

## 6.8 Rate Equations and Cross Sections

In many cases the fastest process in the atom-field interaction dynamics is the dephasing of the dipole moment, i. e.  $T_2 \rightarrow 0$ . For example, in semi-conductors  $T_2 < 50fs$ . In those cases the magnitude of the dipole moment relaxes instantaneously into the steady state and follows the magnitude of the intensity envelope  $I(t)$  of the electromagnetic field, which evolves on a much slower time scale. From Eq.(6.104) we obtain with the steady state solutions for the dipole moment (6.107) and (6.108) for the time dependent inversion in the atomic system after adiabatic elimination of dipole moment

$$\dot{w} = -\frac{w(t) - w_0}{T_1} - \frac{w(t)}{T_1 I_s} L(\omega) I(t), \quad (6.125)$$

where  $I(t)$  is the intensity of the electromagnetic wave interaction with the two-level atom. In this limit the Bloch Equations became simple rate equations. We only take care of the counting of population differences due to spontaneous and stimulated emissions.

The interaction of an atom with light at a given transition with the stream of photons on resonance, i.e.  $\omega = \omega_{eg}$  is often described by the mass action law, that is that the number of induced transitions, for example from the excited to the ground state, is proportional to the product of the number of atoms in the excited state and the photon flux density  $I_{ph} = I/\hbar\omega_{eg}$

$$\dot{w}|_{induced} = -\sigma w I_{ph} = -\frac{w}{T_1 I_s} I. \quad (6.126)$$

Thus the interaction cross section can be expressed in terms of the saturation intensity as

$$\sigma = \frac{\hbar\omega_{eg}}{T_1 I_s} \quad (6.127)$$

$$= \frac{2\omega_{eg} T_2 Z_F}{\hbar} |\vec{M}^* \cdot \vec{e}_p|^2. \quad (6.128)$$

In this chapter, we have introduced the most important spectroscopic quantities that characterize an atomic transition, which are the lifetime of the excited state or often called upper-state lifetime or longitudinal lifetime  $T_1$ , the phase relaxation time or transverse relaxation time  $T_2$  which is the inverse half-width at half maximum of the line and the interaction cross-section that only depends on the dipole matrix element and the linewidth of the transition.

# Bibliography

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