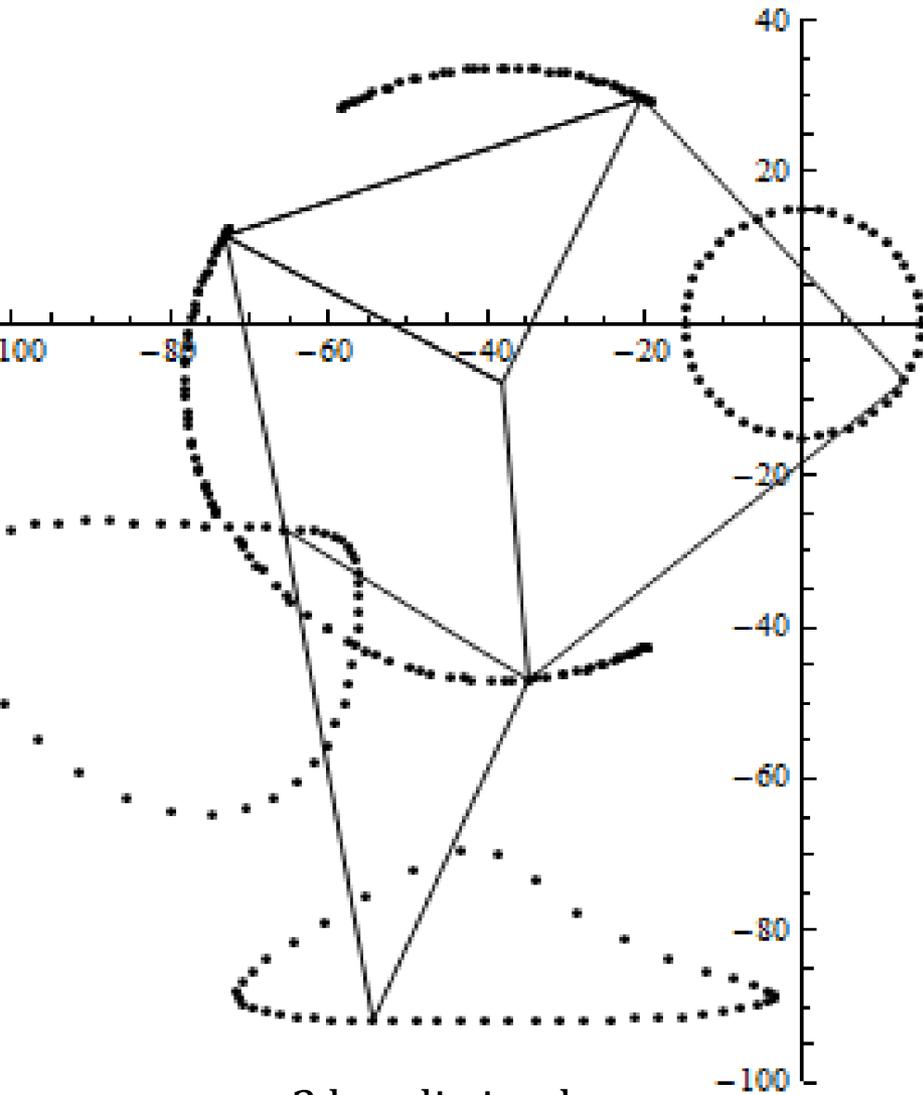
Theo Jansen's *Strandbeests*

<http://youtu.be/NM4q-f68TIY>

“strandbeests ... mechanism”

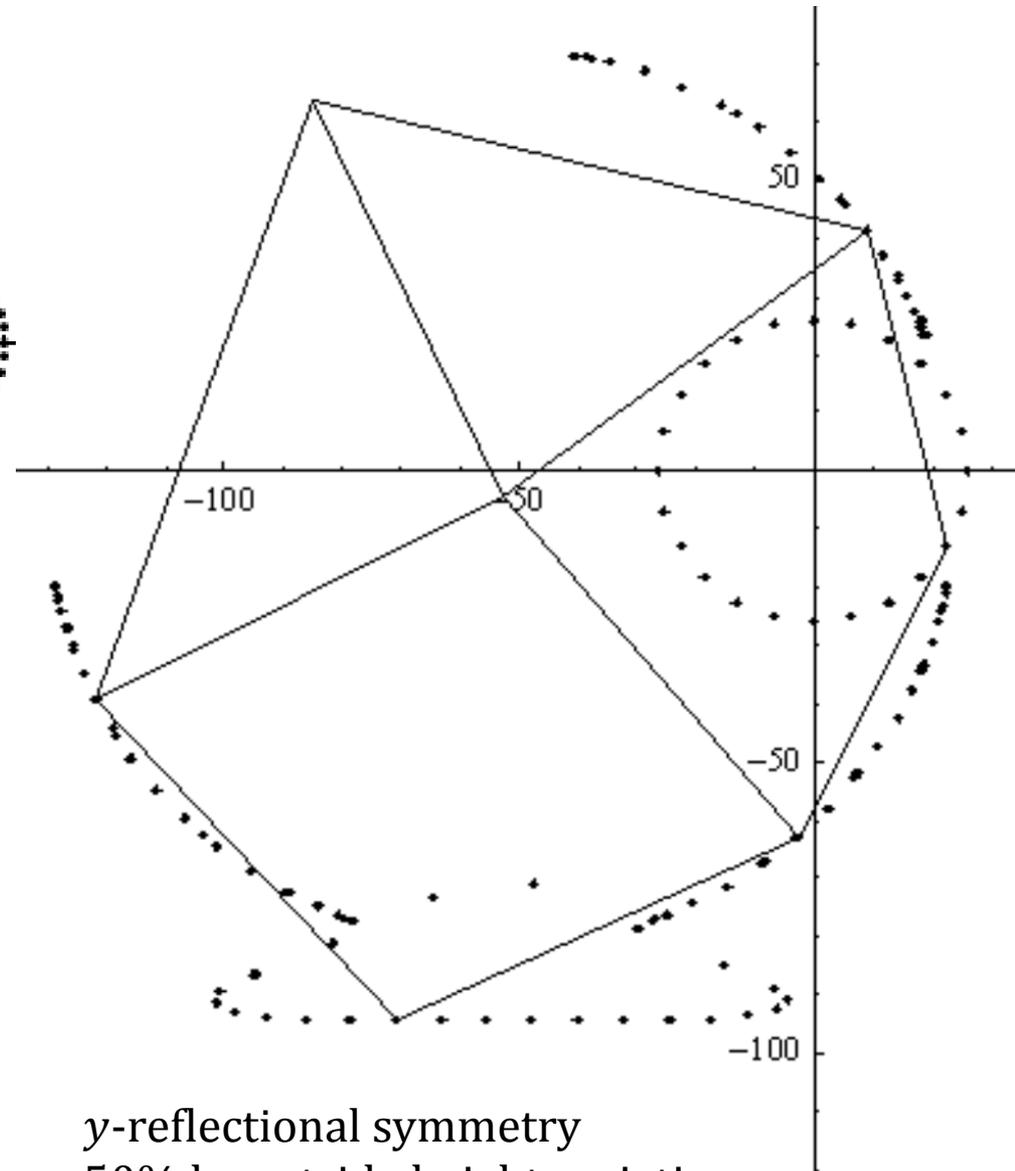
petabyte99

Jansen mechanism



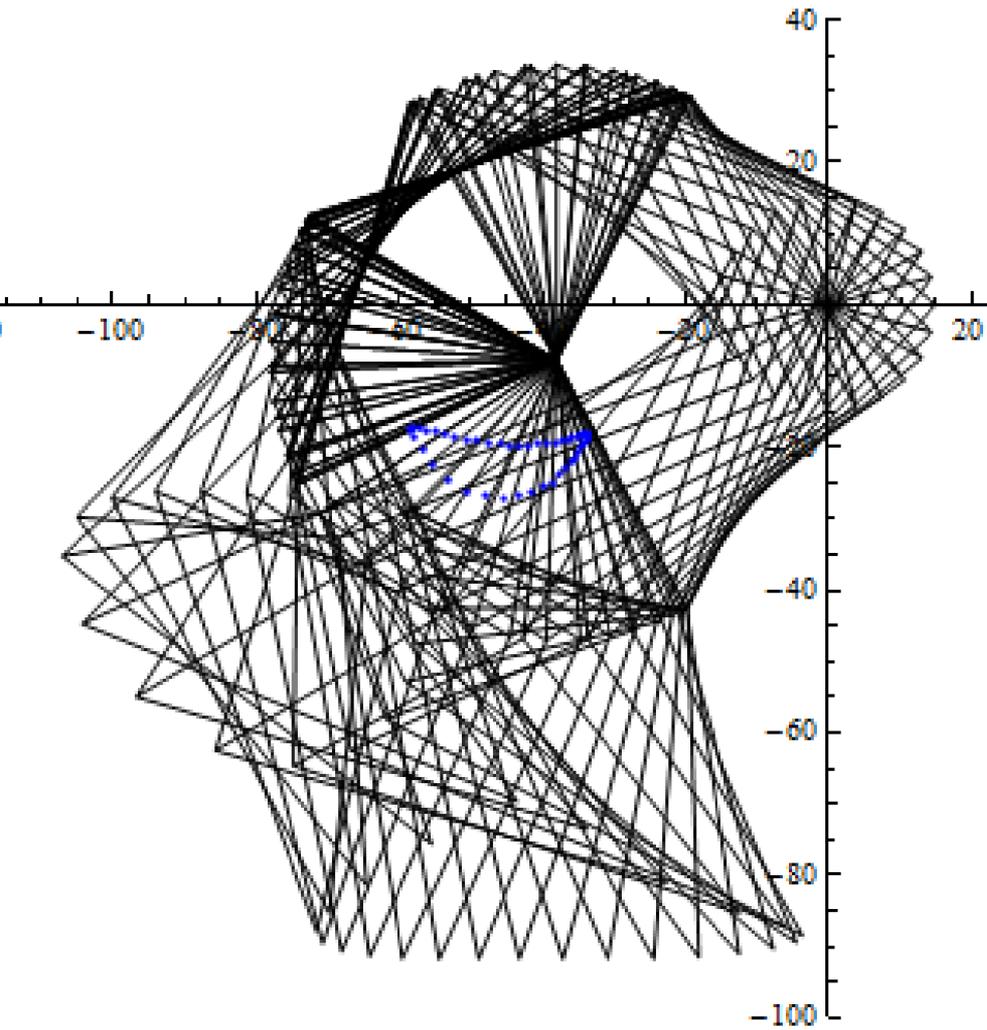
2 legs lie in plane
33% higher step
4% faster stride

Ghassaei mechanism [2011]

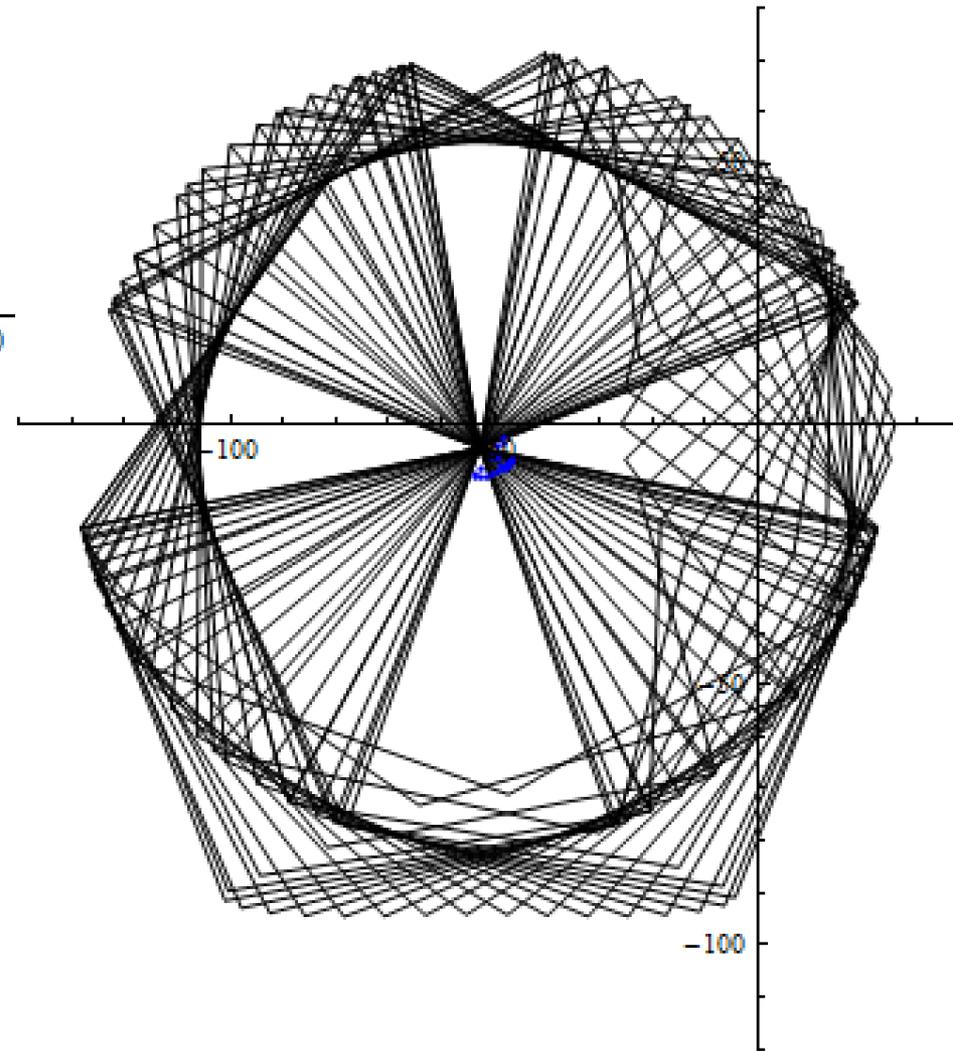


y-reflectal symmetry
50% less stride height variation
50% less stride x velocity variation

Jansen mechanism



Ghassaei mechanism [2011]



Courtesy of Amanda Ghassaei. Used with permission.

center of mass

85% less center of
mass movement

Theo Jansen's *Strandbeests*

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<http://vimeo.com/24278413>

“Animaris Gubernare — Tumble”

Theo Jansen's *Strandbeests*

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<http://vimeo.com/51811740>

“Animaris Adulari”

Theo Jansen's *Strandbeests*

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<http://vimeo.com/52745220>

“about the wings”

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<http://vimeo.com/44057387>

“Adulari lifting itself ...”

Theo Jansen's *Strandbeests*

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“Wagging Neck”

Theo Jansen's *Strandbeests*

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<http://vimeo.com/14646877>

“Untitled”

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<http://vimeo.com/11150979>

“Rhinoceros”

Photographs of Theo Jansen assembly kits removed due to copyright restrictions.
Refer to: http://www.strandbeest.com/shop/index_usa.php.
To view video: <http://www.youtube.com/watch?v=tHXy1nmVXg4> &
<http://www.youtube.com/watch?v=i8KVXy-vluU>.

Photographs of Theo Jansen 3D-printed models removed due to copyright restrictions.
Refer to: http://www.strandbeest.com/shop/beasts_3d.php.

Kinetic Creatures

Land Crawler eXtreme Locomotion Demo Video

<http://youtu.be/U5dpGAw4c0U>

vagabondworks

Poster for "Ocean Beasts" exhibit at The Simons Center (July-August 2012) removed due to copyright restrictions.

Machine with 23 Scraps of Paper

Margot's Other Cat

Arthur Ganson

Machine with Roller Chain

Machine with Oil

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6.849 Geometric Folding Algorithms: Linkages, Origami, Polyhedra
Fall 2012

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