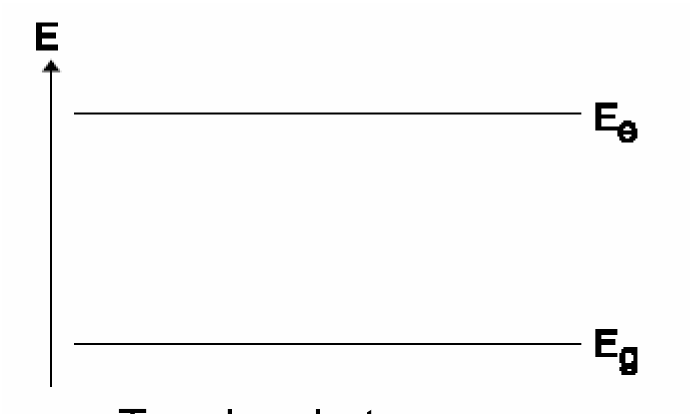
- <u>Background</u>
  - basic quantum optics (two-level atoms, Maxwell-Bloch equation)
  - pulse propagation and compression

→ Real fun just begins now.

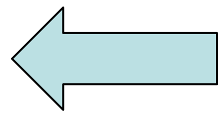
- <u>Generation and Characterization of Ultrashort Pulses</u>
  - CW laser and Q-switching
  - Mode-locking (Active and Passive)
  - KLM, Semiconductor saturable absorber
  - Characterization: Autocorrelator, SPIDER, FROG

- <u>Applications</u>
  - Time-resolved spectroscopy (pump-probe)
  - Optical frequency metrology
  - Optical sampling with electronic-photonic integrated circuits
  - Micromachining
  - Imaging
  - Amplifications; High-harmonic generation and X-ray pulses

Atom-field interaction



Two-level atom



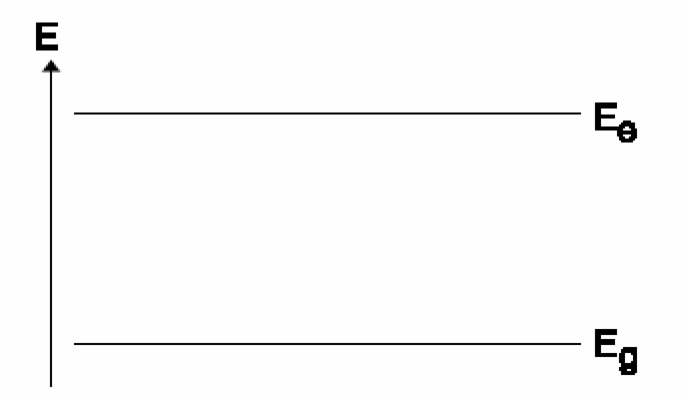
Electromagnetic field

Interaction energy is determined by electric dipole moment.

$$\boxed{\mathrm{H}_{A-F} = -\vec{\mathrm{p}} \cdot \vec{E}(\vec{x}_A, t).}$$

(under dipole approximation: EM wavelength >> atom size → E-field is constant)

Isolated two-level atom



Two-level atom

$$\dot{\rho} = \frac{1}{j\hbar}[\mathrm{H}, \rho].$$

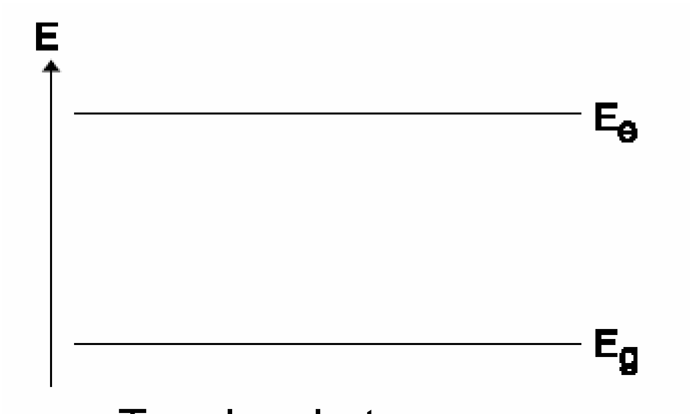
H = H_A

$$\begin{aligned}
\dot{\rho}_{\in e} &= 0, \\
\dot{\rho}_{gg} &= 0,
\end{aligned} \longrightarrow \text{Population constant}$$

$$\begin{aligned}
\dot{\rho}_{eg} &= -j\omega_{eg}\rho_{eg} &\rightarrow\quad \rho_{eg}(t) = e^{-j\omega_{eg}t}\rho_{eg}(0), \\
\dot{\rho}_{ge} &= j\omega_{eg}\rho_{ge} &\rightarrow\quad \rho_{ge}(t) = e^{j\omega_{eg}t}\rho_{ge}(0).
\end{aligned}$$

Dipole moment oscillates with ω_{eg}

Two-level atom interacting with field (thermal light)



Two-level atom

Von Neumann equation
(including incoherent or dissipative process)

$$\dot{\rho} = \frac{1}{j\hbar}[\mathrm{H}, \rho].$$

$H = H_A + H_F + H_{A\text{-}F}$

$$\frac{d}{dt}|c_e(t)|^2 = \frac{d}{dt}\rho_{ee} = -\Gamma_e\rho_{ee} + \Gamma_a\rho_{gg}$$

$$= -\frac{1}{\tau_{sp}}\rho_{ee} - \frac{n_{th}}{\tau_{sp}}\rho_{ee} + \frac{n_{th}}{\tau_{sp}}\rho_{gg}$$

Spontaneous emission Stimulated emission Absorption

$$\Gamma_e = \frac{1}{\tau_{sp}}(n_{th}+1),$$

$$\Gamma_a = \frac{1}{\tau_{sp}}n_{th}.$$

$$n_{th} = 1/(\exp(\hbar\omega_{eg}/kT) - 1)$$

Two-level atom interacting with field (thermal light)

$$\frac{d}{dt}|c_e(t)|^2 = \frac{d}{dt}\rho_{ee} = -\Gamma_e\rho_{ee} + \Gamma_a\rho_{gg}$$

$$\frac{d}{dt}\rho_{gg} = -\frac{d}{dt}\rho_{ee} = \Gamma_e\rho_{ee} - \Gamma_a\rho_{gg}.$$

$$\frac{d}{dt}\rho_{ge} = \mathrm{j}\omega_{eg}\rho_{eg} - \frac{\Gamma_e + \Gamma_a}{2}\rho_{ge}.$$

Two-level atom interacting with field (thermal light)

Dipole moment $$\dot{d} = \dot{\rho}_{ge} = (\mathrm{j}\omega_{eg} - \frac{1}{T_2})d,$$

Inversion $$\dot{w} = \dot{\rho}_{ee} - \dot{\rho}_{gg} = -\frac{w - w_0}{T_1},$$

where $$\frac{1}{T_1} = \frac{2}{T_2} = \Gamma_e + \Gamma_a = \frac{2n_{th} + 1}{\tau_{sp}}$$

T_1 = energy relaxation time
T_2 = phase relaxation time

Equilibrium inversion $$w_0 = \frac{\Gamma_a - \Gamma_e}{\Gamma_a + \Gamma_e} = \frac{-1}{1 + 2n_{th}} = -\tanh\left(\frac{\hbar\omega_{eg}}{2kT}\right).$$

Relaxation times

Dipole moment $\dot{d} \;=\; \dot{\rho}_{ge} = (\mathrm{j}\omega_{eg} - \frac{1}{T_2})d,$

Inversion $\dot{w} \;=\; \dot{\rho}_{ee} - \dot{\rho}_{gg} = -\frac{w - w_0}{T_1},$

where $$\frac{1}{T_1} = \frac{2}{T_2} = \Gamma_e + \Gamma_a = \frac{2n_{th} + 1}{\tau_{sp}}$$

T1 = energy relaxation time
T2 = phase relaxation time

Ideally, $T_2 = 2T_1$

In reality, $T_2 << 2T_1$

Population inversion

Dipole moment $$\dot{d} = \dot{\rho}_{ge} = (\mathrm{j}\omega_{eg} - \frac{1}{T_2})d,$$

Inversion $$\dot{w} = \dot{\rho}_{ee} - \dot{\rho}_{gg} = -\frac{w - w_0}{T_1},$$

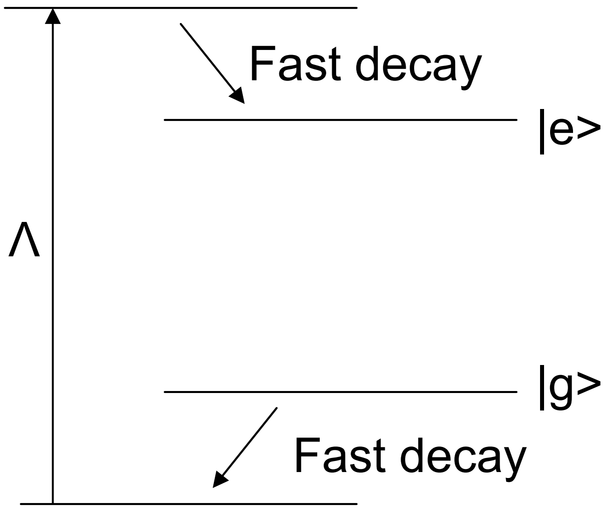
Equilibrium inversion $$w_0 = \frac{\Gamma_a - \Gamma_e}{\Gamma_a + \Gamma_e} = \frac{-1}{1 + 2n_{th}} = -\tanh\left(\frac{\hbar\omega_{eg}}{2kT}\right).$$

→ Always negative for T>0!!!
Population inversion is impossible with incoherent thermal light if there are only two levels!!!

→ Needs "pumping" with incoherent light to an atom with additional levels (to achieve artificial "negative" temperature).

Population inversion

Example: 4-level



$$w_0 = \frac{\overset{\Lambda}{\Gamma_a} - \Gamma_e}{\underset{\Lambda}{\Gamma_a} + \Gamma_e}$$

Λ: pump rate

$$w_0 = \frac{\Lambda - \Gamma_e}{\Lambda + \Gamma_e}.$$ >0 when Λ>Γe

3-level and 4-level systems
will be dealt later again.

Two-level atom interacting with coherent classical external field

Dipole moment $$\dot{d} = \dot{\rho}_{ge} = (j\omega_{eg} - \frac{1}{T_2})d,$$

Inversion $$\dot{w} = \dot{\rho}_{ee} - \dot{\rho}_{gg} = -\frac{w - w_0}{T_1},$$

Adding $\frac{1}{j\hbar}[H_E, \rho]$ term to the previous result:

Bloch Equations:

$$\dot{d} = -(\frac{1}{T_2} - j\omega_{eg})d + \underline{\frac{1}{2j\hbar}\vec{M}\vec{E}^{(+)}w},$$

$$\dot{w} = -\frac{w - w_0}{T_1} + \underline{\frac{1}{j\hbar}(\vec{M}^*\vec{E}^{(-)}d - \vec{M}\vec{E}^{(+)}d^*)}.$$

Describing the dynamics of an atom interacting with a classical E-field

In the stationary state…

(driven by monochromatic field: $\vec{E}(t)^{(+)} = \hat{\vec{E}} e^{j\omega t}$,)

$$\dot{d} = -(\frac{1}{T_2} - j\omega_{eg})d + \frac{1}{2j\hbar}\vec{M}\vec{E}^{(+)}w, \longrightarrow d = \hat{d}e^{j\omega t},$$

$$\dot{w} = -\frac{w - w_0}{T_1} + \frac{1}{j\hbar}(\vec{M}^*\vec{E}^{(-)}d - \vec{M}\vec{E}^{(+)}d^*). \longrightarrow$$

0 and w → w_s

$$\hat{d} = \frac{-j}{2\hbar}\frac{w_s}{1/T_2 + j(\omega - \omega_{eg})}\vec{M}\hat{\vec{E}},$$

$$w_s = \frac{w_0}{1 + \frac{T_1}{\hbar^2}\frac{1/T_2\,|\vec{M}\hat{\vec{E}}|^2}{(1/T_2)^2 + (\omega_{eg} - \omega)^2}}.$$

Saturated inversion, Saturated intensity

$$\hat{d} = \frac{-\mathrm{j}}{2\hbar} \frac{w_s}{1/T_2 + \mathrm{j}(\omega - \omega_{eg})} \vec{M}\hat{\vec{E}},$$

$$w_s = \frac{w_0}{1 + \frac{T_1}{\hbar^2} \frac{1/T_2 \, |\vec{M}\hat{\vec{E}}|^2}{(1/T_2)^2 + (\omega_{eg} - \omega)^2}}.$$

Lorentzian lineshape: $$L(\omega) = \frac{(1/T_2)^2}{(1/T_2)^2 + (\omega_{eg} - \omega)^2},$$

Saturated inversion: $$\boxed{w_s = \frac{w_0}{1 + \frac{I}{I_s} L(\omega)}.}$$

Saturation intensity: $$\boxed{I_s = \left[\frac{2 T_1 T_2 Z_F}{\hbar^2} \frac{\left| \vec{M}\hat{\vec{E}} \right|^2}{\left| \hat{\vec{E}} \right|^2} \right]^{-1}}$$

Maxwell-Bloch Equations

$$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}^{(+)}(z,t) = \mu_0 \frac{\partial^2}{\partial t^2}\vec{P}^{(+)}(z,t),$$

$$\vec{P}^{(+)}(z,t) = -2N\vec{M}^* d(z,t)$$

$$\dot{d} = -\left(\frac{1}{T_2} - \mathrm{j}\omega_{eg}\right)d + \frac{1}{2\mathrm{j}\hbar}\vec{M}\vec{E}^{(+)}w,$$

$$\dot{w} = -\frac{w - w_0}{T_1} + \frac{1}{\mathrm{j}\hbar}\left(\vec{M}^*\vec{E}^{(-)}d - \vec{M}\vec{E}^{(+)}d^*\right).$$

What happens if an EM wave with a slowly varying envelope propagates?

$$\vec{E}(z,t)^{(+)} = \sqrt{2Z_{F_0}}A(z,t)e^{\mathrm{j}(\omega_{eg}t - k_0 z)}\vec{e},$$

Maxwell-Bloch Equations

What happens if an EM wave with a slowly varying envelope propagates?

$$\vec{E}(z,t)^{(+)} = \sqrt{2Z_{F_0}}\,A(z,t)e^{\mathrm{j}(\omega_{eg}t-k_0 z)}\vec{e},$$

$$\downarrow$$

Excites dipole moment in the atomic medium

$$d(z,t) = \hat{d}(z,t)e^{\mathrm{j}(\omega_{eg}t-k_0 z)},$$

$$\downarrow$$ Maxwell-Bloch Eqs:

$$\left(\frac{\partial}{\partial z}+\frac{1}{c_0}\frac{\partial}{\partial t}\right)A(z,t) = \mathrm{j}N\vec{e}^{T}\vec{M}^{*}\sqrt{\frac{Z_{F_0}}{2}}\hat{d}\,(z,t)\,, \tag{2.166}$$

$$\frac{\partial}{\partial t}d(z,t) = -\frac{1}{T_2}\hat{d}+\frac{\sqrt{2Z_{F_0}}}{2\mathrm{j}\hbar}\left(\vec{M}\vec{e}\right)A(t)w \tag{2.167}$$

$$\frac{\partial}{\partial t}w(z,t) = -\frac{w-w_0}{T_1}+\frac{\sqrt{2Z_{F_0}}}{\mathrm{j}\hbar}\left(\left(\vec{M}^{*}\vec{e}^{*}\right)A^{*}(t)\hat{d}-\left(\vec{M}\vec{e}\right)A(t)\hat{d}\right) \tag{2.168}$$

Maxwell-Bloch Equations

when T2 is much shorter than the variation of envelope A(t)

$$\frac{\partial}{\partial t} d(z,t) = -\frac{1}{T_2}\hat{d} + \frac{\sqrt{2Z_{F_0}}}{2\mathrm{j}\hbar}\left(\vec{M}\vec{e}\right)A(t)w \longrightarrow 0$$

$$\longrightarrow \quad \hat{d} = T_2 \frac{\sqrt{2Z_{F_0}}}{2\mathrm{j}\hbar}\left(\vec{M}\vec{e}\right)A(t)w,$$

$$\frac{\partial}{\partial t} w(z,t) = -\frac{w - w_0}{T_1} + \frac{\sqrt{2Z_{F_0}}}{\mathrm{j}\hbar}\left(\left(\vec{M}^*\vec{e}^*\right)A^*(t)\hat{d} - \left(\vec{M}\vec{e}\right)A(t)\hat{d}^*\right) \quad (2.168)$$

$$\longrightarrow \quad \boxed{\dot{w} = -\frac{w - w_0}{T_1} - \frac{|A(z,t)|^2}{E_s} w(z,t)}$$

Es = IsT1 : saturation fluence (J/cm^2)

Rate Equations

Where is dipole moment?

Now we can express only with population difference (of atom) and field amplitude (of wave) !!

$$\boxed{\left(\frac{\partial}{\partial z} + \frac{1}{c_0}\frac{\partial}{\partial t}\right) A(z,t) \;=\; \frac{N\hbar}{4T_2 E_s} w\left(z,t\right) A(z,t),}$$

→ Obtaining gain for inverted (w>0) medium!!!

$$\boxed{\dot{w} \;=\; -\frac{w - w_0}{T_1} - \frac{\left|A(z,t)\right|^2}{E_s} w(z,t)}$$

$$\longrightarrow \qquad w_S = \frac{w_0}{1 + \dfrac{T_1 \,|\, A \,|^2}{E_S}}$$

Gain saturates with the EM power density

Now let’s jump into the new chapter.

Rate Equations

Back to the result of Maxwell-Bloch equation,

$$\dot{w} = -\frac{w - w_0}{T_1} - \frac{|A(z,t)|^2}{E_s} w(z,t)$$

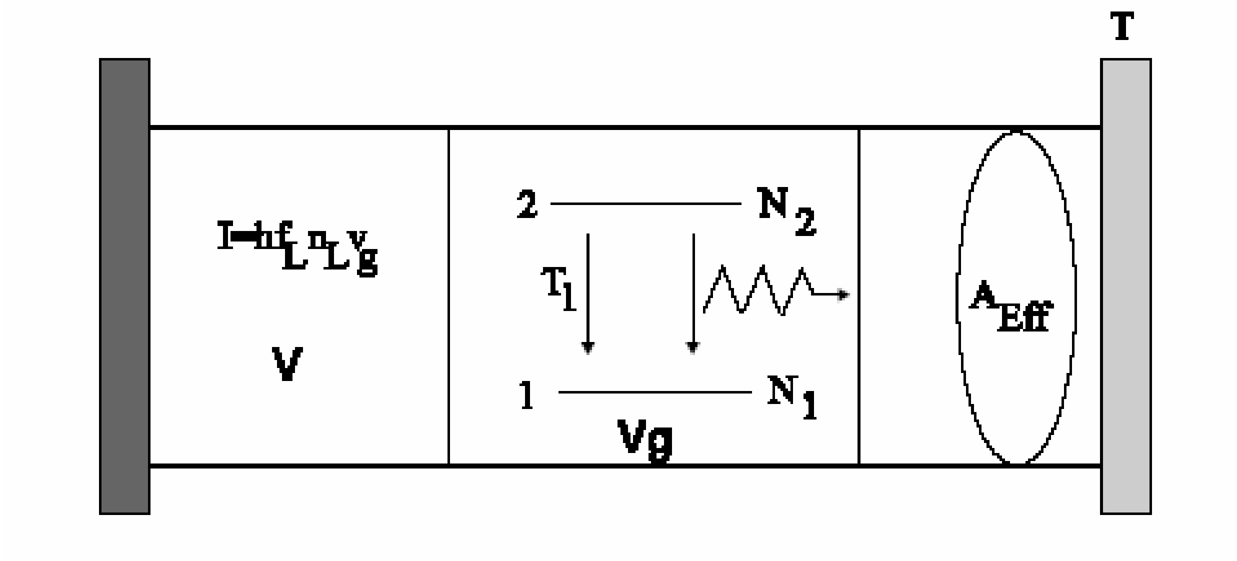
Rate equation for population inversion →

$$\boxed{\frac{dw}{dt} = -\frac{w - w_0}{T_1} - \frac{I}{T_1 I_S} w}$$

where Saturation intensity

$$I_s = \left[\frac{2 T_1 T_2 Z_F}{\hbar^2} \frac{\left| \vec{M} \hat{\vec{E}} \right|^2}{\left| \hat{\vec{E}} \right|^2} \right]^{-1}$$

Rate equations for two-level atom



Terminology for treating single-mode laser

N_1 and N_2 : Populations in level 1 and 2

N_L: number of photons

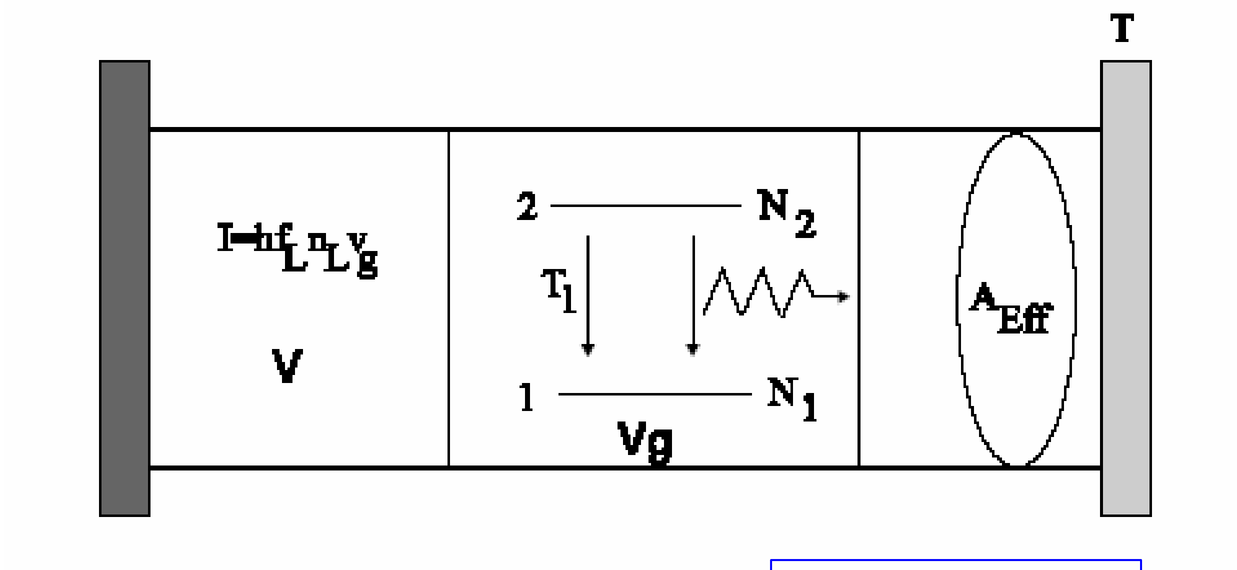
n_L: density of photons in the mode interacting with the atoms

vg: group velocity of photons

V: mode volume

Vg: volume of the active gain medium

Rate equations for two-level atom



One definition: Cross section $\boxed{\sigma = \frac{hf_L}{2^* I_s T_1},}$ ~ $|M|^2 \, T_2$

Physical meaning? $\Delta P_{abs} = \sigma \cdot I$

(dipole antenna capturing radiation?)

Rate equations for two-level atom

$$I = \frac{1}{2^*} h f_L n_L v_g$$ and $$I_S = \frac{hf_L}{2^* \sigma T_1} \longrightarrow \frac{I}{I_S} = \sigma v_g n_L T_1$$

$$w \Rightarrow (N_2 - N_1) \qquad \frac{w_0}{T_1} \Rightarrow R_p$$

$$\frac{dw}{dt} = -\frac{w - w_0}{T_1} - \frac{I}{T_1 I_S} w$$

$$\longrightarrow \boxed{\frac{d}{dt}(N_2 - N_1) = -\frac{(N_2 - N_1)}{T_1} - \sigma (N_2 - N_1) v_g n_L + R_p}$$

(Rate equation for population)

Rate equations for two-level atom

Ok, then how about the rate equation for photon density?

Recall the result of Maxwell-Bloch equation:

$$\left(\frac{\partial}{\partial z} + \frac{1}{c_0}\frac{\partial}{\partial t}\right) A(z,t) \;=\; \frac{N\hbar}{4T_2 E_s} w\,(z,t)\, A(z,t),$$

$\longrightarrow$ Not straightforward to solve.. (boundary condition, non-uniform field)

In fact, we only concentrated to solve the problem from the view point of “atom” in Chapter 2 (photon dynamics is not properly dealt with).

$$\boxed{\frac{d}{dt} n_L = -\frac{n_L}{\tau_p} + \frac{l_g}{L}\frac{\sigma v_g}{V_g}\left[N_2\left(n_L + \frac{1}{V}\right) - N_1 n_L\right].}$$

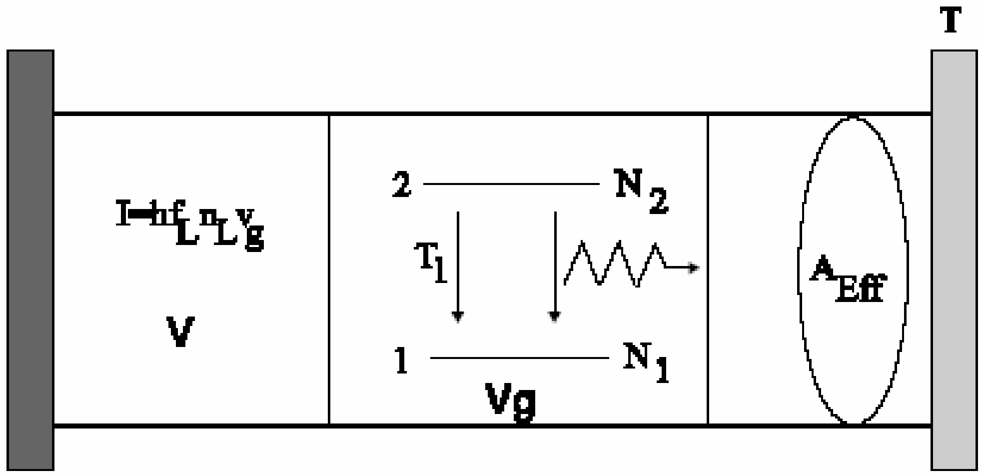
(Rate equation for photon density)

Rate equations for two-level atom

$$\frac{d}{dt}(N_2 - N_1) = -\frac{(N_2 - N_1)}{T_1} - \sigma (N_2 - N_1) v_g n_L + R_p$$

$$\frac{d}{dt} n_L = -\frac{n_L}{\tau_p} + \frac{l_g}{L} \frac{\sigma v_g}{V_g} \left[N_2 \left(n_L + \frac{1}{V} \right) - N_1 n_L \right].$$

Loss in the cavity



$$\frac{d}{dt}n_L = -\frac{n_L}{\tau_p} + \frac{l_g}{L}\frac{\sigma v_g}{V_g}\left[N_2\left(n_L + \frac{1}{V}\right) - N_1 n_L\right].$$

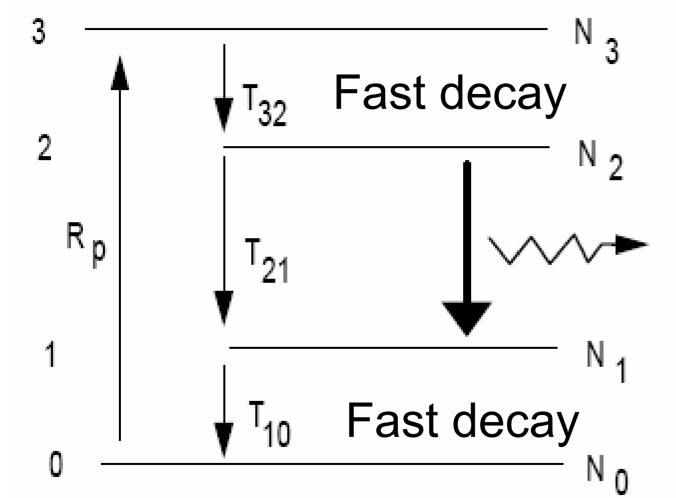
1. Loss from output coupler: power transmission T

Power loss $2l = -\ln(1-T) \approx T$ per round-trip

$\longrightarrow$ $\tau_p = T_R / 2l$ where $T_R = 2^{*} L / c_0$ and $L = l_a + n_g^{group} l_g$

2. In reality, there is internal loss too. $$\frac{1}{\tau_p} = \frac{1}{\tau_{p,OC}} + \frac{1}{\tau_{p,\text{int}}}$$

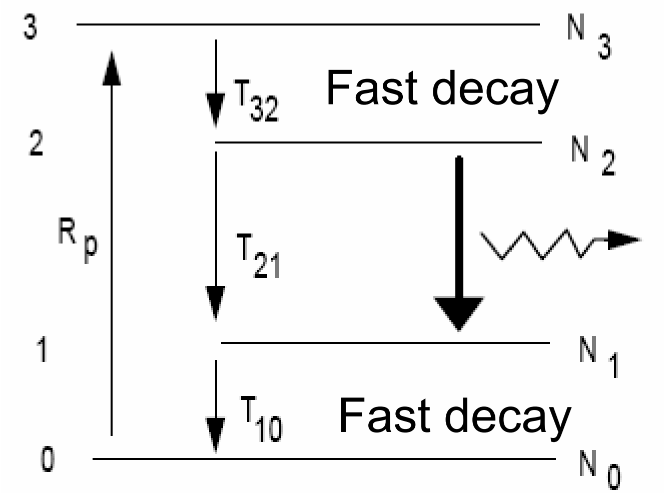
Four-level system



$$\frac{d}{dt}\left(N_2 - \cancelto{0}{N_1}\right) = -\frac{\left(N_2 - \cancelto{0}{N_1}\right)}{T_1} - \sigma\left(N_2 - \cancelto{0}{N_1}\right) v_g n_L + R_p$$

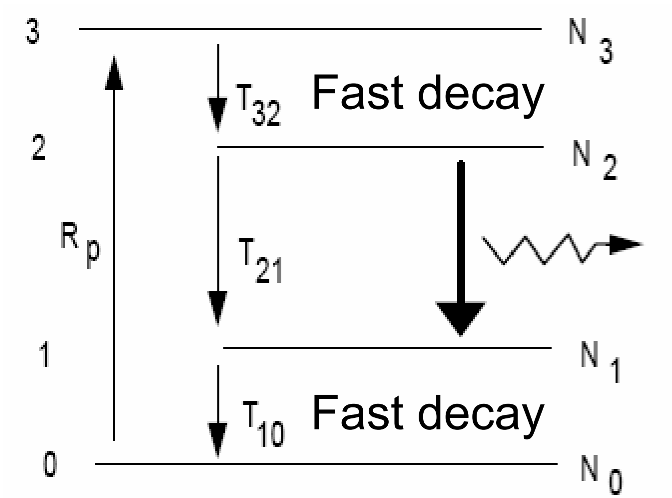
$$\frac{d}{dt} n_L = -\frac{n_L}{\tau_p} + \frac{l_g}{L}\frac{\sigma v_g}{V_g}\left[N_2\left(n_L + \frac{1}{V}\right) - \cancelto{0}{N_1} n_L\right].$$

Four-level system



$$
\begin{aligned}
\frac{d}{dt}N_2 &= -\frac{N_2}{\tau_L} - \sigma v_g N_2 n_L + R_p \\
\frac{d}{dt}n_L &= -\frac{n_L}{\tau_p} + \frac{l_g}{L}\frac{\sigma v_g}{V_g}N_2\left(n_L+1\right).
\end{aligned}
$$

Four-level system



$$
\begin{aligned}
\frac{d}{dt}N_2 &= -\frac{N_2}{\tau_L} - \sigma v_g N_2 n_L + R_p \\
\frac{d}{dt}n_L &= -\frac{n_L}{\tau_p} + \frac{l_g}{L}\frac{\sigma v_g}{V_g}N_2\left(n_L + 1\right).
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
\frac{d}{dt}g &= -\frac{g - g_0}{\tau_L} - \frac{gP}{E_{sat}} \\
\frac{d}{dt}P &= -\frac{1}{\tau_p}P + \frac{2g}{T_R}\left(P + P_{vac}\right)
\end{aligned}
}
$$

$$g = \frac{l_g}{L}\frac{\sigma v_g}{2V_g}N_2 T_R$$

$$P = I \cdot A_{eff}$$

$$
\begin{aligned}
E_s &= I_s A_{eff}\tau_L = \frac{hf_L}{2^*\sigma} \\
P_{sat} &= E_{sat}/\tau_L \\
P_{vac} &= hf_L v_g/2^* L = hf_L/T_R \\
g_0 &= \frac{2^* v_g R_p}{2A_{eff}c_0}\sigma\tau_L
\end{aligned}
$$

$$\sigma = \frac{h f_L}{I_{sat} T_1} = \frac{2T_2}{\hbar^2 Z_F} \frac{|\vec{M}\vec{\tilde{E}}|^2}{|\tilde{E}|^2}$$

Laser Medium	Wave-length $\lambda_0 (nm)$	Cross Section σ (cm^2)	Upper-St. Lifetime τ_L (μs)	Linewidth $\Delta f_{FWHM} = \frac{2}{T_2}$(THz)	Typ	Refr. index n
Nd^{3+}:YAG	1,064	$4.1 \cdot 10^{-19}$	1,200	0.210	H	1.82
Nd^{3+}:LSB	1,062	$1.3 \cdot 10^{-19}$	87	1.2	H	1.47 (ne)
Nd^{3+}:YLF	1,047	$1.8 \cdot 10^{-19}$	450	0.390	H	1.82 (ne)
Nd^{3+}:YVO_4	1,064	$2.5 \cdot 10^{-19}$	50	0.300	H	2.19 (ne)
Nd^{3+}:glass	1,054	$4 \cdot 10^{-20}$	350	3	H/I	1.5
Er^{3+}:glass	1,55	$6 \cdot 10^{-21}$	10,000	4	H/I	1.46
Ruby	694.3	$2 \cdot 10^{-20}$	1,000	0.06	H	1.76
Ti^{3+}:Al_2O_3	660-1180	$3 \cdot 10^{-19}$	3	100	H	1.76
Cr^{3+}:LiSAF	760-960	$4.8 \cdot 10^{-20}$	67	80	H	1.4
Cr^{3+}:LiCAF	710-840	$1.3 \cdot 10^{-20}$	170	65	H	1.4
Cr^{3+}:LiSGAF	740-930	$3.3 \cdot 10^{-20}$	88	80	H	1.4
He-Ne	632.8	$1 \cdot 10^{-13}$	0.7	0.0015	I	$\sim$1
Ar^+	515	$3 \cdot 10^{-12}$	0.07	0.0035	I	$\sim$1
CO_2	10,600	$3 \cdot 10^{-18}$	2,900,000	0.000060	H	$\sim$1
Rhodamin-6G	560-640	$3 \cdot 10^{-16}$	0.0033	5	H	1.33
semiconductors	450-30,000	$\sim 10^{-14}$	$\sim$ 0.002	25	H/I	3 - 4

Building-Up of Laser Oscillation

From spontaneous emission noise to the saturation power…

1. From vacuum fluctuation, $P(0)=P_{vac}$

2. When $P_{vac} \ll P \ll P_{sat}$, $g=g_0$

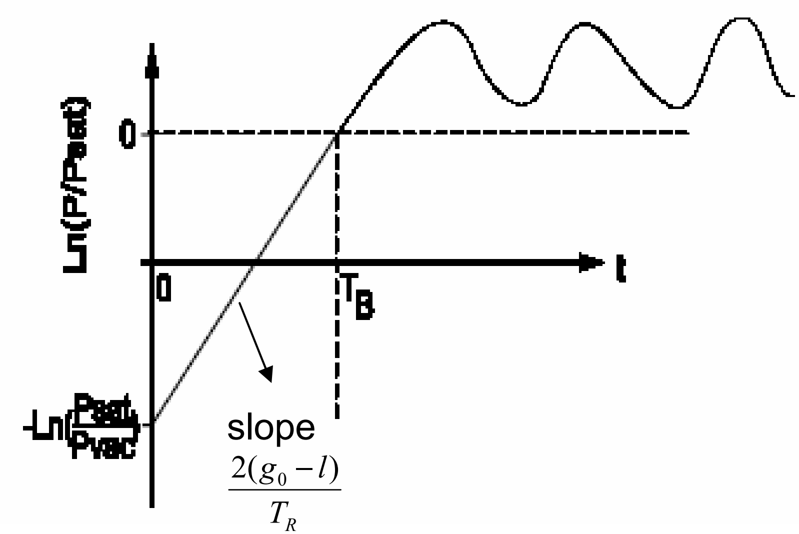
$$\frac{d}{dt}g = -\frac{g-g_0}{\tau_L} - \frac{gP}{E_{sat}} = 0$$

3. $$\frac{d}{dt}P = -\frac{1}{\tau_p}P + \frac{2\overset{g_0}{g}}{T_R}\left(P+\cancel{P_{vac}}\right)$$

and $\tau_p = T_R / 2l$

$$\longrightarrow \frac{dP}{P} = 2\left(g_0 - l\right)\frac{dt}{T_R}$$

$$\longrightarrow P(t) = P(0)e^{2(g_0-l)\frac{t}{T_R}}$$



Built-up time:

$$T_B = \frac{T_R}{2\left(g_0-l\right)}\ln\frac{P_{sat}}{P_{vac}} = \frac{T_R}{2\left(g_0-l\right)}\ln\frac{A_{eff}T_R}{\sigma\tau_L}.$$

4. P(t) reaches P_{sat} and then P_s.

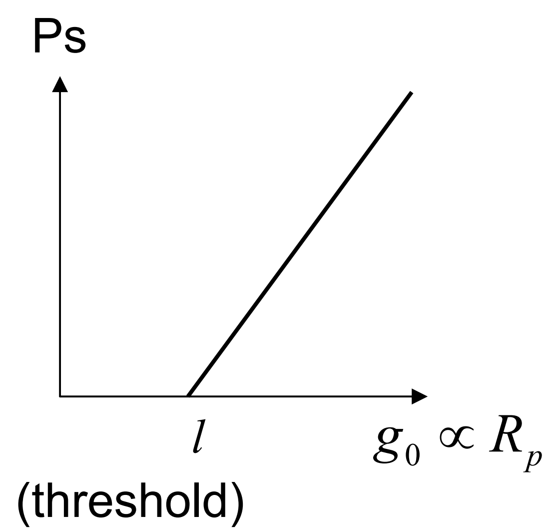
Building-Up of Laser Oscillation

In the steady-state,

$$\frac{d}{dt}g = -\frac{g-g_0}{\tau_L} - \frac{gP}{E_{sat}} \longrightarrow 0$$

$$\frac{d}{dt}P = -\frac{1}{\tau_p}P + \frac{2g}{T_R}\left(P + P_{vac}\right) \longrightarrow 0$$

$$\boxed{\begin{aligned} g_s &= \frac{g_0}{1+\frac{P_s}{P_{sat}}} = l \\ P_s &= P_{sat}\left(\frac{g_0}{l} - 1\right) \end{aligned}}$$



Stability of CW laser

The laser reached the steady-state…

What happens if perturbation occurs?

$$
\begin{aligned}
g &= g_s + \Delta g \\
P &= P_s + \Delta P
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\Delta P}{dt} &= +2\frac{P_s}{T_R}\Delta g \\
\frac{d\Delta g}{dt} &= -\frac{g_s}{E_{sat}}\Delta P - \frac{1}{\tau_{stim}}\Delta g
\end{aligned}
$$

$$
\frac{1}{\tau_{stim}} = \frac{1}{\tau_L}\underbrace{\boxed{\left(1+\frac{P_s}{P_{sat}}\right)}}_{= r}
$$

Stability of CW laser

$$\frac{d\Delta P}{dt} = +2\frac{P_s}{T_R}\Delta g$$
$$\frac{d\Delta g}{dt} = -\frac{g_s}{E_{sat}}\Delta P - \frac{1}{\tau_{stim}}\Delta g$$

Laplace transform:

$$\begin{pmatrix} -s & 2\frac{P_s}{T_R} \\ -\frac{T_R}{E_{sat}2\tau_p} & -\frac{1}{\tau_{stim}} - s \end{pmatrix} \begin{pmatrix} \Delta P_0 \\ \Delta g_0 \end{pmatrix} = 0.$$

$$\longrightarrow \quad s\left(\frac{1}{\tau_{stim}} + s\right) + \frac{P_s}{E_{sat}\tau_p} = 0$$

$$\longrightarrow \quad s_{1/2} = -\frac{1}{2\tau_{stim}} \pm \sqrt{\left(\frac{1}{2\tau_{stim}}\right)^2 - \frac{P_s}{E_{sat}\tau_p}}.$$

Stability of CW laser

$$s_{1/2} = -\frac{1}{2\tau_{stim}} \pm \sqrt{\left(\frac{1}{2\tau_{stim}}\right)^2 - \frac{P_s}{E_{sat}\tau_p}}.$$

Employing pump parameter: $r = 1 + \frac{P_s}{P_{sat}}$

$$\begin{aligned} s_{1/2} &= -\frac{1}{2\tau_{stim}}\left(1 \pm j\sqrt{\frac{4(r-1)}{r}\frac{\tau_{stim}}{\tau_p} - 1}\right) \\ &= -\frac{r}{2\tau_L} \pm j\sqrt{\frac{(r-1)}{\tau_L\tau_p} - \left(\frac{r}{2\tau_L}\right)^2} \end{aligned}$$

What is the meaning?

Stability of CW laser

$$s_{1/2} = -\frac{r}{2\tau_L} \pm j\sqrt{\frac{(r-1)}{\tau_L\tau_p} - \left(\frac{r}{2\tau_L}\right)^2}$$

1. The stationary states are always stable: Re{s} < 0

2. **Relaxation oscillation** occurs when r is sufficiently larger than 1.

$$\sqrt{\frac{r}{\tau_L\tau_p}\left(1-\frac{\tau_p r}{4\tau_L}\right)} \approx \sqrt{\frac{r}{\tau_L\tau_p}} = \sqrt{\frac{1}{\tau_{stim}\tau_p}}$$

(typically $\tau_p = T_R / 2l \quad << \quad \tau_L = T_{21}$)

$$s_{1/2} = -\frac{1}{2\tau_{stim}} \pm j\sqrt{\frac{1}{\tau_{stim}\tau_p}}$$

Relaxation oscillation

$$\omega_R = \sqrt{\frac{1}{\tau_{stim}\tau_p}}$$

Stability of CW laser

$$s_{1/2} = -\frac{1}{2\tau_{stim}} \pm j\sqrt{\frac{1}{\tau_{stim}\tau_p}}$$

$$\boxed{\omega_R = \sqrt{\frac{1}{\tau_{stim}\tau_p}}}$$

$$\sqrt{\frac{r}{\tau_L\tau_p}\left(1-\frac{\tau_p r}{4\tau_L}\right)} \approx \sqrt{\frac{r}{\tau_L\tau_p}} = \sqrt{\frac{1}{\tau_{stim}\tau_p}}$$

What happens if the pump is really strong?

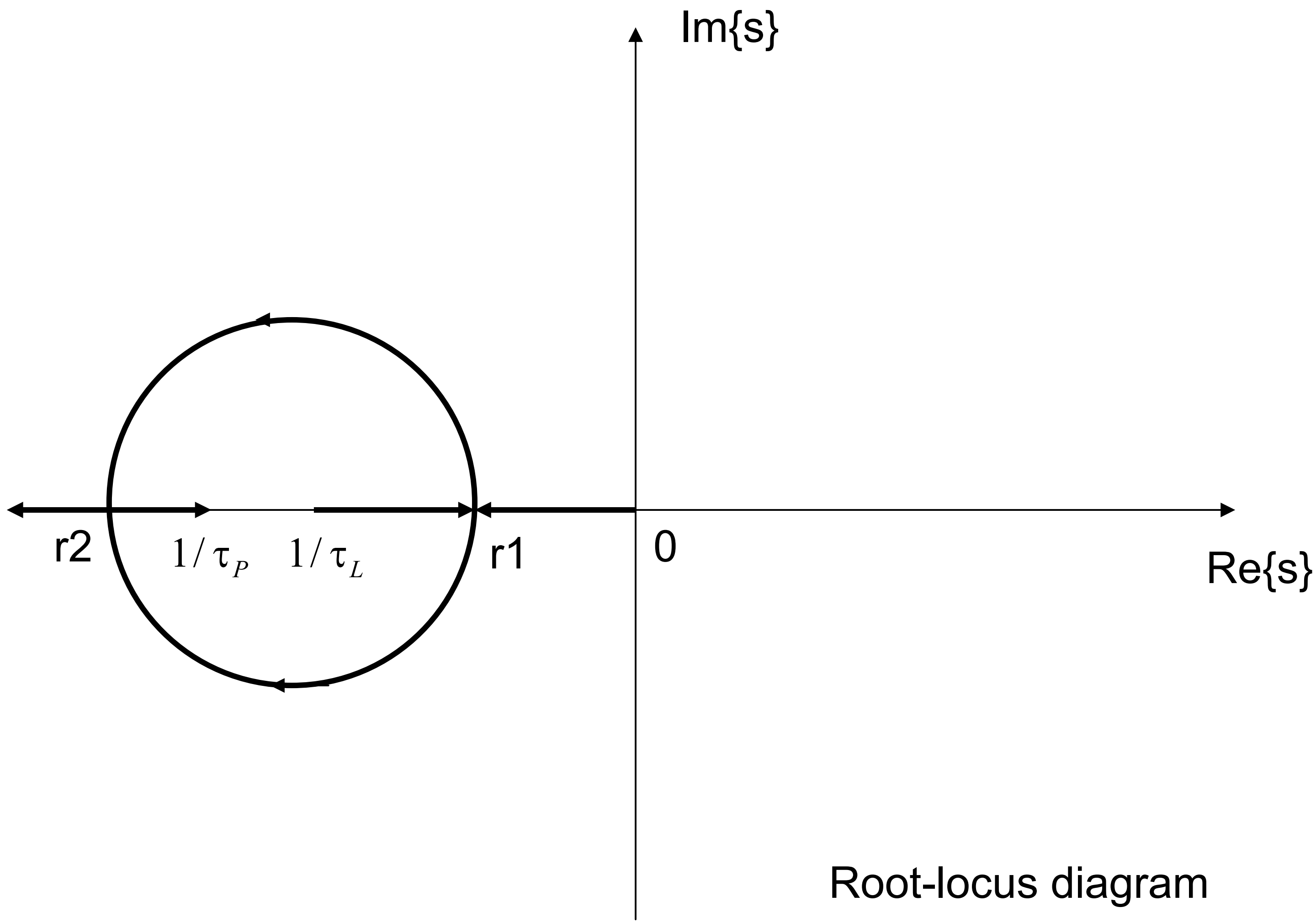
→ Stimulated lifetime can be as short as cavity decay time

→ $\frac{\tau_p r}{\tau_L} = \frac{\tau_p}{\tau_{stim}}$ is no longer much smaller than 1

$$\sqrt{\frac{r}{\tau_L\tau_p}\left(1-\frac{\tau_p r}{4\tau_L}\right)} \approx \cancel{\sqrt{\frac{r}{\tau_L\tau_p}} = \sqrt{\frac{1}{\tau_{stim}\tau_p}}}$$

→ Relaxation oscillation vanishes.

Relaxation oscillation



Root-locus diagram

Relaxation oscillation

Example: diode-pumped Nd:YAG-Laser

$$\lambda_0 = 1064 \text{ nm}, \sigma = 4 \cdot 10^{-20} cm^2, \; A_{eff} = \pi \left(100\mu m \times 150\mu m\right), r = 50$$
$$\tau_L = 1.2 \; ms, \; l = 1\%, \; T_R = 10ns$$

From Eq.(4.4) we obtain:

$$I_{sat} = \frac{hf_L}{\sigma \tau_L} = 3.9 \frac{kW}{cm^2}, \; P_{sat} = I_{sat} A_{eff} = 1.8 \; W, \; P_s = 91.5W$$
$$\tau_{stim} = \frac{\tau_L}{r} = 24\mu s, \; \tau_p = \cancel{1\mu s}, \omega_R = \sqrt{\frac{1}{\tau_{stim}\tau_p}} = 2 \cdot 10^5 s^{-1}.$$

500 ns

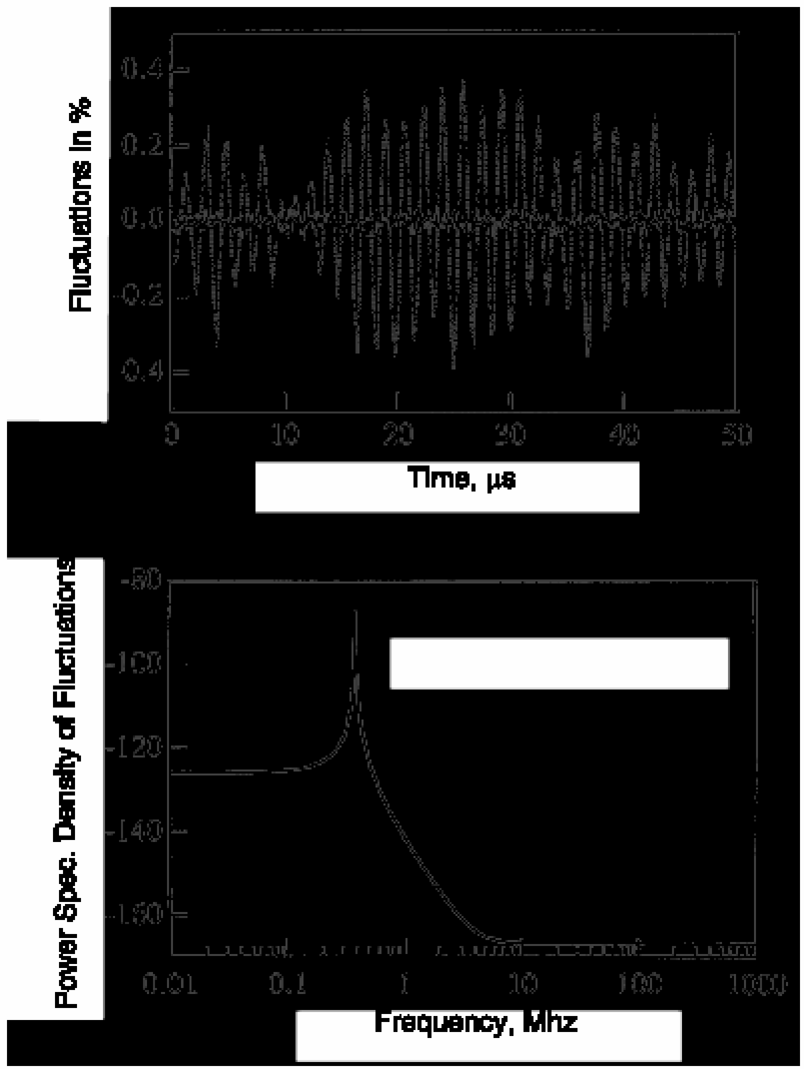
f_R = 30 kHz

Quality factor

$$Q = \sqrt{\frac{4\tau_L}{\tau_p} \frac{(r-1)}{r^2}}$$

Can be several thousands for solid-state lasers with long upper-state lifetime of ms range (then $\tau_L >> \tau_p$)

Relaxation oscillation



RIN measurement