2.7 Waveguides and Integrated Optics

As with electronics, miniaturization and integration of optics is desired to reduce cost while increasing functionality and reliability. One essential element is the guiding of the optical radiation in waveguides for integrated optical devices and optical fibers for long distance transmission. Waveguides can be as short as a few millimeters. Guiding of light with exceptionally low loss in fiber (0.1dB/km) can be achieved by using total internal reflection. Figure 2.83 shows different optical waveguides with a high index core material and low index cladding. The light will be guided in the high index core. Similar to the Gaussian beam the guided mode is made up of mostly paraxial plane waves that hit the high/low-index interface at grazing incidence and therefore undergo total internal reflections. The concomittant lensing effect overcomes the diffraction of the beam that would happen in free space and leads to stationary mode profiles fof the radiation.

Depending on the index profile and geometry one distinguishes between different waveguide types. Figure 2.83 (a) is a planar slab waveguide, which guides light only in one direction. This case is analyzed in more detail, as it has simple analytical solutions that show all phenomena associated with waveguiding such as cutoff, dispersion, single and multimode operation, coupling of modes and more, which are used later in devices and to achieve certain device properties. The other two cases show complete waveguiding in the transverse direction; (b) planar strip waveguide and (c) optical fiber.

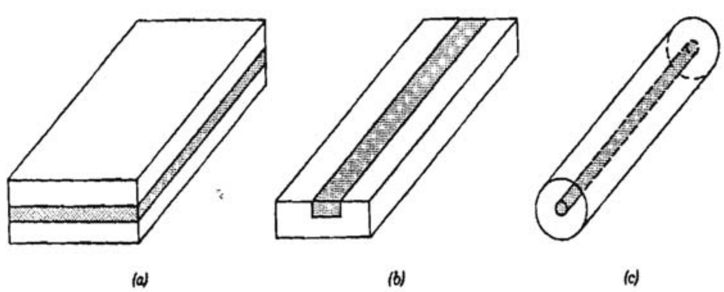

Figure 2.83: Dark shaded area constitute the high index regions. (a) planar slab waveguide; (b) strip waveguide; (c) optical fiber [6], p. 239.

In integrated optics many components are fabricated on a single substrate, see Figure 2.84 with fabrication processes similar to those in microelectronics.

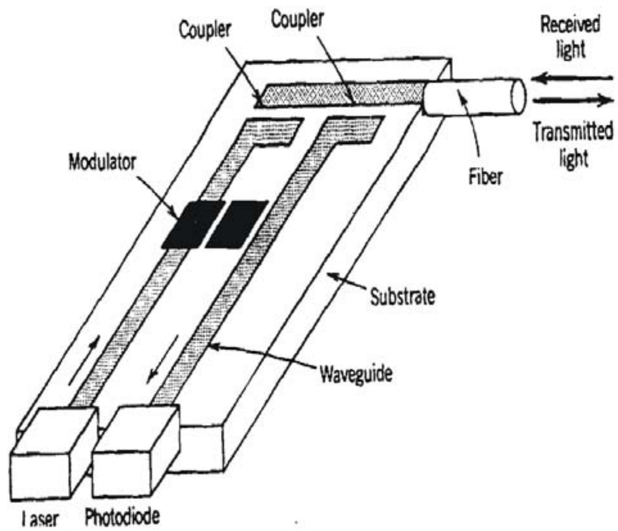

Figure 2.84: Integrated optical device resembling an optical transmitter/receiver, [6], p. 2.83.

As this example shows, the most important passive component to understand in an integrated optical circuit are waveguides and couplers.

2.7.1 Planar Waveguides

To understand the basic physics and phenomena in waveguides, we look at a few examples of guiding in one transverse dimension. These simple cases can be treated analytically.

Planar-Mirror Waveguides

The planar mirror waveguide is composed of two ideal metal mirrors a distance d apart, see Figure 2.85

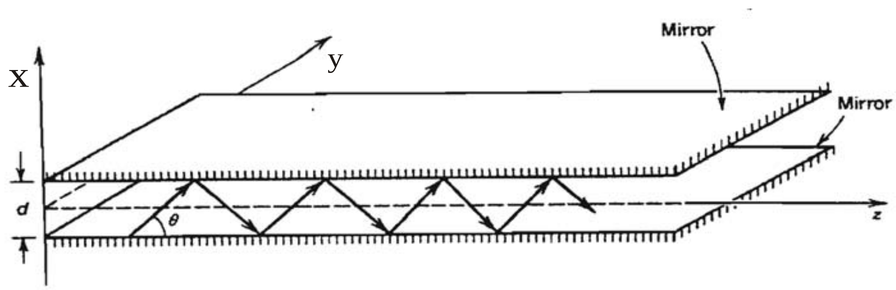

Figure 2.85: Planar mirror waveguide, [6], p. 240.

We consider a TE-wave, whose electric field is polarized in the $y-$direction and that propagates in the $z-$direction. The reflections of the light at the ideal lossless mirrors will guide or confine the light in the $x-$direction. The field will be homogenous in the $y-$direction, i.e. will not depend on y. Therefore, we make the following trial solution for the electric field of a monochromatic complex TE-wave

$$\underline{\vec{E}}(x,z,t) = \underline{E}_y(x,z)\ e^{j\omega t}\ \vec{e}_y. \tag{2.304}$$

Note, this trial solution also satisfies the condition $\nabla \cdot \vec{E} = 0$, see (2.12)

Modes of the planar waveguide Furthermore, we are looking for solutions that do not change their field distribution transverse to the direction of propagation and experience only a phase shift during propagation. We call such solutions modes of the waveguide, because they don't change its transverse field profile. The modes of the above planar waveguide can be expressed as

$$\underline{\vec{E}}_y(x,z) = u(x)\ e^{-j\beta z}\ \vec{e}_y, \tag{2.305}$$

where β is the propagation constant of the mode. This solution has to obey the Helmholtz Eq.(2.18) in the free space section between the mirrors

$$\frac{d^2}{dx^2}u_y(x) = \left(\beta^2 - k^2\right) u_y(x) \text{ with } k^2 = \frac{\omega^2}{c^2}. \tag{2.306}$$

The presence of the metal mirrors requires that the electric fields vanish at the metal mirrors, otherwise infinitely strong currents would start to flow to shorten the electric field.

$$u_y(x = \pm d/2) = 0 \tag{2.307}$$

.Note, that Eq.(2.306) is an eigenvalue problem to the differential operator $\frac{d^2}{dx^2}$

$$\frac{d^2}{dx^2}u(x) = \lambda u(x) \text{ with } u(x = \pm d/2) = 0. \tag{2.308}$$

in a space of functions u, that satisfies the boundary conditions (2.307). The eigenvalues λ are for the moment arbitrary but constant numbers. Depending on the sign of the eigenvalues the solutions can be sine or cosine functions ($\lambda < 0$) or exponentials with real exponents for ($\lambda > 0$). In the latter case, it is impossible to satiesfy the boundary conditions. Therefore, the eigensolutions are

$$u_m(x) = \begin{cases} \sqrt{\frac{2}{d}}\cos\left(k_{x,m}x\right) \text{ with } , k_{x,m} = \frac{\pi}{d}m, \ m = 1,3,5,..., \text{ even modes} \\ \sqrt{\frac{2}{d}}\sin\left(k_{x,m}x\right) \text{ with } , k_{x,m} = \frac{\pi}{d}m, \ m = 2,4,6,..., \text{ odd modes} \end{cases} \tag{2.309}$$

Propagation Constants The propagation constants for these modes follow from comparing (2.306) with (2.308) to be

$$\beta^2 = k^2 - k_{x,m}^2 \tag{2.310}$$

or

$$\beta = \pm\sqrt{\frac{\omega^2}{c^2} - \left(\frac{\pi}{d}m\right)^2} = \pm\sqrt{\left(\frac{2\pi}{\lambda}\right)^2 - \left(\frac{\pi}{d}m\right)^2} \tag{2.311}$$

where $\lambda = \lambda_0/n(\lambda_0)$ is the wavelength in the medium between the mirrors. This relationship is shown in Figure 2.86. The lowest order mode with index $m = 1$ has the smallest k-vector component in x-direction and therefore the largest k-vector component into z-direction. The sum of the squares of both components has to be identical to the magnitude sqaure of the k-vector in the medium k. Higher order modes have increasingly more nodes in the x-direction, i.e. largest k_x-components and the wave vector component in z-direction decreases, until there is no real solution anymore to Eq.(**??**) and the corresponding propagation constants β_m become imaginary. That is, the corresponding waves become evanescent waves, i..e they can not propagate in a waveguide with the given dimensions.

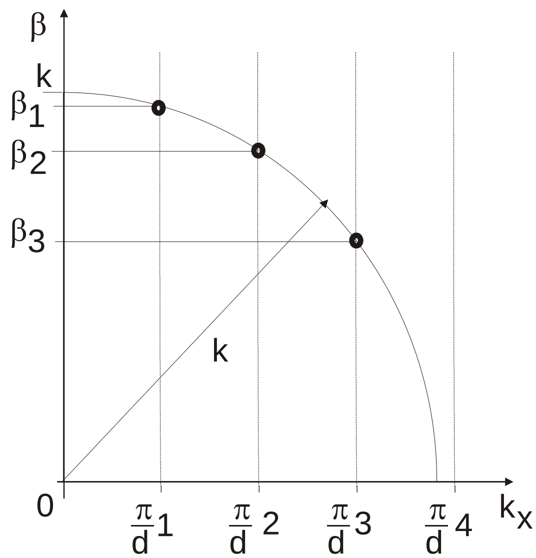

Figure 2.86: Determination of propagation constants for modes

Field Distribution The transverse electric field distributions for the various TE-modes is shown in Figure 2.87

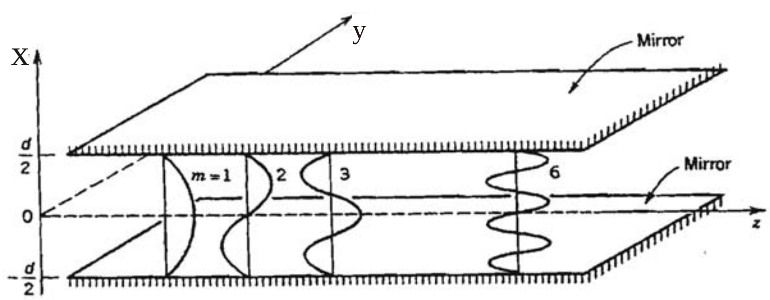

Figure 2.87: Field distributions of the TE-modes of the planar mirror waveguide [6], p. 244.

Cutoff Wavelength/Frequency For a given planar waveguide with separation d, there is a lowest frequency, i.e. longest wavelength, beyond which no propagating mode exists. This wavelenth/frequency is refered to as cutoff wavelength/frequency which is

$$\lambda_{cutoff} = 2d \quad (2.312)$$

$$f_{cutoff} = \frac{c}{2d}. \quad (2.313)$$

The physical origin for the existence of a cutoff wavelength or frequency is that the guided modes in the mirror waveguide are a superposition of two plane waves, that propagate under a certain angle towards the z-axis, see Figure 2.88

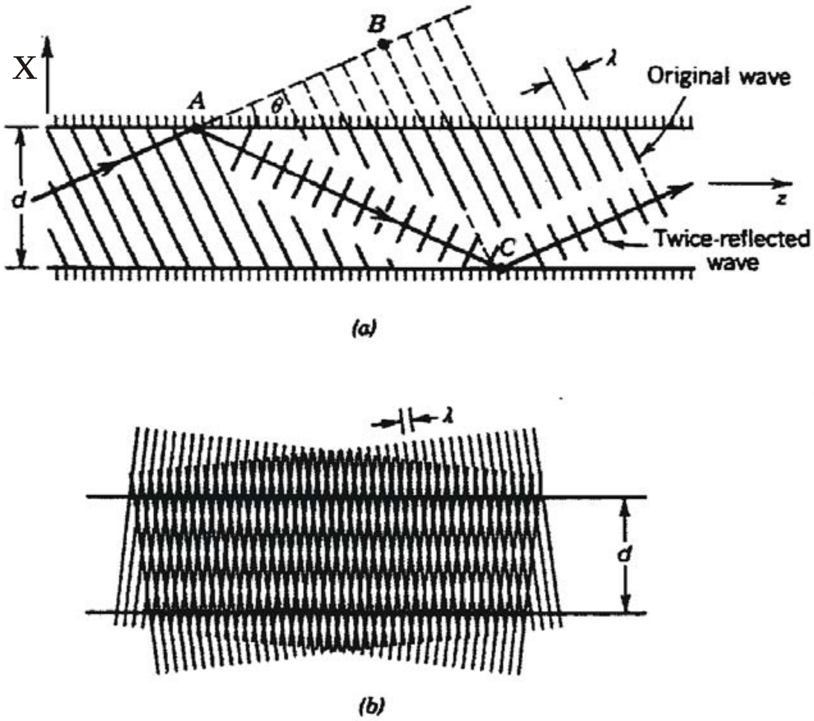

Figure 2.88: (a) Condition for self-consistency: as a wave reflects twice it needs to be in phase with the previous wave. (b) The angles for which self-consistency is achieved determine the x-component of the $\vec{k}$-vectors involved. The corresponding two plane waves setup an interference pattern with an extended node at the position of the metal mirrors satisfying the boundary conditions, [6], p. 241.

In order that the sum of the electric field of the two plane waves fulfills the boundary conditions, the phase of one of the plane waves after reflection on both mirrors needs to be inphase with the other plane wave, i.e. the x-component of the $\vec{k}$-vectors involved, k_x, must be a multiple of 2π

$$2k_x d = \pm 2\pi m.$$

If we superimpose two plane waves with $k_{x,m} = \pm\pi m/d$, we obtain an interference pattern which has nodes along the location of the metal mirrors, which obviously fulfills the boundary conditions. It is clear that the minimum distance between these lines of nodes for waves of a given wavelength λ is $\lambda/2$, hence the separation d must be greater than $\lambda/2$ otherwise no solution is possible.

Single-Mode Operation For a given separation d, there is a wavelength range over which only a single mode can propagate, we call this wavelength range single-mode operation. From Figure 2.86 it follows for the planar mirror waveguide

$$\frac{\pi}{d} < k < \frac{\pi}{d}2 \tag{2.314}$$

or

$$d < \lambda < 2d \tag{2.315}$$

Waveguide Dispersion Due to the waveguiding, the relationship between frequency and propagation constant is no longer linear. This does not imply that the waveguide core, i.e. here the medium between the plan parallel mirrors, has dispersion. For example, even for $n = 1$, we find for phase and group velocity of the m-th mode

$$\frac{1}{v_p} = \frac{\beta(\omega)}{\omega} = \frac{1}{c}\sqrt{1 - \left(\frac{c\pi}{d\omega}m\right)^2} \tag{2.316}$$

$$= \frac{1}{c}\sqrt{1 - \left(\frac{\lambda}{2d}m\right)^2} \tag{2.317}$$

and

$$\frac{1}{v_g} = \frac{d\beta(\omega)}{d\omega} = \frac{1}{2\sqrt{\frac{\omega^2}{c^2} - \left(\frac{\pi}{d}m\right)^2}} 2\frac{\omega}{c^2} \tag{2.318}$$

or

$$v_g \cdot v_p = c^2. \tag{2.319}$$

Thus different modes have different group and phase velocities. Figure 2.89 shows group and phase velocity for the different modes as a function of the normalized wave number kd/π.

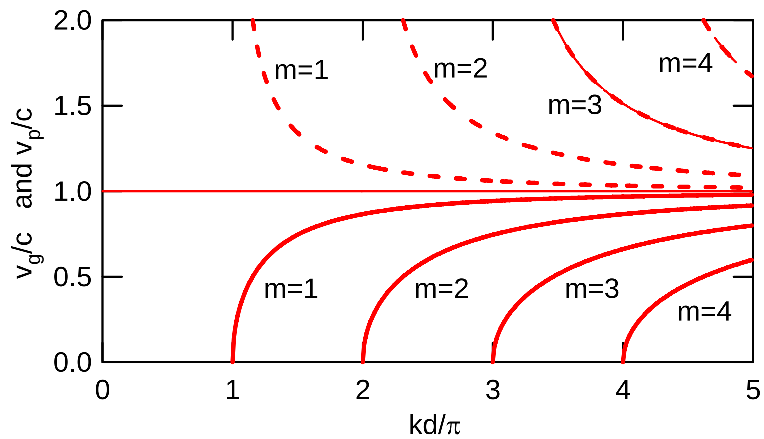

Figure 2.89: Group and phase velocity of propagating modes with index m as a function of normailzed wave number.

TM-Modes The planar mirror waveguide does not only allow for TE-waves to propagate. There are also TM-waves, which have only a magnetic field component transverse to the propagation direction and parallel to the mirrors, i.e. in y-direction

$$\underline{\vec{H}}(x,z,t) = \underline{H}_y(x,z)\; e^{j\omega t}\; \vec{e}_y, \tag{2.320}$$

and now $\underline{H}(x,z)$ has to obey the Helmholtz equation for the magnetic field. The corresponding electric field can be derived from Ampere's law

$$\underline{\vec{E}}(x,z) = \frac{-1}{j\omega\varepsilon} \nabla \times \left(\underline{H}_y(x,z)\; \vec{e}_y\right) \tag{2.321}$$

$$= \frac{1}{j\omega\varepsilon} \frac{\partial \underline{H}_y(x,z)}{\partial z} \vec{e}_x + \frac{-1}{j\omega\varepsilon} \frac{\partial \underline{H}_y(x,z)}{\partial x} \vec{e}_z. \tag{2.322}$$

The electric field tangential to the metal mirrors has to vanish again, which leads to the boundary condition

$$\frac{\partial \underline{H}_y(x,z)}{\partial x}(x = \pm d/2) = 0. \tag{2.323}$$

After an analysis very similar to the discussion of the TE-waves we find for the TM-modes with

$$\underline{H}_y(x,z) = u(x)\ e^{-j\beta z}\ \vec{e}_y, \tag{2.324}$$

the transverse mode shapes

$$u_m(x) = \begin{cases} \sqrt{\frac{2}{d}}\cos\left(k_{x,m}x\right) \text{ with } , k_{x,m} = \frac{\pi}{d}m,\ m = 2,4,6,..., \text{ even modes} \\ \sqrt{\frac{2}{d}}\sin\left(k_{x,m}x\right) \text{ with } , k_{x,m} = \frac{\pi}{d}m,\ m = 1,3,5,..., \text{ odd modes} \end{cases} \tag{2.325}$$

Note, that in contrast to the electric field of the TE-waves being zero at the metal surface, the transverse magnetic field of theTM-waves is at a maximum at the metal surface. We will not consider this case further, because the discussion of cutoff frequencies and dispersion can be worked out very analogous to the case for TE-modes.

Multimode Propagation Depending on the boundary conditions at the input of the waveguide at $z = 0$ many modes may be excited. Eventually there are even excitations with such high transverse wavevectors k_x present, that are below cutoff. Depending on the excitation amplitudes of each mode, the total field in the waveguide will be the superposition of all modes. Lets assume that there are only TE-modes excited, then the total field is

$$\underline{\vec{E}}(x,z,t) = \sum_{m=1}^{\infty}\left(\underline{a}_m\ e^{-j\beta_m z} + \underline{b}_m\ e^{j\beta_m z}\right)\ u_m(x)\ e^{j\omega t}\ \vec{e}_y, \tag{2.326}$$

where the amplitudes $\underline{a}_m$ and $\underline{b}_m$ are the excitations of the m-th mode in forward and backward direction, respectively. It is easy to show that these excitation amplitudes are determined by the transverse electric and magnetic fields at $z = 0$ and $t = 0$. In many cases, the excitation of the waveguide will be such that only the forward propagating modes are excited.

$$\underline{\vec{E}}(x,z,t) = \sum_{m=1}^{\infty}\underline{a}_m\ u_m(x)\ e^{-j\beta_m z}\ e^{j\omega t}\ \vec{e}_y, \tag{2.327}$$

When many modes are excited, the transverse field distribution will change during propagation, see Figure 2.90

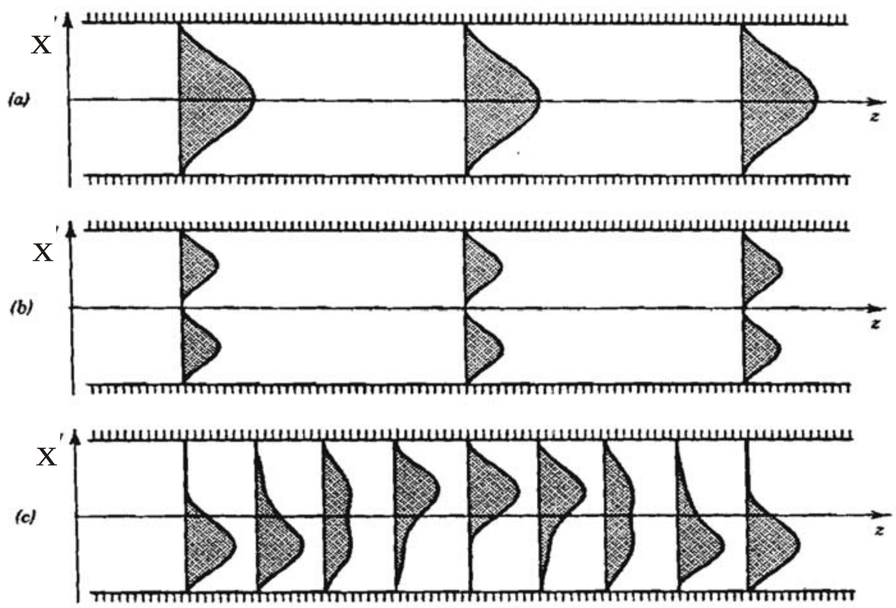

Figure 2.90: Variation of the intensity distribution in the transverse direction x at different distances z. Intensity profile of (a) the fundamental mode $m = 1$, (b) the second mode with $m = 2$ and (c) a linear combination of the fundamental and second mode, [6], p. 247.

Modes which are excited below cutoff will decay rapidly as evanescent waves. The other modes will propagate, but due to the different propagation constants these modes superimpose differently at different propagation distances along the waveguide. This dynamic can be used to build many kinds of important integrated optical devices, such as multimode interference couplers (see problem set 5). Depending on the application, undesired multimode excitation may be very disturbing due to the large group delay difference between the different modes. This effect is called modal dispersion.

Mode Orthogonality

It turns out that the transverse modes determined by the functions $u_m(x)$ build an orthogonal set of basis functions into which any function in a certain function space can be decomposed. This is obvious for the case of the planar-mirror waveguide, where the $u_m(x)$ are a subset of the basis functions for a Fourier series expansion of an arbitrary function $f(x)$ in the interval $[-d/2, 3d/2]$ which is antisymmetric with respect to $x = d/2$ and fullfills the boundary condition $f(x = \pm d/2) = 0$. It is

$$\int_{-d/2}^{d/2} u_m(x)\ u_n(x)\ dx = \delta_{mn}, \tag{2.328}$$

$$f(x) = \sum_m a_m \ \ u_m(x) \tag{2.329}$$

$$\text{with } a_m = \int_{-d/2}^{d/2} u_m(x)\ f(x)\ dx \tag{2.330}$$

From our familiarity with Fourier series expansions of periodic functions, we can accept these relations here without proof. We will return to these equations later in Quantum Mechanics and discuss in which mathematical sense Eqs.(2.328) to (2.329) really hold.

Besides illustrating many important concepts, the planar mirror waveguide is not of much practical use. More in use are dielectric waveguides.

Planar Dielectric Slab Waveguide

In the planar dielectric slab waveguide, waveguiding is not achieved by real reflection on a mirror but rather by total internal reflection at interfaces between two dielectric materials with refractive indices $n_1 > n_2$, see Figure 2.91

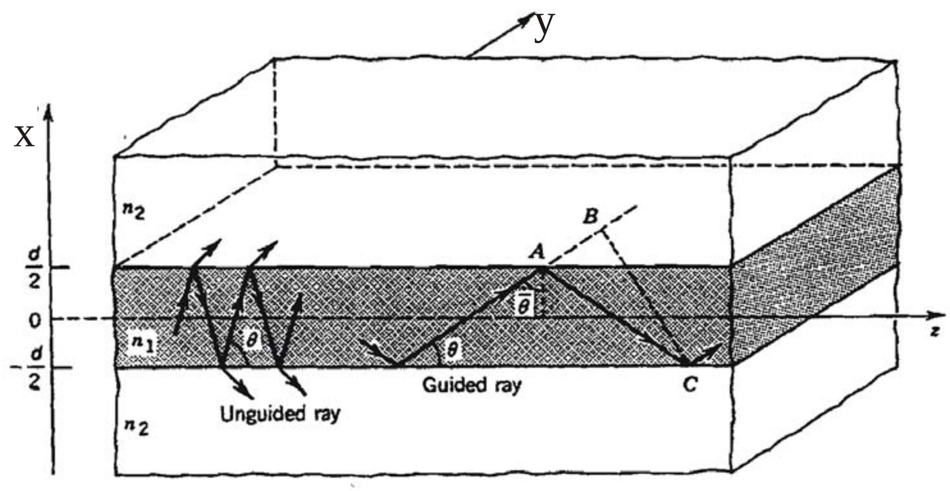

Figure 2.91: Symmetric planar dielectric slab waveguide with $n_1 > n_2$. The light is guided by total internal reflection. The field is evanescent in the cladding material and oscillatory in the core, [6], p. 249.

Waveguide Modes As in the case of the planar mirror waveguide, there are TE and TM-modes and we could find them as a superposition of correspondingly polarized TEM waves propagating with a certain transverse k-vector such that total internal reflection occurs. We do not want to follow this procedure here, but rather use immediately the Helmholtz Equation. We again write the electric field

$$\underline{\vec{E}}_y(x,z) = u(x)\ e^{-j\beta z}\ \vec{e}_y. \tag{2.331}$$

The field has to obey the Helmholtz Eq.(2.18) both in the core and in the cladding

$$\text{core} \ : \ \frac{d^2}{dx^2}u(x) = \left(\beta^2 - k_1^2\right) u(x) \text{ with } k_1^2 = \frac{\omega^2}{c_0^2} n_1^2, \tag{2.332}$$

$$\text{cladding} \ : \ \frac{d^2}{dx^2}u(x) = \left(\beta^2 - k_2^2\right) u(x) \text{ with } k_2^2 = \frac{\omega^2}{c_0^2} n_2^2 \tag{2.333}$$

The boundary conditions are given by the continuity of electric and magnetic field components tangential to the core/cladding interfaces as in section 2.2.

Since the guided fields must be evanescent in the cladding and oscillatory in the core, we rewrite the Helmholtz Equation as

$$\text{core} \quad : \quad \frac{d^2}{dx^2}u(x) = -k_x^2 u(x) \text{ with } k_x^2 = \left(k_1^2 - \beta^2\right), \tag{2.334}$$

$$\text{cladding} \quad : \quad \frac{d^2}{dx^2}u(x) = \kappa_x^2 u(x) \text{ with } \kappa_x^2 = \left(\beta^2 - k_2^2\right) \tag{2.335}$$

where κ_x is the decay constant of the evanescent waves in the cladding. It is obvious that for obtaining guided modes, the propagation constant of the mode must be between the two propagation constants for core and cladding

$$k_2^2 < \beta^2 < k_1^2. \tag{2.336}$$

Or by defining an effective index for the mode

$$\beta = k_0 n_{eff}, \text{ with } k_0 = \frac{\omega}{c_0} \tag{2.337}$$

we find

$$n_1 > n_{eff} > n_2, \tag{2.338}$$

and Eqs.(2.334), (2.335) can be rewritten as

$$\text{core} \quad : \quad -\frac{d^2}{dx^2}u(x) - k_0^2\left(n_1^2 - n_{eff}^2\right)u(x) = 0 \tag{2.339}$$

$$\text{cladding} \quad : \quad -\frac{d^2}{dx^2}u(x) + \kappa_0^2\left(n_{eff}^2 - n_2^2\right)u(x) = 0 \tag{2.340}$$

For reasons, which will become more obvious later, we draw in Figure 2.92 the negative refractive index profile of the waveguide.

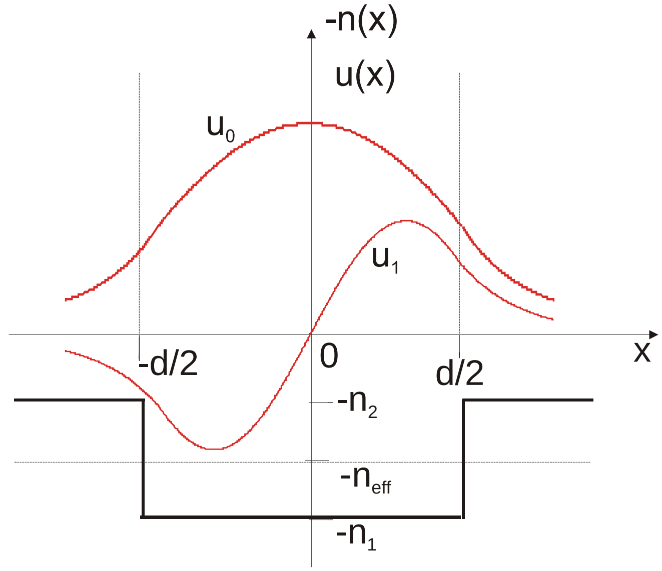

Figure 2.92: Negative refractive index profile and shape of electric field for the fundamental and first higher order transverse TE-mode

From Eq.(2.339) we find that the solution has the general form

$$u(x)=\begin{cases} A\exp\left(-\kappa_x x\right)\ +B\exp\left(\kappa_x x\right), & \text{for } x<-d/2 \\ C\cos\left(k_x x\right)\ +D\sin\left(k_x x\right), & \text{for } |x|<d/2 \\ E\exp\left(-\kappa_x x\right)\ +F\exp\left(\kappa_x x\right), & \text{for } |x|>d/2 \end{cases} \tag{2.341}$$

For a guided wave, i.e. $u_m(x\to\pm\infty)=0$ the coefficients A and F must be zero. It can be also shown from the symmetry of the problem, that the solutons are either even or odd (proof later)

$$u^{(e)}(x)=\begin{cases} B\exp\left(\kappa_x x\right), & \text{for } x<-d/2 \\ C\cos\left(k_x x\right), & \text{for } |x|<d/2 \\ E\exp\left(-\kappa_x x\right), & \text{for } |x|>d/2 \end{cases}, \tag{2.342}$$

$$u^{(o)}(x)=\begin{cases} B\exp\left(\kappa_x x\right), & \text{for } x<-d/2 \\ D\sin\left(k_x x\right), & \text{for } |x|<d/2 \\ E\exp\left(-\kappa_x x\right), & \text{for } |x|>d/2 \end{cases}. \tag{2.343}$$

The coefficients B and E in each case have to be determined from the boundary conditions. From the continuity of the tangential electric field $\underline{E}_y$, and the tangential magnetic field $\underline{H}_z$, which follows from Faraday's Law to be

$$\underline{H}_z(x) = \frac{1}{-j\omega\mu_0}\frac{\partial \underline{E}_y}{\partial x} \sim \frac{du}{dx} \tag{2.344}$$

we obtain the boundary conditions for $u(x)$

$$u(x = \pm d/2 + \epsilon) = u(x = \pm d/2 - \epsilon), \tag{2.345}$$

$$\frac{du}{dx}(x = \pm d/2 + \epsilon) = \frac{du}{dx}(x = \pm d/2 - \epsilon). \tag{2.346}$$

Note, these are four conditions determining the coefficients B, D, E and the propagation constant β or refractive index n_{eff}. These conditions solve for the parameters of even and odd modes separately. For the case of the even modes, where $B = E$, we obtain

$$B\exp\left(-\kappa_x \frac{d}{2}\right) = C\cos\left(k_x \frac{d}{2}\right) \tag{2.347}$$

$$B\ \kappa_x \exp\left(-\kappa_x \frac{d}{2}\right) = Ck_x \sin\left(k_x \frac{d}{2}\right) \tag{2.348}$$

or by division of the both equations

$$\kappa_x = k_x \tan\left(k_x \frac{d}{2}\right). \tag{2.349}$$

Eqs.(2.334) and (2.335) can be rewritten as one equation

$$k_x^2 + \kappa_x^2 = \left(k_1^2 - k_2^2\right) = k_0^2\left(n_1^2 - n_2^2\right) \tag{2.350}$$

Eq.(2.349) together with Eq.(2.350) determine the propagation constant β via the two relations.

$$\kappa_x \frac{d}{2} = k_x \frac{d}{2}\tan\left(k_x \frac{d}{2}\right), \text{ and} \tag{2.351}$$

$$\left(k_x \frac{d}{2}\right)^2 + \left(\kappa_x \frac{d}{2}\right)^2 = \left(k_0 \frac{d}{2} NA\right)^2 \tag{2.352}$$

where

$$NA = \sqrt{(n_1^2 - n_2^2)} \tag{2.353}$$

is called the numerical apperture of the waveguide. We will discuss the physical significance of the numerical apperture shortly. A graphical solution of these two equations can be found by showing both relations in one plot, see Figure 2.93.

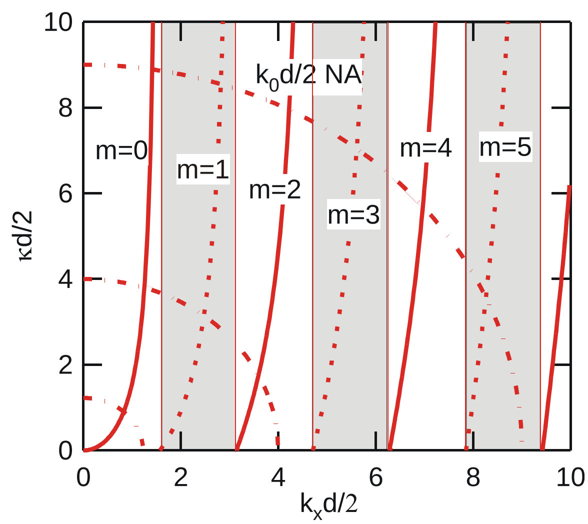

Figure 2.93: Graphical solution of Eqs.(2.351) and (2.352), solid line for even modes and Eq.(2.354) for the odd modes. The dash dotted line shows (2.352) for different values of the product $\left(k_0 \frac{d}{2}\right) NA$

Each crossing in Figure 2.93 of a solid line (2.351) with a circle (2.352) with radius $k_0 \frac{d}{2} NA$ represents an even guided mode. Similarly one finds for the odd modes from the boundary conditions the relation

$$\kappa_x \frac{d}{2} = -k_x \frac{d}{2} \cot\left(k_x \frac{d}{2}\right), \tag{2.354}$$

which is shown in Figure 2.93 as dotted line. The corresponding crossings with the circle indicate the existence of an odd mode.

There are also TM-modes, which we don't want to discuss for the sake of brevity.

Numerical Aperture Figure 2.93 shows that the number of modes guided is determined by he product $k_0 \frac{d}{2} NA$, where NA is the numerical apperture defined in Eq.(2.353)

$$M = Int\left[k_0 \frac{d}{2} NA/(\pi/2)\right] + 1, \tag{2.355}$$

$$= Int\left[2 \frac{d}{\lambda_0} NA\right] + 1, \tag{2.356}$$

where the function $Int[x]$means the largest integer not greater than x. Note, that there is always at least one guided mode no matter how small the sized and the refractive index contrast between core and cladding of the waveguide is. However, for small size and index contrast the mode may extend very far into the cladding and the confinement in the core is low.

The numberical apperture also has an additional physical meaning that becomes obvious from Figure 2.94.

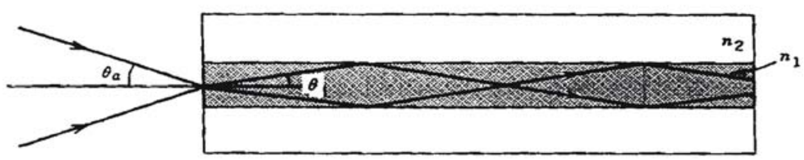

Figure 2.94: Maximum angle of incoming wave guided by a waveguide with numerical apperture NA, [6], p. 262.

The maximum angle of an incoming ray that can still be guided in the waveguide is given by the numerical apperture, because according to Snell's Law

$$n_0 \sin(\theta_a) = n_1 \sin(\theta), \tag{2.357}$$

where n_0 is the refractive index of the medium outside the waveguide. The maximum internal angle θ where light is still guided in the waveguide by total internal reflection is determined by the critical angle for total internal reflection (2.126) , i.e. $\theta_{\max} = \pi/2 - \theta_{tot}$ with

$$\sin(\theta_{tot}) = \frac{n_2}{n_1}. \tag{2.358}$$

Thus for the maximum angle of an incoming ray that can still be guided we find

$$n_0 \sin(\theta_{a,\max}) = n_1 \sin(\theta_{\max}) = n_1 \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} = NA. \tag{2.359}$$

Most often the external medium is air with $n_0 \approx 1$ and the refractive index contrast is week, so that $\theta_{a,\max} \ll 1$ and we can replace the sinusoid with its argument, which leads to

$$\theta_{a,\max} = NA. \tag{2.360}$$

Field Distributions Figure 2.95 shows the field distribution for the TE guided modes in a dielectric waveguide. Note, these are solutions of the second order differential equations (2.339) and (2.340) for an effective index n_{eff}, that is between the core and cladding index. These guided modes have a oscillatory behavior in those regions in space where the negative effective index is larger than the negative local refractive index, see Figure 2.92 and exponentially decaying solutions where the negative effective index is smaller than than the negative local refractive index.

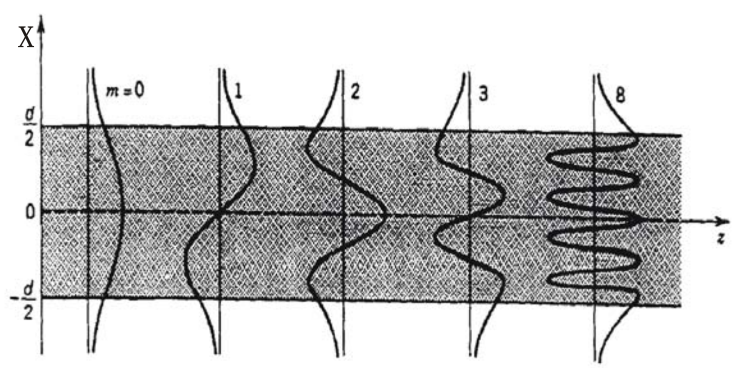

Figure 2.95: Field distributions for TE guided modes in a dielectric waveguide. These results should be compared with those shown in Figure 2.87 for the planar-mirror waveguide [6], p. 254.

Figure 2.96 shows a comparison of the guided modes in a waveguide with a Gaussian beam. In contrast to a the Gaussian beam which diffracts, in a waveguide diffraction is balanced by the guiding action of the index discontinuity, i.e. total internal reflection. Most importantly the cross section of a waveguide mode stays constant and therefore a waveguide mode can efficiently interact with the medium constituting the core or a medium that is incorporated in the core.

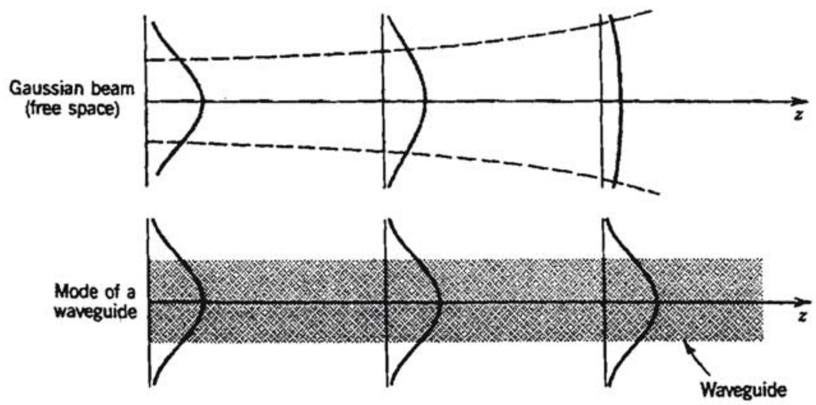

Figure 2.96: Comparison of Gaussian beam in free space and a waveguide mode, [6], p. 255.

Besides integration, this prolong interaction disctance is one of the major reasons for using waveguides. The interaction lenght can be arbitrarily long, only limited by the waveguide loss, in contrast to a Gaussian beam, which stays focused only over the confocal distance or Rayleigh range.

As in the case of a planar-mirror waveguide, one can show that the transverse mode functions are orthogonal to each other. At first, a striking difference here is that we have only a finite number of guided modes and one might worry about the completeness of the transverse mode functions. The answer is that in addition to the guided modes, there are unguided modes or leaky modes, which together with the guided modes from a complete set. Each initial field can be decomposed into these modes. The leaky modes rapidly loose energy because of radiation and after a relatively short propagation distance only the field of guided modes remains in the waveguide. We will not pursue this further in this introductory class. The interested reader should consult with [11].

Confinement Factor

A very important quantity for a waveguide mode is its confinement in the core, which is called the confinement factor

$$\Gamma_m = \frac{\int_0^{d/2} u_m^2(x)\ dx}{\int_0^{\infty} u_m^2(x)\ dx}. \tag{2.361}$$

The confinement factor quantifies the fraction of the mode energy propagating in the core of the waveguide. This is very important for the interaction of the mode with the medium of the core, which may be used to amplify the mode or which may contain nonlinear media for frequency conversion.

Waveguide Dispersion

For the guided modes the effective refractive indices of the modes and therefore the dispersion relations must be between the indices or dispersion relations of core and cladding, see Figure 2.97

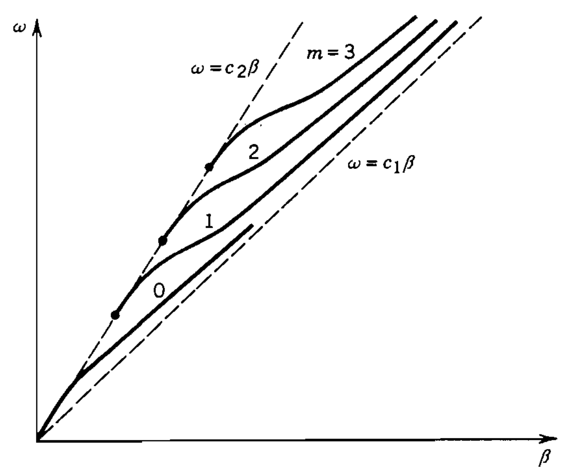

Figure 2.97: Dispersion relations for the different guided TE-Modes in the dielectric slab waveguide.

The different slopes $d\omega/d\beta$ for each mode indicate the difference in group velocity between the modes. Note, that there is at least always one guided mode.

2.7.2 Two-Dimensional Waveguides

Both the planar-mirror waveguide and the planar dielectric slab waveguide confine light only in one direction. It is straight forward to analyze the modes of the two-dimensional planar-mirror waveguide, which you have already done in 6.013. Figure 2.98 shows various waveguides that are used in praxis for various devices. Here, we do not want to analyze them any further, because this is only possible by numerical techniques.

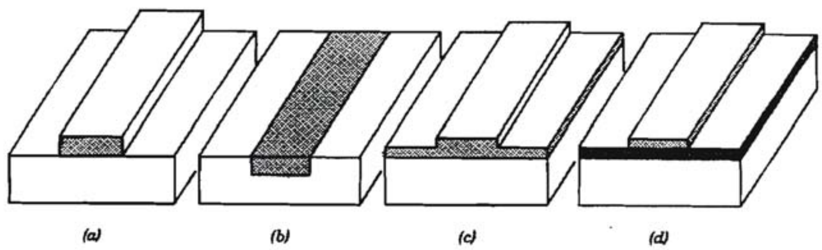

Figure 2.98: Various types of waveguide geometries: (a) strip: (b) embedded strip: (c) rib ro ridge: (d) strip loaded. The darker the shading, the higher the refractive index [6], p. 261.