8.01L SUMMARY OF EQUATIONS

Note: Quantities shown in **bold** are vectors. $\mathbf{v} = d\mathbf{r}/dt$ $\mathbf{a} = d\mathbf{v}/dt$

Circular motion at constant speed $a = \frac{v^2}{r} = \omega^2 r$ (Centripetal acceleration, points towards center of circle, ω is angular speed in radians per second)

Adding relative velocities ("wrt" is short for "with respect to"): $\mathbf{v}_A + \mathbf{v}_B = \mathbf{v}_A$ $\frac{\mathbf{w}_T + \mathbf{v}_B}{\mathbf{w}_T + \mathbf{v}_C} = \mathbf{v}_A$

 $\sum \mathbf{F} = 0 \iff \mathbf{a} = 0$ (Newton's first law) $\overline{\mathbf{F}} = \mathbf{m}\mathbf{a}$ or $\mathbf{F} = d\mathbf{p}/dt$ (Newton's second law) $\mathbf{F}_{AB} = -\mathbf{F}_{BA}$ (Newton's third law) $\mathbf{p} = m\mathbf{v}$ (momentum) $\mathbf{J} = \int_{t_1}^{t_2} \mathbf{F} dt = \int_{t_1}^{t_2} \frac{d\mathbf{p}}{dt} dt = \mathbf{p}_2 - \mathbf{p}_1 \quad (\text{impulse})$ $\mathbf{r}_{cm} = \frac{\sum m_i \mathbf{r}_i}{\sum m_i}$ (position of center of mass) $\mathbf{F} = -\mathbf{k}\mathbf{x}$ (spring force) $f \le \mu N$ (Friction force relative to Normal force) $\mathbf{F} = -\frac{GMm}{r^2}\hat{\mathbf{r}}$ (gravitational force between two particles) $W = \int \mathbf{F} \cdot d\mathbf{r}$ (work done by force \mathbf{F}) $W_{other} = \Delta E = E_F - E_I$ E = KE + PE (work-energy theorem) $F_x = -\frac{\mathrm{d}U}{\mathrm{d}x}$ (force derived from potential energy) Potential Energies: $U = \frac{1}{2}kx^2$ (spring force) $U = \frac{-GMm}{r}$ (gravitational, general) U = mgh (gravitational, near Earth) $\omega = \sqrt{k/m}$ $x = A\cos(\omega t + \phi)$ (Equations for Simple Harmonic Motion) $v = -A\omega\sin(\omega t + \phi)$ $T = 2\pi/\omega$ $P_2 + \rho_{gy_2} = P_1 + \rho_{gy_1}$ (Pascal's Law: pressure versus height in a liquid with velocity = 0) $P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$ (Bernoulli's equation) $A_2v_2 = A_1v_1$ (continuity equation) PV = NkT = nRT (ideal-gas law) $\left(\frac{1}{2}mv^2\right) = \frac{3}{2}kT$ (definition of kinetic temperature) $F_B = \rho_f V_f g$ (buoyancy force, f=fluid displaced)

Quantity	Translational	Rotational (about axis)
Velocity, acceleration	<i>v</i> , <i>a</i>	$\boldsymbol{\omega}, \boldsymbol{\alpha} (v=R\omega, a=R\alpha)$
Mass	$M = \sum_{i} m_{i}$	$I = \sum_{i} m_{i} R_{i}^{2}$
Kinetic energy	$\frac{1}{2}Mv^2$	$\frac{1}{2}I\omega^2$
Net force	$\sum_{i} \mathbf{F}^{\text{ext}} = M \mathbf{a}_{\text{cm}}$	$\sum_{i} \tau^{\text{ext}} = I\alpha$
Momentum	$\mathbf{p} = m\mathbf{v}$	$\mathbf{L} = \mathbf{r} \times \mathbf{p}$ or $\mathbf{L} = \mathbf{I} \mathbf{O}$

 $\mathbf{\tau} = \mathbf{r} \times \mathbf{F} = \frac{d\mathbf{L}}{dt} = \mathbf{I}\boldsymbol{\alpha} \quad |\boldsymbol{\tau}| = \mathbf{r}F\sin(\phi) = Fr_{\perp} \quad \text{(torque equations)}$

 $L = r \times p$ $|L| = mvrsin(\phi)$ (angular momentum of point particle)

 $L=I \ \ \, \textbf{(angular momentum for solid object)}$

 $I_1 = I_{c.m.} + Md^2$ (parallel axis theorum)

 $I = \frac{1}{2}MR^2$ (cylinder around center) $I = \frac{2}{5}MR^2$ (solid sphere around center) $I = \frac{1}{12}ML^2$ (rod around center) $I = \frac{1}{3}ML^2$ (rod around end)

 $KE = \frac{1}{2}M_{Tot}v_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$ (kinetic energy for object moving and rolling)

 $KE = \frac{1}{2}I_{Pivol}\omega^2$ (kinetic energy for object rotating around a fixed pivot) <u>Physical Constants:</u>

g = 9.8 m/s² Use the approximate value g = 10 m/s² where told to do so. G = 6.67×10^{-11} N m²/kg² k = 1.38×10^{-23} J/K R = 8.31 J/(mol. K) 0° C = 273° K Density of water = 1,000 kg/m³ Atmospheric pressure = 1.0×10^{5} Pa <u>Conversion reminder:</u>

 π radians = 180°

Lazy Physicist 's Favorite Angle: (to be used when calculators are not allowed):

 36.9° and 53.1° are the angles of a 3-4-5 right triangle so: $\sin(36.9^{\circ}) = \cos(53.1^{\circ}) = 0.60$ $\cos(36.9^{\circ}) = \sin(53.1^{\circ}) = 0.80$

 $\tan(36.9^\circ) = 0.75$ $\tan(53.1^\circ) = 1.33$

Other (possibly) Useful Trig Functions:

 $\cos(30^{\circ}) = \sin(60^{\circ}) = \frac{\sqrt{3}}{2} \qquad \sin(30^{\circ}) = \cos(60^{\circ}) = \frac{1}{2}$ $\cos(45^{\circ}) = \sin(45^{\circ}) = \frac{1}{\sqrt{2}}$

<u>Solution to a Quadratic Equation:</u> If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$