2.7.6 Optical Fibers

Optical fibers are cylindrical waveguides, see Figure 2.108, made of low-loss materials such as silica glass.

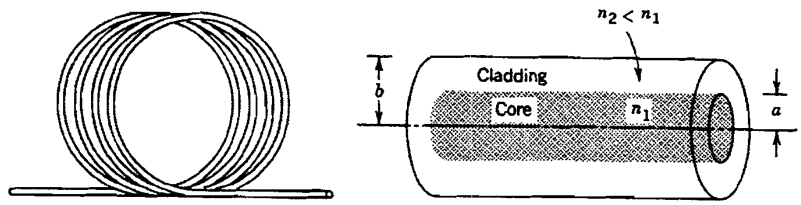

Figure 2.108: Optical fibers are cylindrical dielectric waveguides, [6], p. 273.

Similar to the waveguides studied in the last section the most basic fibers consist of a high index core and a lower index cladding. Today fiber technology is a highly developed art which has pushed many of the physical parameters of a waveguide to values which have been thought to be impossible a few decades ago:

- Fiber with less than 0.16dB/km loss
- Photonic crystal fiber (Nanostructured fiber)
- Hollow core fiber
- Highly nonlinear fiber
- Er-doped fiber for amplifiers
- Yb-doped fiber for efficient lasers and amplifiers
- Raman gain fiber
- Large area single mode fibers for high power (kW) lasers.

Figure 2.109 shows the ranges of attenuation coefficients of silica glass single-mode and multimode fiber.

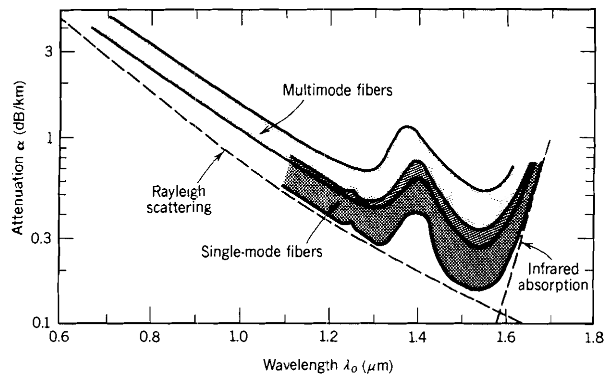

Figure 2.109: Ranges of attenuation coefficients of silica glass single-mode and multimode fiber, [10], p. 298.

For the purpose of this introductory class we only give an overview about the mode structure of the most basic fiber, the step index fiber, see Figure 2.110 (b)

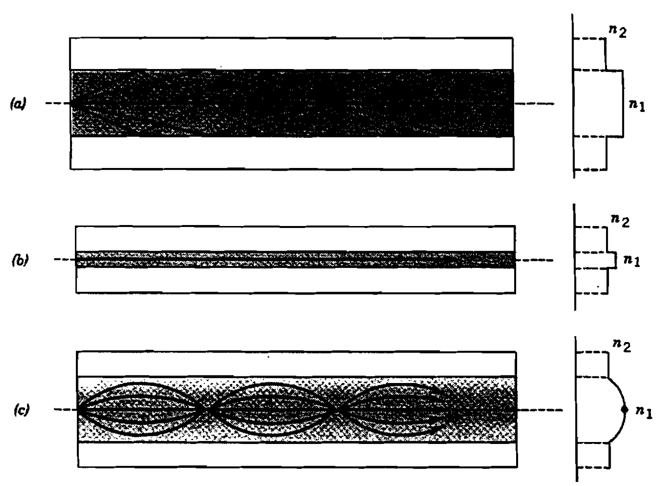

Figure 2.110: Geometry, refractive index profile, and typical rays in: (a) a multimode step-index fiber, (b) a single-mode step-index fiber, (c) a multimode graded-index fiber [6], p. 274

Step-index fiber is a cylindrical dielectric waveguide specified by its core and cladding refractive indices, n_1 and n_2 and the core radius a, see Figure 2.108. Typically the cladding is assumed to be so thick that the finite cladding radius does not need to be taken into account. The guided modes need to be sufficiently decaded before reaching the cladding boundary, which is usually strongly scattering or absorbing. In standard fiber, the cladding indices differ only slightly, so that the relative refractive-index difference

$$\Delta = \frac{n_1 - n_2}{n_1} \tag{2.382}$$

is small, typically $10^{-3} < \Delta < 2 \cdot 10^{-2}$. Most fibers currently used in medium to long optical communication systems are made of fused silica glass (SiO_2) of high chemical purity. The increase in refractive index of the core is achieved by doping with titanium, germanium or boron, among others. The refractive index n_1 ranges from 1.44 to 1.46 depending on the wavelength utilized in the fiber. The acceptance angle of the rays coupling from free space into guided modes of the waveguide is determined by the numerical apperture as already discussed for the dielectric slab waveguide, see Figure 2.111

$$\theta_a \sim \sin(\theta_a) = NA = \sqrt{n_1^2 - n_2^2} \approx n_1 \sqrt{2\Delta}. \tag{2.383}$$

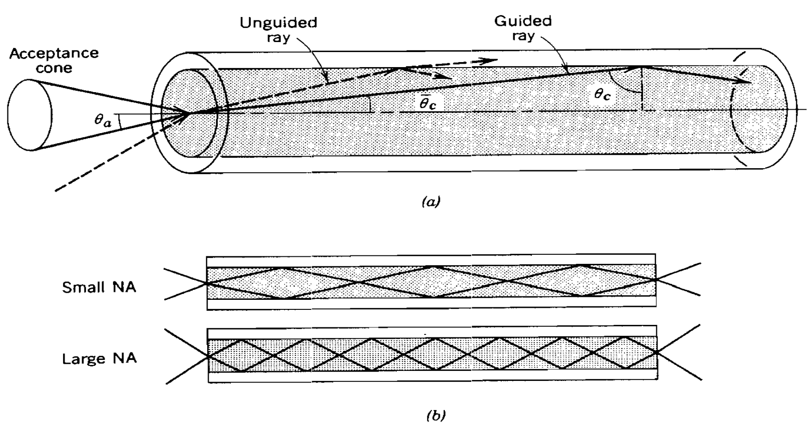

Figure 2.111: The acceptance angle of a fiber and numerical aperture NA [6], p. 276.

Guided Waves

Again the guided waves can be found by looking at solutions of the Helmholtz equations in the core and cladding where the index is homogenous and by additionally requesting the continuity of the tangential electric and magnetic fields at the core-cladding boundary. In general the fiber modes are not anylonger pure TE or TM modes but rather are hybrid modes, i.e. the modes have both transverse and longitudinal electric and magnetic field components. Only the radial symmetric modes are still TE or TM modes. To determine the exact mode solutions of the fiber is beyond the scope of this class and the interested reader may consult reference [2]. However, for weakly guiding fibers, i.e. $\Delta \ll 1$, the modes are actually very much TEM like, i.e. the longitudinal field components are much smaller than the radial field components. The linear in x and y directions polarized modes form orthogonal polarization states. The linearly polarized (l, m) mode is usually denoted as the LP_{lm}-mode.The two polarizations of the mode with indices (l, m) travel with the same propagation constant and have identical intensity distributions.

The generic solutions to the Helmhotz equation in cylindrical coordinates are the ordinary, $J_m(kr)$, and modified, $K_m(kr)$, Bessel functions (analogous to the $\cos(x)/\sin(x)$ and exponential functions $e^{\pm\kappa x}$, that are solutions to the Helmholtz equation in cartesian coordinates). Thus, a generic mode function for a cylinder symmetric fiber has the form

$$u_{l,m}(r,\varphi) = \begin{cases} J_l(k_{l,m}r)\begin{cases}\cos(l\varphi)\\ \sin(l\varphi)\end{cases}, \text{ for } r < a, \text{ core} \\ K_l(k_{l,m}r)\begin{cases}\cos(l\varphi)\\ \sin(l\varphi)\end{cases}, \text{ for } r > a, \text{ cladding}\end{cases} \tag{2.384}$$

For large r, the modified Bessel function approaches an exponential, $K_l(k_{l,m}r) \sim e^{-\kappa_{\ln m}r}\begin{cases}\cos(l\varphi)\\ \sin(l\varphi)\end{cases}$. The propagation constants for this two dimensional waveguide have to fullfil the additional constraints

$$k_{l,m}^2 = \left(n_1^2k_0^2 - \beta^2\right), \tag{2.385}$$

$$\kappa_{l,m}^2 = \left(\beta^2 - n_2^2k_0^2\right), \tag{2.386}$$

$$k_{l,m}^2 + \kappa_{l,m}^2 = k_0^2NA^2. \tag{2.387}$$

Figure 2.112 shows the radial dependence of the mode functions

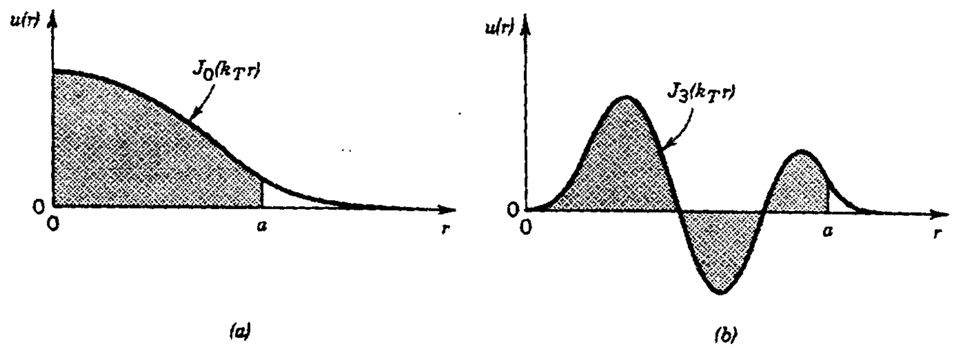

Figure 2.112: Radial dependence of mode functions $u(r)$,[6], p.279.

The transverse intensity distribution of the linearly polarized $LP_{0,1}$ and $LP_{3,4}$ modes are shown in Figure 2.113.

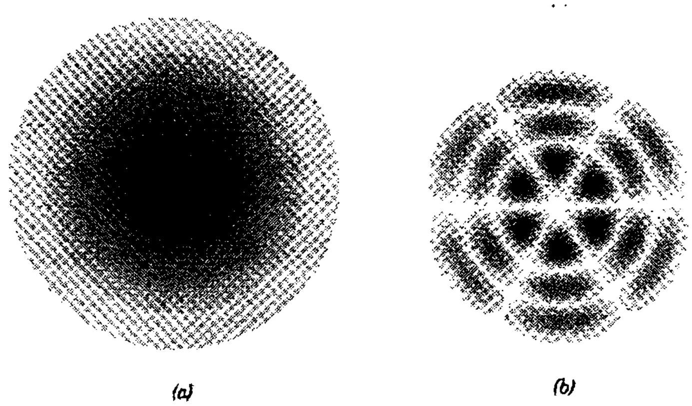

Figure 2.113: Intensity distribtuion of the (a) LP_{01} and (b) $\mathrm{LP}_{3,4}$ modes in the transverse plane. The LP_{01} has a intensity distribution similar to the Gaussian beam, [6], p. 283.

Number of Modes

It turns out, that as in the case of the dielectric slab waveguide the number of guided modes critically depends on the numerical aperture or more precisely on the V-parameter, see Eq.(2.355)

$$V = k_0 \frac{d}{2} NA. \tag{2.388}$$

Without proof the number of modes is

$$M \approx \frac{4}{\pi^2} V^2, \text{ for } V \gg 1. \tag{2.389}$$

which is similar to Eq.(2.355) for the one-dimensional dielectric slab waveguide, but the number of modes here is now related to the square of the V-parameter, because of the two-dimensional transverse confinement of the modes in the fiber. As in the case of the dielectric waveguide, there is always at least one guided mode (two polarizations). However, the smaller the V-parameter the more the mode extends into the cladding and the guiding properties become weak, i.e. small bending of the fiber may already lead to high loss.