# **Grammar Induction**

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#### Three non-NLP questions

- Which is the odd number out?
   625,361,256,197,144
- 2. Insert the missing letter:

B,E,?,Q,Z

3. Complete the following number sequence:
4, 6, 9, 13
7, 10, 15, ?

#### How do you solve these questions?

• Guess a pattern that generates the sequence

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Insert the missing letter:
B,E,?,Q,Z
2,5,?,17, 26
k^2 + 1
```

• Select a solution based on the detected pattern  $k = 3 \rightarrow 10$ th letter of the alphabet  $\rightarrow J$ 

#### More Patterns to Decipher: Byblos Script

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# More Patterns to Decipher: Lexicon Learning

Ourenemiesareinnovativeandresourceful, and so arewe.

Theyneverstopthinkingaboutnewwaystoharmourcountry and our people, and neitherdowe.

Which is the odd word out?

Ourenemies . . . Enemies . . . We . . .

# More Patterns to Decipher: Natural Language Syntax

Which is the odd sentence out?

The cat eats tuna.

The cat and the dog eats tuna.

# Today

- Vocabulary Induction
  - Word Boundary Detection
- Grammar Induction
  - Feasibility of language acquisition
  - Algorithms for grammar induction

# **Vocabulary Induction**

Task: Unsupervised learning of word boundary segmentation

• Simple:

Ourenemiesareinnovativeandresourceful, and so arewe.

Theyneverstop thinking about new ways to harmour country and our people, and neither dowe.

• More ambitious:

Image of Byblos script removed for copyright reasons.

## Word Segmentation (Ando&Lee, 2000)

Key idea: for each candidate boundary, compare the frequency of the n-grams adjacent to the proposed boundary with the frequency of the n-grams that straddle it.



For N = 4, consider the 6 questions of the form: "Is  $\#(s_i) \ge \#(t_j)$ ?", where #(x) is the number of occurrences of x

Example: Is "TING" more frequent in the corpus than "INGE"?

## **Algorithm for Word Segmentation**

- $\begin{array}{ll} s_1^n & \text{non-straddling n-grams to the left of location } k \\ s_2^n & \text{non-straddling n-grams to the right of location } k \\ t_j^n & \text{straddling n-gram with } j \text{ characters to the right of location } k \\ I_{>}(y, z) & \text{indicator function that is 1 when } y \geq z, \text{ and 0 otherwise.} \end{array}$
- 1. Calculate the fraction of affirmative answers for each n in N:

$$v_n(k) = \frac{1}{2*(n-1)} \sum_{i=1}^{2} \sum_{j=1}^{n-1} I_{\geq}(\#(s_i^n), \#(t_j^n))$$

2. Average the contributions of each n - gram order

$$v_N(k) = \frac{1}{N} \sum_{n \in N} v_n(k)$$

## Algorithm for Word Segmentation (Cont.)

Place boundary at all locations *l* such that either:

- *l* is a local maximum:  $v_N(l) > v_N(l-1)$  and  $v_N(l) > v_N(l+1)$
- $v_N(l) \ge t$ , a threshold parameter



#### **Experimental Framework**

- Corpus: 150 megabytes of 1993 Nikkei newswire
- Manual annotations: 50 sequences for development set (parameter tuning) and 50 sequences for test set
- Baseline algorithms: Chasen and Juman morphological analyzers (115,000 and 231,000 words)

#### Evaluation

- Precision (P): the percentage of proposed brackets that exactly match word-level brackets in the annotation
- Recall (R): the percentage of word-level annotation brackets that are proposed by the algorithm

• 
$$F = 2 \frac{PR}{(P+R)}$$

• F = 82% (improvement of 1.38% over Jumann and of 5.39% over Chasen)

#### **Grammar Induction**

- Task: Unsupervised learning of a language's syntax from a corpus of observed sentences
  - Ability to uncover an underlying grammar
  - Ability to parse
  - Ability to judge grammaticality

## **Plato's Problem**

Logical problem of language acquisition: (Chomsky 1965, Pinker 1994, Pullum 1996)

- A child hears a finite number of utterances from a target language
- This finite experience is consistent with infinitely many targets
- The child manages to select the correct target language

## **Gold's Formalization(1967)**

- Given: A target language L from a set  $\mathcal{L}$  of possible languages
- A learner C is shown a set of positive examples  $[s_i], s_i \in L$
- *C* is never given negative examples
- Each s ∈ L will be presented at some point i (no guarantees on the order or frequency of examples)
- *C* maintains a hypothesis  $L(C, [s_0, \ldots, s_n]) \in \mathcal{L}$

## Identifiability in the Limit

- A language family *L* is identifiable in the limit if for any target language and example sequence, the learner's hypothesis is eventually correct
- A language family *L* is identifiable in the limit if there is some learner *C* such that, for any *L* ∈ *L* and any legal presentation of examples [*s<sub>i</sub>*], there is some point *k* such that for all *j* > *k*, *L*(*C*, [*s*<sub>0</sub>, ..., *s<sub>k</sub>*]) = *L*

Example:  $\mathcal{L} = \{\{a\}, \{a, b\}\}$ 

## **Gold's Results**

A wide variety of language families are not learnable (proof based on recursive function theory)

- Superfinite family (all the finite languages and at least one infinite language)
- Family of regular languages
- Family of context-free languages

#### **Issues to Consider (Pullman 2003)**

- Learners may receive considerable information about which strings are not grammatical (perhaps indirectly)
- It is not clear that real language learners ever settle on a grammar at all
- Learners could *approximate* rather than exactly identify grammars
- The learner may operate over strings paired with meaning
- Learning can be viewed as partial characterization of linguistic structure (rather than defining a unique set of grammatical strings)

Horning(1969): probabilistic context free grammars are learnable if some Gold's constraints are relaxed

#### Nativism

- *Poverty of stimulus* (Chomsky, 1965): the lack of crucial relevant data in the learner's experience
- Richness of constraint: human languages are highly constrained, since the actual family of human languages is relatively small

## **Grammar Induction: Evaluation**

- Evaluation
  - Compare grammars
  - Compare trees
- Baselines
  - Random trees
  - Left- and Right-Branching Trees

#### **Grammar Induction: Approaches**

- Structure search
  - Add productions to a context-free grammar
  - Select HMM topology
- Parameter search
  - Determine parameters for a fixed PCFG

#### Structure search: Example

- Input: {*ab*, *abab*}
- Possible output:  $L = (ab)^n$



# **Model Merging**

- A method to construct an initial model from data
- A way to merge submodels
- An error measure to compare the goodness of various candidates for merging and to limit generalization
- A strategy to pick merging operators, search the model space

# Model Merging (Stolcke&Omohundro, 1994)

- **Data Incorporation:** Given a body of data X, build an initial model *M*<sub>0</sub> by explicitly accommodating each data point individually
- Generalization: Build a sequence of new models, obtaining M<sub>i+1</sub> from M<sub>i</sub> by applying a merging operator m that coalesces substructures in M<sub>i</sub>, M<sub>i+1</sub> = m(M<sub>i</sub>)
- Utility function: Maximize posterior probability P(M|X)
- **Search:** Greedy or beam search through the space of possible merges

### **HMM Topology Induction**

- Data Incorporation: For each observed sample, create a unique path between the initial and final states by assigning a new state to each symbol token in the sample
- **Generalization:** Two HMM states are replaced by a single new state, which inherits the union of the transitions and emissions from the old states









#### **Posterior Computation**

Goal: maximize posterior  $P(M|X) = \frac{P(M)P(X|M)}{P(X)}$ 

- We will maximize  $P(M|X) \propto P(M)P(X|M)$
- We know how to compute P(X|M)
- We need to compute prior P(M)

#### **Prior Distribution**

Model M is defined by topology  $M_s$  and  $\theta_M$ 

 $P(M) = P(M_s)P(\theta_M|M_s)$ 

- $P(M_s) \propto \exp(-l(M_s))$ , where  $l(M_s)$  is the number of bits required to encode  $M_s$ 
  - Each transition is encoded using  $\log(|Q|+1)$  bits, where |Q| is the number of states
  - The total description length for all transitions from state q is  $n_t^{(q)} \log(|Q| + 1)$  bits, where  $n_t^{(q)}$  the number of transitions from state q

- The total emission length for state q is
   n<sub>e</sub><sup>(q)</sup> log(|Σ| + 1) bits, where n<sub>e</sub><sup>(q)</sup> the number of state q emissions, and |Σ| is the size of the alphabet
- The resulting prior

$$P(M_s^{(q)}) \propto (|Q|+1)^{-n_t^{(q)}} (|\Sigma|+1)^{-n_e^{(q)}}$$

•  $P(\theta_M|M_s)$  are defined as Dirichlet priors

# Algorithm

- 1. Build the initial, maximum-likelihood model  $M_0$  from the dataset X
- 2. Let i := 0. Loop:
  - (a) Compute a set of candidate merges K among the states of model  $M_i$
  - (b) For each candidate  $k \in K$  compute the merged model  $k(M_i)$ , and its posterior probability  $P(k(M_i)|X)$
  - (c) Let  $k^*$  be the merge that mazimizes  $P(k(M_i)|X)$ . Then let  $M_{i+1} := k^*(M_i)$
  - (d) If  $P(M_{i+1}|X) > P(M_i|X)$ , return  $M_i$  as the induced model.
  - (e) Let i := i + 1

#### Evaluation

Method	Cross-Entropy	Language
Merging	2.158	$ac^*a \cup bc^*b$
Baum-Welch+	2.105	
Baum-Welch-	2.825	
Merging	5.623	$a^+b^+a^+b^+$
Baum-Welch+	5.688	
Baum-Welch-	8.395	

## **Learning PCFGs**

(Carroll&Charniak, 1992)

Goal: Learning grammars for natural language

- Divide the corpus into two parts: the rule corpus and the training corpus.
- For all the sentences in the rule corpus, generate all rules which might be used to parse the sentence, subject to constraints which we will specify later.
- Estimate the probabilities for the rules.
- Using the training corpus, improve our estimate of probabilities.
- Delete all rules with probability  $\leq \delta$  for some small  $\delta$ .

#### **Rule Generation: Dependency Format**

Informally, a dependency grammar produces a set of terminals connected by a set of directed arcs — one arc for every terminal except the root terminal



#### **Dependency Grammar**

- Assumption: POS tags are provided
- Theorem: A sentence of length n, consisting of all distinct terminals will have  $n(2^{n-1} + 1)$  dependency grammar rules to confirm to it

#### **Rule Generation**

We have to prune rule space!

- Order sentences by length and generate rules incrementally
- Do not consider rules that were discarded on previous stages
- Limit the number of symbols on the right-hand side of the rule

# Algorithm

Loop for i from 2 until *i* > sentence-length-stopping point

- Add rules required for the sentences with length *i* from the rule creation subset
- Estimate the probabilities for all rules, based upon all sentences of length  $\leq i$  from the rule training subset
- Remove any rules with probability  $\leq \delta$  if its probability doesn't increase

#### Reestimation

• We have sentences  $S_1, \ldots, S_n$ . Trees are hidden variables.

$$L(\theta) = \sum_{i} \log \sum_{T} P(S_i, T|\theta)$$

• Basic quantity needed for re-estimating with EM:

$$\theta_{\alpha \to \beta} = \frac{\sum_{i} Count(S_i, \alpha \to \beta)}{\sum_{i} \sum_{s \in R(\alpha)} Count(S_i, s)}$$

• There are efficient algorithms for calculating

$$Count(S_i, r) = \sum_{T} P(T|S_i, \theta^{t-1}) Count(S_i, T, r)$$

for a PCFG. See Inside-Outside algorithm (Baker, 1979)

#### Example

Induce PCFG, given the following corpus:

"noun verb"

"verb noun"

"verb"

"det noun verb"

"verb det noun"

		Rule	1 ITER	6 ITER	20 ITER
S	$\rightarrow$	$d\overline{e}t$	0.181818	0.0	0.0
S	$\rightarrow$	noūn	0.363636	0.0	0.0
S	$\rightarrow$	verb	0.454545	1.0	1.0
$d\overline{e}t$	$\rightarrow$	det	0.250000	1.0	1.0
$d\overline{e}t$	$\rightarrow$	$det \ no \overline{u} n$	0.250000	0.0	0.0
$d\overline{e}t$	$\rightarrow$	det verb	0.125	0.0	0.0
$d\overline{e}t$	$\rightarrow$	$verb \ dar et$	0.125	0.0	0.0
$d\overline{e}t$	$\rightarrow$	verb~dar et~nar oun	0.125	0.0	0.0
$n o \overline{u} n$	$\rightarrow$	noun	0.333333	0.781317	0.998847
$n o \overline{u} n$	$\rightarrow$	$d \bar{e} t \ noun$	0.166667	0.218683	0.01153
$var{e}rb$	$\rightarrow$	$n o \overline{u} n \ ver b$	0.153846	0.286749	0.200461
verb	$\rightarrow$	verb noūn	0.153846	0.288197	0.200461

## **Experiment 1**

- Use grammar from the handout
- Randomly generate 1000 words for the rule corpus, and 9000 for the training corpus
- Evaluation: compare the output with the generated grammar
- Constraint: rules were required to have fewer than five symbols on their right-hand side

## Results

- Successfully minimizes a cross entropy (1.245 bits/word on the training of the learned grammar vs. 1.220 bits/word of the correct grammar)
- Miserably fails to recover the correct grammar
  - 300 unsuccessful attempts
- .220  $pron \rightarrow pron verb$
- .214  $pron \rightarrow prep pron$
- .139  $pron \rightarrow pron \ verb \ det$
- .118  $pron \rightarrow verb pron$

#### Experiment 2

Place more restrictions on the grammar

Specify what non-terminals may appear on the right-hand side of a rule with a particular non-terminal on the left

• The algorithm converges to the correct grammar

	noun	verb	pron	det	prep	adj	wh	•
noun				+	+	+	+	
verb	+		+		+			
pron		_						
det						_		

# Adding Knowledge to Grammar Induction Algorithms

- Carrol&Charniak (1992): restrictions on the rule format
- Magerman&Marcus (1990): use a di-stituent grammar to eliminate undesirable rules
- Pereira&Schabes (1992): use partially bracketed corpora

#### Learning Constituents

Are syntactic patterns evident in a corpus? (Klein, 2005)

• Compute context for each POS

Tag	Top Context by Frequency
DT	(IN-NN), (IN-JJ), (IN-NNP), (VB-NN)
JJ	(DT-NN), (IN-NNS), (IN-NN), (JJ-NN)

• Cluster POS based on their context

#### Learning Constituents

The most similar POS pairs based on their context

Rank	Tag Pairs
1	(VBZ, VBD)
2	(DT, PRP\$)
3	(NN, NNS)
4	(WDT, WP)
5	(VBG, VBN)

#### Learning Constituents

The most similar POS sequence pairs based on their context

Rank	Tag Pairs
1	(NNP NNP, NNP NNP NNP)
2	(DT JJ NN IN, DT NN IN)
3	(NNP NNP NNP NNP, NNP NNP NNP)
4	(DT NNP NNP, DT NNP)
5	(IN DT JJ NN, IN DT NN)

#### Learning Constituents (Clark, 2001)

- Identify frequent POS sequences in a corpus
- Cluster them based on their context
- Filter out spurious candidates
  - Based on mutual information before the candidate constituent and the symbol after they are not independent

#### Summary

- Language acquisition problem
- Three unsupervised induction algorithms:
  - Vocabulary Induction
  - HMM-topology induction
  - PCFG induction