2.5 Rays and Optical Systems

Now, that we understand how a beam of finite size as a solution of Maxwell's Equations can be constructed, we are interested how such a beam can be imaged by an optical system. Propagation of a Gaussian beam in free space leads to spreading of the beam because of the diffraction. We need means to focus the beam again. The output beam from a laser may have a certain size but we may need a different size for a given experiment. We can change the size or focus the beam by an optical imaging system. Optical systems are studied and analyzed using ray optics. What is a ray? We have already discussed that diffraction of a beam is similar to dispersion of an optical pulse. Dispersion of a pulse we understood because of the different group velocity of different frequency components or sub-pulses. It turns out that these sub-pulses are the temporal analog to the rays. In the same way we can construct a short pulse by a superposition of sub-pulses with different center frequencies, we can construct a Gaussian beam by sub-beams with different center transverse k-vectors and a very narrow spread in transverse k-vectors. These are Gaussian beams with a large beam diameter such that diffraction is not any longer important. These beams are called rays. The ray only experiences a phase shift during propagation depending on the local refractive index $n(r)$. Therefore, we can completely understand the imaging of Gaussian beams in paraxial optical systems by the imaging properties of rays.

2.5.1 Ray Propagation

A ray propagating in an optical system, see Figure 2.64, can be described by its position r with respect to the optical axis and its inclination with respect to the optical axis r'. It is advantageous to use not (r, r') as the ray coordinates but the combination $(r, n\ r')$, where n is the local refractive index at the position of the ray. Due to propagation, the ray coordinates may change, which can be desribed by a marix, that maps initial position and inclination into the corresponding quantitaties after the propagation

$$\begin{pmatrix} r_2 \\ n_2 r_2' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} r_1 \\ n_1 r_1' \end{pmatrix}. \tag{2.242}$$

This imaging matrix is called an ABCD-matrix.

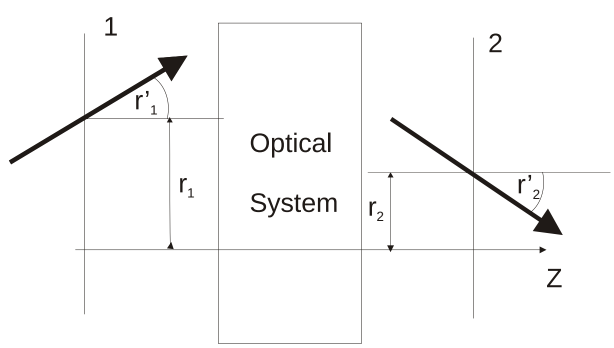

Figure 2.64: Description of optical ray propagation by its distance and inclination from the optical axis

The advantage in using $(r, n\, r')$ as the ray coordinates is that it preserves the phase space volume, i.e. for lossless optical systems the determinant of the ABCD-matrix must be 1. Also Snell's law for paraxial rays has then a simple form, see Figure 2.65. For paraxial rays the angles to the interface normal, θ_1 and θ_2, are much smaller than 1, and we can write

$$r_1' = \tan\theta_1 \approx \sin\theta_1 \approx \theta_1, \text{ and } r_2' = \tan\theta_2 \approx \sin\theta_2 \approx \theta_2.$$

Then Snell's law is

$$n_1\, r_1' = n_2\, r_2'. \tag{2.243}$$

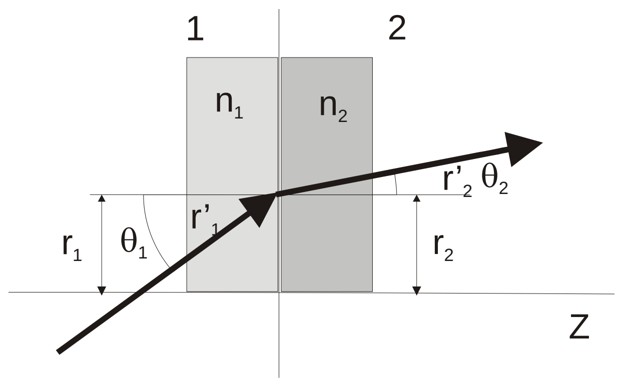

Figure 2.65: Snell's law for paraxial rays

The ABCD-matrix describing a ray going from a medium with index n_1 to a medium with index n_2 is the unity matrix

$$r_2 = r_1 \tag{2.244}$$
$$n_2\, r_2' = n_1\, r_1'. \tag{2.245}$$

Free space propagation

For propagation in free space, see Figure 2.66, the relationship between input and output ray parameters is

$$\begin{aligned} r_2 &= r_1 + r_1' \cdot L \\ r_2' &= r_1' \end{aligned}$$

or the propagation matrix is

$$\mathbf{M} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}. \tag{2.246}$$

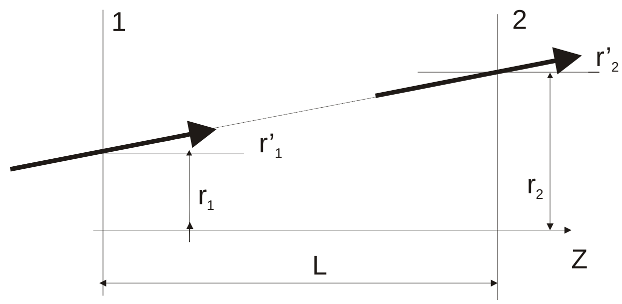

Figure 2.66: Free space propagation

Propagation in medium with length L and index n

Free propagation through a medium with index n does result in a reduced position shift with respect to the optical axis in comparison to free space, because the beam is first bent to the optical axis according to Snell's law, see Figure 2.67. Therefore the corresponding ABCD-matrix is

$$\mathbf{M} = \begin{pmatrix} 1 & L/n \\ 0 & 1 \end{pmatrix}. \tag{2.247}$$

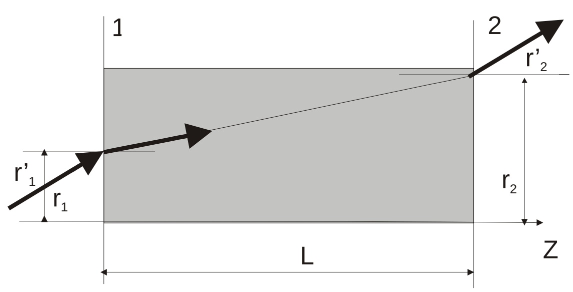

Figure 2.67: Ray propagation through a medium with refractive index n, shortens the path length of the beam by a factor of n.

Parbolic surface or thin lens

Plano-Convex Lens When a ray penetrates a parabolic surface between two media with refractive indices n_1 and n_2, it changes its inclination. A parabolic surface can be closely approximated by the surface of a sphere, see Figure 2.68. Snells law in paraxial approximation is

$$n_1 \ (r_1' + \alpha) = n_2 \ (r_2' + \alpha) \,. \tag{2.248}$$

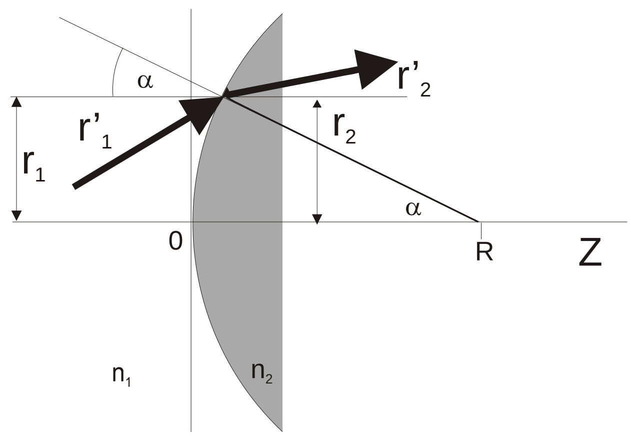

Figure 2.68: Derivation of ABCD-matrix of a thin plano-convex lens.

The small angle α can be approximated by $\alpha \approx r_1/R$. In total we then obtain the mapping

$$r_2 = r_1 \tag{2.249}$$

$$n_2 \, r_2' = n_1 \, r_1' + \frac{n_1 - n_2}{R} r_1 \tag{2.250}$$

or

$$\mathbf{M} = \begin{pmatrix} 1 & 0 \\ \frac{n_1 - n_2}{R} & 1 \end{pmatrix} . \tag{2.251}$$

Note, the second normal interface does not change the ray propagation matrix and therefore Eq.(2.251) describes correctly the ray propagation through a thin plano-convex lens.

Biconvex Lens If the lens would have a second convex surface, this would refract the ray twice as strongly and we would obtain

$$\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 2\frac{n_1-n_2}{R} & 1 \end{pmatrix}. \tag{2.252}$$

The quantity $2\frac{n_2-n_1}{R}$ is called the refractive strength of the biconvex lense or inverse focal length $1/f$.Because the system of a thin lens plus free space propagation results in the matrix (calculated in the reverse order)

$$\mathbf{M}_{tot} = \begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} = \begin{pmatrix} 0 & f \\ -\frac{1}{f} & 1 \end{pmatrix}, \tag{2.253}$$

which ensures that each ray parallel to the optical axis goes through the on axis focal point at the end of the free space section, see Figure 2.69.

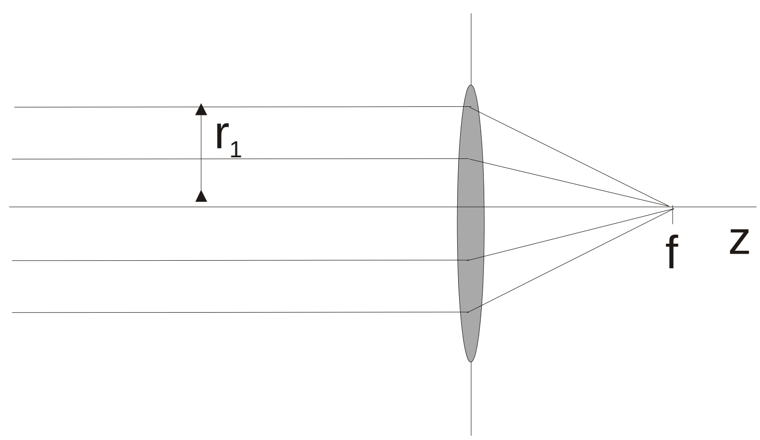

Figure 2.69: Imaging of parallel rays through a lens with focal length f.

Curved Mirrors

Other often used optical components in imaging systems are curved mirrors with radius of curvature $ROC = R$, see Figure 2.70. The advantage of reflective optics is that the rays don't have to pass through dispersive material like through a lense, which is very disturbing for ultrashort pulses.

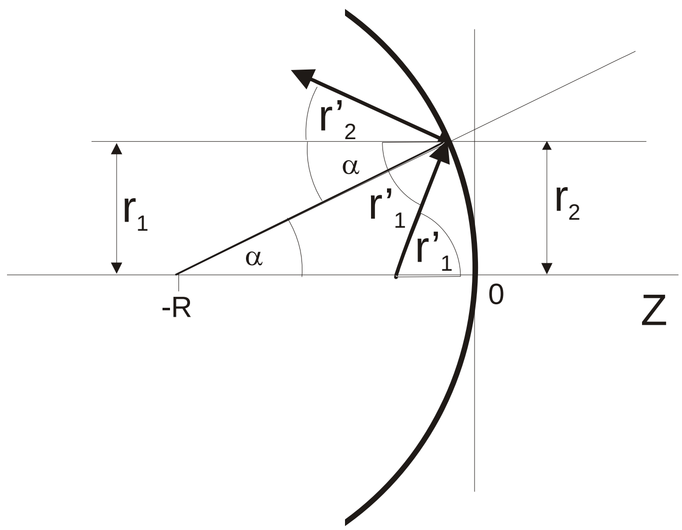

Figure 2.70: Derivation of ray matrix for concave mirror with Radius R.

As in the case of the thin lens,e the imaging does not change the distance of the ray from the optical axis, however, the slope of the rays obey

$$r_1' - \alpha = r_2' + \alpha. \tag{2.254}$$

with $\alpha \approx r_1/R$ in paraxial approximation. Therefore the ABCD matrix describing the reflection of rays at a curved mirror with $ROC = R$ is

$$\mathbf{M} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}, \text{ with } f = \frac{R}{2}. \tag{2.255}$$

2.5.2 Gauss' Lens Formula

As a simple application of the ray matrices for optical system design, we derive Gauss' lens formula, which says that all rays emitted from an orignial placed a distance d_1 from a lens with focal length f form an image at a distance d_2, which is related to d_1 by

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f}, \tag{2.256}$$

see Figure 2.71.

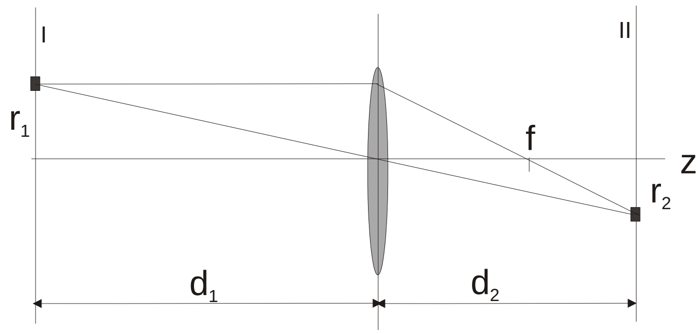

Figure 2.71: Gauss' lens formula.

The magnification of the lens system is $M_r = \frac{r_2}{r_1} = \frac{d_2}{d_1} = \left|\frac{f}{d_1-f}\right|$. The ray matrix that describes the imagaing from the orignal plane I to the image plane II is described by the product

$$\begin{aligned}\begin{pmatrix} A & B \\ C & D \end{pmatrix} &= \begin{pmatrix} 1 & d_2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}\begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1-\frac{d_2}{f} & \left(1-\frac{d_2}{f}\right)d_1 + d_2 \\ -\frac{1}{f} & 1-\frac{d_1}{f} \end{pmatrix}. \end{aligned} \tag{2.257}$$

In order that the distance r_2 only depends on r_1, but not on r_1', B must be 0, which is Eq. (2.256). Thus in total we have

Magnification	$M_r =$	$\frac{f}{d_1-f}$
Distance to focus	$d_2 - f = M_r^2\,(d_1 - f)$	

(2.258)

More complicated imaging systems, such as thick lenses, can be described by ray matrices and arbitrary paraxial optical systems can be analyzed with them, which shall not be pursued further here. Rather, we want to study how Gaussian beams are imaged by paraxial optical systems