# 6.852: Distributed Algorithms Fall, 2009

Class 24

# Today's plan

- Self-stabilization
- Self-stabilizing algorithms:
  - Breadth-first spanning tree
  - Mutual exclusion
- Composing self-stabilizing algorithms
- Making non-self-stabilizing algorithms self-stabilizing
- Reading:
  - [Dolev, Chapter 2]
- Next time:
  - Partially synchronous distributed algorithms
  - Clock synchronization
  - Reading:
    - Chapters 23-25
    - [Attiya, Welch] Section 6.3, Chapter 13

### Self-stabilization

- A useful fault-tolerance property for distributed algorithms.
- Algorithm can start in any state---arbitrarily corrupted.
- From there, if it runs normally (usually, without any further failures), it eventually gravitates back to correct behavior.
- [Dijkstra 73: Self-Stabilizing Systems in Spite of Distributed Control]
  - Dijkstra's most important contribution to distributed computing theory.
  - [Lamport talk, PODC 83] Reintroduced the paper, explained its importance, popularized it.
  - Became (still is) a major research direction.
  - Won PODC Influential Paper award, in 2002.
  - Award renamed the Dijkstra Prize.
- [Dolev book, 00] summarizes main ideas of the field.

# Today...

- Basic ideas, from [ Dolev, Chapter 2 ]
- Rest of the book describes:
  - Many more self-stabilizing algorithms.
  - General techniques for designing them.
  - Converting non-SS algorithms to SS algorithms.
  - Transformations between models, preserving SS.
  - SS in presence of ongoing failures.
  - Efficient SS.
  - Etc.

#### Self-Stabilization: Definitions

### **Self-stabilization**

- [Dolev] considers:
  - Message-passing models, with FIFO reliable channels.
  - Shared-memory models, with read/write registers.
  - Asynchronous and synchronous models.
- To simplify, avoids internal process actions---combines these with sends, receives, or register access steps.
- Sometimes considers message losses ("loss" steps).
- Many models, must continually specify which is used.
- Defines executions:
  - Like ours, but needn't start in initial state.
  - Same as our "execution fragments".
- Fair executions:
  - Described informally.
  - Our task-based definition is fine.

# Legal execution fragments

- Given a distributed algorithm A, define a set L of legal execution fragments of A.
- L can include both safety and liveness conditions.
- Example: Mutual exclusion problem
  - L might be the set of all fragments  $\alpha$  satisfying:
    - Mutual exclusion:
      - No two processes are in the critical region, in any state in  $\boldsymbol{\alpha}.$
    - Progress:
      - If in some state of  $\alpha$ , someone is in T and no one is in C, then sometime thereafter, someone  $\rightarrow$  C.
      - If in some state of  $\alpha$ , someone is in E, then sometime thereafter, someone  $\rightarrow$  R.

# Self-stabilization: Definition

- A global state s of algorithm A is safe with respect to legal set L, provided that every fair execution fragment of A that starts with s is in L.
- Algorithm A is self-stabilizing for legal set L if every fair execution fragment α of A contains a state s that is safe with respect to L.
  - Implies that the suffix of  $\alpha$  starting with s is in L.

α

- Also, any other fair execution fragment starting with s is in L.



• Weaker definition: Algorithm A is self-stabilizing for legal set L if every fair execution fragment  $\alpha$  has a suffix in L.

In L

# Stronger vs. weaker definition of self-stabilization

- Stronger definition: Algorithm A is self-stabilizing for legal set L if every fair execution fragment of A contains a state s that is safe with respect to L.
- Weaker definition: Algorithm A is self-stabilizing for legal set L if every fair execution fragment has a suffix in L.
- [Dolev] generally uses the stronger definition; so will we.
- But occasionally, he appears to be using the weaker definition; we'll warn when this arises.
- Q: Equivalent definitions? Not in general. LTTR.

# Non-termination

- Self-stabilizing algorithms for nontrivial problems don't terminate.
- E.g., consider message-passing algorithm A:
  - Suppose A is self-stabilizing for legal set L, and A has a terminating global state s.
    - All processes quiescent, all channels empty.
  - Consider a fair execution fragment  $\alpha$  starting with s.
  - $-\alpha$  contains no steps---just global state s.
  - Since A is self-stabilizing with respect to L,  $\alpha$  must contain a safe state.
  - So s must be a safe state.
  - Then the suffix of  $\alpha$  starting with s is in L; that is, just s itself is in L.
  - So L represents a trivial problem---doing nothing satisfies it.
- Similar argument for shared-memory algorithms.

Self-Stabilizing Algorithm 1: Self-Stabilizing Breadth-First Spanning Tree Construction

# Breadth-first spanning tree

- Shared-memory model
- Connected, undirected graph G = (V,E).
- Processes  $P_1, \dots, P_n$ ,  $P_1$  a designated root.
- Permanent knowledge (built into all states of the processes):
  - P<sub>1</sub> always knows it's the root.
  - Everyone always knows who their neighbors are.
- Neighboring processes in G share registers in both directions:
  - r<sub>ij</sub> written by P<sub>i</sub>, read by P<sub>j</sub>.
- Output: A breadth-first spanning tree, recorded in the r<sub>ii</sub> registers:
  - $r_{ii}$ .parent = 1 if j is i's parent, 0 otherwise.
  - $-r_{ij}$ .dist = distance from root to i in the BFS tree = smallest number of hops on any path from 1 to i in G.
  - Values in registers should remain constant from some point onward.



# In terms of legal sets...

- Define execution fragment  $\alpha$  to be legal if:
  - The registers have correct BFS output values, in all states in  $\alpha$ .
  - Registers never change.
- L = set of legal execution fragments.
- Safe state s:
  - Global state from which all extensions have registers with correct, unchanging BFS output values.
- SS definition says:
  - Any fair execution fragment  $\alpha$ , starting from any state, contains some safe state s.
  - That is, one from which all extensions have registers with correct, unchanging BFS output values.
  - Implies that any fair execution fragment  $\alpha$  has a suffix in which the register contents represent a fixed BFS tree.

# **BFS Algorithm strategy**

- The system can start in any state, with
  - Any values (of the allowed types) in registers,
  - Any values in local process variables.
- Processes can't assume that their own states and output registers are initially correct.
- Repeatedly recalculate states and outputs based on inputs from neighbors.
- In case of tie, use some default rule for selecting parent.
- Prove correctness, stabilization time, using induction on distance from root.

### Root process P<sub>1</sub>



- Keep writing (0,0) everywhere.
- Access registers in fixed, round-robin order.

# Non-root process P<sub>i</sub>

- Maintains local variables Ir<sub>ji</sub> to hold latest observed values of incoming registers r<sub>ji</sub>.
- First loop:
  - Read all the  $r_{ii}$ , copy them into  $Ir_{ii}$ .
- Use this local info to calculate new best distance dist, choose a parent that yields this distance.
  - Use default rule, e.g., smallest index, so always break ties the same way.
  - Needed to ensure stabilization to a fixed tree.
- Second loop:
  - Write dist to all outgoing registers.
  - Notify new parent.

# Non-root process P<sub>i</sub>

- do forever
  - for every neighbor m do
    - $Ir_{mi} := read(r_{mi})$
  - dist := min({ Ir<sub>mi</sub>.dist }) + 1
  - found := false
  - for every neighbor m do
    - if not found and dist = Ir<sub>mi</sub>.dist + 1 then
      - write r<sub>im</sub> := (1,dist)
      - found := true
    - else
      - write  $r_{im} := (0, dist)$
- Note:
  - P<sub>i</sub> doesn't take min of its own dist and neighbors' dists.
  - Unlike non-SS relaxation algorithms.
  - Ignores its own dist, recalculates solely from neighbors' dists.
  - Because its own value could be erroneous.

#### Correctness

- Prove this stabilizes to a particular "default" BFS tree.
- Define the default tree to be the unique BFS tree where ties in choosing parent are resolved using the rule:
  - Choose the smallest index yielding the shortest distance.
- Prove that, from any starting global state, the algorithm eventually reaches and retains the default BFS tree.
- More precisely, show it reaches a safe state, from which any execution fragment retains the default BFS tree.
- Show this happens within bounded time: O(diam ∆ I), where
  - diam is diameter of G (max distance from  $P_1$  to anyone is enough).
  - $\Delta$  is maximum node degree
  - I is upper bound on local step time
  - The constant in the big-O is about 4.

#### Correctness

- Uses a lemma marking progress through distances 0, 1, 2,..., diam, as for basic AsynchBFS.
- New complication: Erroneous, too-small distance estimates.
- Define a floating distance in a global state to be a value of some r<sub>ij</sub> dist that is strictly less than the actual distance from P<sub>1</sub> to P<sub>i</sub>.
  - Can't be correct.
- Lemma: For every k ≥ 0, within time (4k+1)∆I, we reach a configuration such that:
  - 1. For any i with dist( $P_1, P_i$ )  $\leq k$ , every  $r_{ij}$ .dist is correct.
  - 2. There is no floating distance < k.
- Moreover, these properties persist after this configuration.

# Proof of lemma

- Lemma: For every k ≥ 0, within time (4k+1)∆I, we reach a configuration such that:
  - 1. For any i with dist( $P_1, P_i$ )  $\leq k$ , every  $r_{ii}$  dist is correct.
  - 2. There is no floating distance < k.
- **Proof:** Induction on k.
  - k = 0: P<sub>1</sub> writes (0,0) everywhere within time  $\Delta I$ .
  - Assume for k, prove for k+1:
    - Property 1:
      - Consider  $P_i$  at distance k+1 from  $P_1$ .
      - In one more interval of length 4∆I, P<sub>i</sub> has a chance to update its local dist and outgoing register values.
      - By inductive hypothesis, these updates are based entirely on:
        - » Correct distance values from nodes with distance  $\leq$  k from P<sub>1</sub>, and
        - » Possibly some floating values, but these must be  $\geq k$ .
      - So P<sub>i</sub> will calculate a correct distance value.
    - Property 2:
      - For anyone to calculate a floating distance < k+1, it must see a floating distance < k.</li>
      - Can't, by inductive hypothesis.

# Proof, cont'd

- We have proved:
  - Lemma: For every  $k \ge 0$ , within time  $(4k+1)\Delta I$ , we reach a configuration such that:
    - 1. For any i with dist( $P_1, P_i$ )  $\leq k$ , every  $r_{ij}$ .dist is correct.
    - 2. There is no floating distance < k.
- So within time (4 diam +1) ∆I, all the r<sub>ij</sub>.dist values become correct.
- Persistence is easy to show.
- Once all the r<sub>ij</sub>.dist values are correct, everyone will use the default rule and always obtain the default BFS tree.
- Ongoing failures:
  - If arbitrary failures occur from time to time, not too frequently, the algorithm gravitates back to correct behavior in between failures.
  - Recovery time depends on size (diameter) of the network.

Self-Stabilizing Algorithm 2: Self-Stabilizing Mutual Exclusion

# Self-stabilizing mutual exclusion

- [Dijkstra 73]
- Ring of processes, each with output variable x<sub>i</sub>.
- Large granularity: In one atomic step, process P<sub>i</sub> can read both neighbors' variables, compute its next value, and write it to variable x<sub>i</sub>.



- P<sub>1</sub> tries to make its variable one more than its predecessor's (mod n+1).
- Each other process tries to make its variable equal to its predecessor's

### **Mutual exclusion**

- In what sense does this "solve mutual exclusion"?
- Definition: "P<sub>i</sub> is enabled" (or "P<sub>i</sub> can change its state") in a configuration, if the variables are set so P<sub>i</sub> can take a step and change the value of its variable x<sub>i</sub>.
- Legal execution fragment  $\alpha$ :
  - In any state in  $\alpha$ , exactly one process is enabled.
  - For each i,  $\alpha$  contains infinitely many states in which P<sub>i</sub> is enabled.
- Use this to solve mutual exclusion:
  - Say  $P_i$  interacts with requesting user  $U_i$ .
  - $P_i$  grants U<sub>i</sub> the critical section when:
    - U<sub>i</sub> has requested it, and
    - P<sub>i</sub> is enabled.
  - When  $U_i$  returns the resource,  $P_i$  actually does its step, changing  $x_i$ .
  - Guarantees mutual exclusion, progress.
  - Also lockout-freedom.

- Legal α:
  - In any state in  $\alpha$ , exactly one process is enabled.
  - For each i,  $\alpha$  contains infinitely many states in which P<sub>i</sub> is enabled.
- Lemma 1: A configuration in which all the x variables have the same value is safe.
- This means that, from such a configuration, any fair execution fragment is legal.
- Proof: Only P<sub>1</sub> can change its state, then P<sub>2</sub>, then P<sub>3</sub>, ..., and so on around the ring (forever).
- Remains to show: Starting from any state, the algorithm eventually reaches a configuration in which all the x values are the same.
- This uses some more lemmas.

- Lemma 2: In every configuration, at least one of the potential x values, {0,...,n}, does not appear in any x<sub>i</sub>.
- Proof: Obviously. There are n+1 values and only n variables.

- Lemma 3: In any fair execution fragment (from any configuration c), P<sub>1</sub> changes x<sub>1</sub> at least once every nl time.
- Proof:
  - Assume not---P<sub>1</sub> goes longer than nl without changing x<sub>1</sub> from some value v.
  - Then by time I,  $P_2$  sets  $x_2$  to v,
  - By time 2I,  $P_3$  sets  $x_3$  to v,
  - ...
  - By (n-1)I,  $P_n$  sets  $x_n$  to v.
  - All these values remain = v, as long as  $x_1$  doesn't change.
  - But then by time nI,  $P_1$  sees  $x_n = x_1 = v$ , and increments  $x_1$ .

- Lemma 4: In any fair execution fragment α, a configuration in which all the x values are the same (and so, a safe configuration) occurs within time (n<sup>2</sup> + n)l.
- Proof:
  - Let c = initial configuration of  $\alpha$ .
  - Let v = some value that doesn't appear in any  $x_i$ , in c.
  - Then v doesn't appear anywhere, in  $\alpha$ , unless/until P<sub>1</sub> sets x<sub>1</sub> := v.
  - Within time nI,  $P_1$  changes  $x_1$ , incrementing it by 1, mod (n+1).
  - Within another nI,  $P_1$  increments  $x_1$  again.
  - ...
  - Within  $n^2 I$ ,  $P_1$  increments  $x_1$  to v.
  - At that point, there are still no other v's anywhere else.
  - Then this v propagates all the way around the ring.
  - $P_1$  doesn't change  $x_1$  until v reaches  $x_n$ .
  - Yields all  $x_i = v$ , within time  $(n^2 + n)I$ .

# Putting the pieces together

- Legal execution fragment  $\alpha$ :
  - In any state in  $\alpha$ , exactly one process is enabled.
  - For each i,  $\alpha$  contains infinitely many states in which P<sub>i</sub> is enabled.
- L = set of legal fragments.
- Theorem: Dijkstra's algorithm is self-stabilizing with respect to legal set L.
- In the sense of reaching a safe state.
- Remark:
  - This uses n+1 values for the  $x_i$  variables.
  - A curiosity:
    - This also works with n values, or even n-1.
    - But not with n-2 [Dolev, p. 20].

# Reducing the atomicity

- Dijkstra's algorithm reads x<sub>i-1</sub>, computes, and writes x<sub>i</sub>, all atomically.
- Now adapt this for usual model, in which only individual read/write steps are atomic.
- Consider Dijkstra's algorithm on a 2nprocess ring, with processes Q<sub>j</sub>, variables y<sub>j</sub>. j = 1, 2, ..., 2n.
  - Needs 2n+1 values for the variables.
- Emulate this in the usual n-process ring, with processes P<sub>i</sub>, variables x<sub>i</sub>:
  - $P_i$  emulates both  $Q_{2i-1}$  and  $Q_{2i}$ .
  - $y_{2i-1}$  is a local variable of P<sub>i</sub>.
  - $y_{2i}$  corresponds to  $x_i$ .

 $X_1$   $X_n$   $P_1$   $P_2$   $P_3$  $X_3$ 

# Reducing the atomicity

- Consider Dijkstra's algorithm on a 2nprocess ring, with processes Q<sub>j</sub>, variables y<sub>i</sub>. j = 1, 2, ..., 2n.
- Emulate this in an n-process ring, with processes P<sub>i</sub>, variables x<sub>i</sub>.
  - $P_i$  emulates both  $Q_{2i-1}$  and  $Q_{2i}$ .
  - $y_{2i-1}$  is a local variable of P<sub>i</sub>.
  - $y_{2i}$  corresponds to  $x_i$ .



- To emulate a step of  $Q_{2i-1}$ ,  $P_i$  reads from  $x_{i-1}$ , writes to its local variable  $y_{2i-1}$ .
- To emulate a step of  $Q_{2i}$ ,  $P_i$  reads from its local variable  $y_{2i-1}$ , writes to  $x_i$ .
- Since in each case one variable is internal, can emulate each step with just one ordinary read or write to shared memory.

# Composing Self-Stabilizing Algorithms

# Composing self-stabilizing algorithms

- Consider several algorithms, where
  - $A_1$  is self-stabilizing for legal set L<sub>1</sub>,
  - $A_2$  is SS for legal set  $L_2$ , "assuming  $A_1$  stabilizes for  $L_1$ "
  - $A_3$  is SS for legal set  $L_3$ , "assuming  $A_1$  stabilizes for  $L_1$  and  $A_2$  stabilizes for  $L_2$ "
  - etc.
- Then we should be able to run all the algorithms together, and the combination should be self-stabilizing for L1  $\cap$  L2  $\cap$  L3  $\cap$  ...
- Need composition theorems.
- Details depend on which model we consider.
- E.g., consider two shared memory algorithms,  $A_1$  and  $A_2$ .

# **Composing SS algorithms**

- Consider read/write shared memory algorithms, A<sub>1</sub> and A<sub>2</sub>, where:
  - All of  $A_1$ 's shared registers are written only by  $A_1$  processes.
    - No inputs arrive in A<sub>1</sub>'s registers.
  - All of  $A_2$ 's shared registers are written only by  $A_1$  and  $A_2$  processes.
    - No other inputs arrive in A<sub>2</sub>'s registers.
  - Registers shared between  $\overline{A}_1$  and  $A_2$  are written only by  $A_1$  processes, not by  $A_2$  processes.
  - One-way information flow, from  $A_1$  and  $A_2$ .
  - $A_1$  makes sense in isolation, but  $A_2$  depends on  $A_1$  for some inputs.
- Definition:  $A_2$  is self-stabilizing for  $L_2$  with respect to  $A_1$  and  $L_1$  provided that: If  $\alpha$  is any fair execution fragment of the combination of  $A_1$  and  $A_2$  whose projection on  $A_1$  is in  $L_1$ , then  $\alpha$  has a suffix in  $L_2$ .
- Theorem: If  $A_1$  is SS for  $L_1$  and  $A_2$  is SS for  $L_2$  with respect to  $A_1$  and  $L_1$ , then the combination of  $A_1$  and  $A_2$  is SS for  $L_2$ .

# Weaker definition of SS

- At this point, [Dolev] seems to be using the weaker definition for self-stabilization:
- Instead of:
  - Algorithm A is self-stabilizing for legal set L if every fair execution fragment  $\alpha$  of A contains a state s that is safe with respect to L.
- Now using:
  - Algorithm A is self-stabilizing for legal set L if every fair execution fragment  $\alpha$  has a suffix in L.
- So we'll switch here.

# **Composing SS algorithms**

- Def: A<sub>2</sub> is self-stabilizing for L<sub>2</sub> with respect to A<sub>1</sub> and L<sub>1</sub> provided that any fair execution fragment of the combination of A<sub>1</sub> and A<sub>2</sub> whose projection on A<sub>1</sub> is in L<sub>1</sub>, has a suffix in L<sub>2</sub>.
- **Theorem:** If  $A_1$  is SS for  $L_1$  and  $A_2$  is SS for  $L_2$  with respect to  $A_1$  and  $L_1$ , then the combination of  $A_1$  and  $A_2$  is SS for  $L_2$ .

#### • Proof:

- Let  $\alpha$  be any fair exec fragment of the combination of A<sub>1</sub> and A<sub>2</sub>.
- We must show that  $\alpha$  has a suffix in L<sub>2</sub> (weaker definition of SS).
- Projection of  $\alpha$  on A<sub>1</sub> is a fair execution fragment of A<sub>1</sub>.
- Since  $A_1$  is SS for  $L_1$ , this projection has a suffix in  $L_1$ .
- Therefore,  $\alpha$  has a suffix  $\alpha'$  whose projection on A<sub>1</sub> is in L<sub>1</sub>.
- Since  $A_2$  is self-stabilizing with respect to  $A_1$ ,  $\alpha'$  has a suffix  $\alpha''$  in  $L_2$ .
- So  $\alpha$  has a suffix in L<sub>2</sub>, as needed.
- Total stabilization time is the sum of the stabilization times of A<sub>1</sub> and A<sub>2</sub>.

### Applying the composition theorem

- Theorem supports modular construction of SS algorithms.
- Example: SS mutual exclusion in an arbitrary rooted undirected graph
  - A<sub>1</sub>:
    - Constructs rooted spanning tree, using the SS BFS algorithm.
    - The r<sub>ij</sub> registers contain all the tree info (parent and distance).
  - A<sub>2</sub>:
    - Takes A<sub>1</sub>'s r<sub>ij</sub> registers as input.
    - Solves mutual exclusion using a Dijkstralike algorithm, which runs on the stable tree in the r<sub>ii</sub> registers.
  - Q: But Dijkstra's algorithm uses a ring--how can we run it on a tree?
  - A: Thread the ring through the nodes of the tree, e.g.:



#### Mutual exclusion in a rooted tree

- Use the read/write version of the Dijkstra ring algorithm, with local and shared variables.
- Each process P<sub>j</sub> emulates several processes of Dijkstra algorithm.
- Bookkeeping needed, see [Dolev, p. 24-27].
- Initially, both the tree and the mutex algorithm behave badly.
- After a while (O(diam △ I) time), the tree stabilizes (since the BFS algorithm is SS), but the mutex algorithm continues to behave badly.
- After another while (O(n<sup>2</sup> I) time), the mutex algorithm also stabilizes (since it's SS given that the tree is stable).
- Total time is the sum of the stabilization times of the two algorithms: O(diam  $\Delta$  I) + O(n<sup>2</sup> I) = O(n<sup>2</sup> I).



#### **Self-Stabilizing Emulations**

Self-stabilizing emulations [Dolev, Chapter 4]

- Design a SS algorithm A<sub>2</sub> to solve a problem L<sub>2</sub>, using a model that is more powerful then the "real" one.
- Design an algorithm A<sub>1</sub> using the real model, that "stabilizes to emulate" the powerful model
- Combine  $A_1$  and  $A_2$  to get a SS algorithm for  $L_2$  using the real model.

# Self-stabilizing emulations

- Example 1 [Dolev, Section 4.1]: Centralized scheduler
  - Rooted undirected graph of processes.
  - Powerful model: Process can read several variables, change state, write several variables, all atomically.
  - Basic model: Just read/write steps.
  - Emulation algorithm  $A_1$ :
    - Uses Dijkstra-style mutex algorithm over BFS spanning tree algorithm
    - Process performs steps of A<sub>2</sub> only when it has the critical section (global lock).
    - Performs all steps that are performed atomically in the powerful model, before exiting the critical section.
  - Stabilizes to emulate the more powerful model.
  - Initially, both emulation  $A_1$  and algorithm  $A_2$  behave badly.
  - After a while, emulation begins behaving correctly, yielding mutual exclusion.
  - After another while,  $A_2$  stabilizes for  $L_2$ .

# Self-stabilizing emulations

- Example 2 [Nolte]: Virtual Node layer for mobile networks
  - Mobile ad hoc network: Collection of processes running on mobile nodes, communicating via local broadcast.
  - Powerful model: Also includes stationary Virtual Nodes at fixed geographical locations (e.g., grid points).
  - Basic model: Just the mobile nodes.
  - Emulation algorithm  $A_1$ :
    - Mobile nodes in the vicinity of a Virtual Node's location cooperate to emulate the VN.
    - Uses Replicated State Machine strategy, coordinated by a leader.
  - Application algorithm  $A_2$  running over the VN layer:
    - Geocast, or point-to-point routing, or motion coordination,...
  - Initially, both the emulation  $A_1$  and the application algorithm  $A_2$  behave badly.
  - Then the emulation begins behaving correctly, yielding a VN Layer.
  - Then the application stabilizes.

# Making Non-Self-Stabilizing Algorithms Self-Stabilizing

# Making non-self-stabilizing algorithms self-stabilizing

- [Dolev, Section 2.8]: Recomputation of floating outputs.
  - Method of converting some non-SS distributed algorithms to SS algorithms.
- What kinds of algorithms?
  - Algorithm A, computes a distributed function based on distributed inputs.
  - Assumes processes' inputs are in special, individual input variables, I<sub>i</sub>, whose values never change (e.g., contain fixed information about local network topology).
  - Outputs placed in special, individual output variables O<sub>i</sub>.
- Main idea: Execute A repeatedly, from its initial state, with the fixed inputs, with two kinds of output variables:
  - Temporary output variables o<sub>i</sub>.
  - Floating output variables FO<sub>i</sub>.
- Use the temporary variables o<sub>i</sub> the same way A uses O<sub>i</sub>.
- Write to the floating variables FO<sub>i</sub> only at the end of function computation.
- When restarting A, reset all variables except the floating outputs FO<sub>i</sub>.
- Eventually, the floating outputs should stop changing.

# Example: Consensus

- Start with a simple synchronous, non-fault-tolerant, nonself-stabilizing network consensus algorithm A, and make it self-stabilizing.
- Undirected graph G = (V,E), known upper bound D on diameter.
- Non-SS consensus algorithm A:
  - Everyone starts with Boolean input in  $I_i$ .
  - After D rounds, everyone agrees, and decision value = 1 iff someone's input = 1.
  - At intermediate rounds, process i keeps current consensus proposal in O<sub>i</sub>.
  - At each round, send O<sub>i</sub> to neighbors, resets O<sub>i</sub> to "or" of its current value and received values.
  - Stop after D rounds.
- A works fine, in synchronous model, if it executes once, from initial states.

#### Example: Consensus

- To make this self-stabilizing:
  - Run algorithm A repeatedly, with the FO<sub>i</sub> as floating outputs.
  - While running A, use  $o_i$  instead of  $O_i$ .
  - Copy o<sub>i</sub> to FO<sub>i</sub> at the end of each execution of A.
- This is not quite right...
  - Assumes round numbers are synchronized.
  - Algorithm begins in an arbitrary global state, so round numbers can be off.

# Example: Consensus

- Run algorithm A repeatedly, with the FO<sub>i</sub> as floating outputs.
- While running A, use o<sub>i</sub> instead of O<sub>i</sub>.
- Copy o<sub>i</sub> to FO<sub>i</sub> at the end of each execution of A.
- Must also synchronize round numbers 1,2,...,D.
  - Needs a little subprotocol.
  - Each process, at each round, sets its round number to max of its own and all those of its neighbors.
  - When reach D, start over at 1.
- Eventually, rounds become synchronized throughout the network.
- Thereafter, the next full execution of A succeeds, produces correct outputs in the FO<sub>i</sub> variables.
- Thereafter, the FO<sub>i</sub> will never change.

#### Extensions

- Can make this into a fairly general transformation, for synchronous algorithms.
- Using synchronizers, can extend to some asynchronous algorithms.

#### Making non-SS algorithms SS: Monitoring and Resetting [Section 5.2]

- AKA Checking and Correction.
- Assumes message-passing model.
- Basic idea:
  - Continually monitor the consistency of the underlying algorithm.
  - Repair the algorithm when inconsistency is detected.
- For example:
  - Use SS leader election service to choose a leader (if there isn't already a distinguished process).
  - Leader, repeatedly:
    - Conducts global snapshots,
    - Checks consistency,
    - Sends out corrections if necessary.
- Local monitoring and resetting [Varghese thesis, 92]
  - For some algorithms, can check and restore local consistency predicates.
  - E.g., BFS: Can check that local distance is one more than parent's distance, recalculate dist and parent if not.

# Other stuff in the book

- Discussion of practical motivations.
- Proof methods for showing SS.
- Stabilizing to an abstract specification.
- Model conversions, for SS algorithms:
  - Shared memory  $\rightarrow$  message-passing
  - Synchronous  $\rightarrow$  asynchronous
- SS in presence of ongoing failures.
  Stopping, Byzantine, message loss.
- Efficient "local" SS algorithms.
- More examples.

#### Next time...

- Partially synchronous distributed algorithms
- Reading:
  - Chapters 23-25
  - [Attiya, Welch], Section 6.3, Chapter 13

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