### 8.01L SUMMARY OF EQUATIONS

Note: Quantities shown in bold are vectors.
$\mathbf{v}=\mathrm{dr} / \mathrm{dt} \quad \mathbf{a}=\mathrm{d} \mathbf{v} / \mathrm{dt}$
For constant acceleration $\mathbf{a}$, if at $t=0 \mathbf{r}=\mathbf{r}_{0}$ and $\mathbf{v}=\mathbf{v}_{0}$ :

$$
\begin{aligned}
& \mathbf{v}=\mathbf{v}_{\mathbf{0}}+\mathbf{a} t \\
& \mathbf{r}=\mathbf{r}_{\mathbf{0}}+\mathbf{v}_{\mathbf{0}} t+\frac{1}{2} \mathbf{a} t^{2}
\end{aligned}
$$

Circular motion at constant speed $a=v^{2} / r=\omega^{2} r$ (Centripetal acceleration, points towards center of circle, $\omega$ is angular speed in radians per second)
Adding relative velocities ("wrt" is short for "with respect to"): $\underset{\substack{\mathbf{v}_{A} \\ B}}{w_{t}}+\underset{\substack{w r t \\ C}}{\mathbf{v}_{B}}=\underset{\substack{w \\ w}}{\mathbf{v}_{A}}$
$\sum \mathbf{F}=0 \Leftrightarrow \mathbf{a}=0 \quad$ (Newton's first law)
$\mathbf{F}=$ ma or $\mathbf{F}=\mathrm{d} \mathbf{p} / \mathrm{dt} \quad$ (Newton's second law) $\quad \mathbf{F}_{\mathrm{AB}}=-\mathbf{F}_{\mathrm{BA}} \quad$ (Newton's third law)
$\mathbf{p}=m \mathbf{v} \quad$ (momentum)
$\mathbf{J}=\int_{t_{1}}^{t_{2}} \mathbf{F} \mathrm{~d} t=\int_{t_{1}}^{t_{2}} \frac{\mathrm{~d} \mathbf{p}}{\mathrm{~d} t} \mathrm{~d} t=\mathbf{p}_{2}-\mathbf{p}_{1} \quad$ (impulse)
$\mathbf{r}_{\mathrm{cm}}=\frac{\Sigma m_{i} \mathbf{r}_{i}}{\Sigma m_{i}} \quad$ (position of center of mass)
$\mathbf{F}=-\mathrm{kx} \quad$ (spring force) $\quad \mathrm{f} \leq \mu \mathrm{N} \quad$ (Friction force relative to Normal force)
$\mathbf{F}=-\frac{G M m}{r^{2}} \hat{\mathbf{r}} \quad$ (gravitational force between two particles)
$W=\int \mathbf{F} \cdot \mathrm{d} \mathbf{r} \quad$ (work done by force $\mathbf{F}$ )
$W_{\text {other }}=\Delta E=E_{F}-E_{I} \quad E=K E+P E \quad$ (work-energy theorem)
$F_{x}=-\frac{\mathrm{d} U}{\mathrm{~d} x} \quad$ (force derived from potential energy)
Potential Energies: $U=\frac{1}{2} k x^{2} \quad$ (spring force)
$U=\frac{-G M m}{r}$ (gravitational, general) $\quad U=m g h \quad$ (gravitational, near Earth)
$\omega=\sqrt{\mathrm{k} / \mathrm{m}} \quad \mathrm{x}=\mathrm{A} \cos (\omega \mathrm{t}+\phi)$
$v=-A \omega \sin (\omega \mathrm{t}+\phi) \quad T=2 \pi / \omega$
(Equations for Simple Harmonic Motion)
$P_{2}+\rho g y_{2}=P_{1}+\rho g y_{1} \quad($ Pascal's Law: pressure versus height in a liquid with velocity $=0)$
$P+\frac{1}{2} \rho v^{2}+\rho g y=$ constant (Bernoulli's equation)
$A_{2} v_{2}=A_{1} v_{1} \quad$ (continuity equation)
$P V=N k T=n R T \quad$ (ideal-gas law)
$\left\langle\frac{1}{2} m v^{2}\right\rangle=\frac{3}{2} k T \quad$ (definition of kinetic temperature)
$F_{B}=\rho_{f} V_{f} g \quad$ (buoyancy force, $\mathrm{f}=$ fluid displaced)

| Quantity | Translational | Rotational (about axis) |
| :--- | :--- | :--- |
| Velocity, acceleration | $\boldsymbol{v}, \boldsymbol{a}$ | $\boldsymbol{\omega}, \boldsymbol{\alpha}(v=R \omega, a=R \alpha)$ |
| Mass | $M=\sum_{i} m_{i}$ | $I=\sum_{i} m_{i} R_{i}^{2}$ |
| Kinetic energy | $\frac{1}{2} M v^{2}$ | $\frac{1}{2} I \omega^{2}$ |
| Net force | $\sum_{i} \mathbf{F}^{\mathrm{ext}}=M \mathbf{M a}_{\mathrm{cm}}$ | $\sum_{i} \tau^{\mathrm{ext}}=I \alpha$ |
| Momentum | $\mathbf{p}=m \mathbf{v} \square$ | $\mathbf{L}=\mathbf{r} \times \mathbf{p}$ or $\mathbf{L}=\mathrm{I} \boldsymbol{\omega}$ |

$\tau=\mathbf{r} \times \mathbf{F}=\frac{\mathrm{d} \mathbf{L}}{\mathrm{dt}}=\mathrm{I} \boldsymbol{\alpha} \quad|\tau|=\mathrm{rF} \sin (\phi)=F r_{\perp} \quad$ (torque equations)
$\mathbf{L}=\mathbf{r} \times \mathbf{p} \quad|\mathrm{L}|=\operatorname{mvrsin}(\phi) \quad$ (angular momentum of point particle)
$\mathbf{L}=\mathbf{I} \boldsymbol{\omega} \quad$ (angular momentum for solid object)
$I_{1}=I_{c . m .}+M d^{2} \quad$ (parallel axis theorum)
$I=\frac{1}{2} M R^{2} \quad$ (cylinder around center) $\quad I=\frac{2}{5} M R^{2} \quad$ (solid sphere around center)
$I=\frac{1}{12} M L^{2} \quad\left(\operatorname{rod}\right.$ around center) $\quad I=\frac{1}{3} M L^{2} \quad(\operatorname{rod}$ around end)
$K E=\frac{1}{2} M_{T o t} v_{C M}^{2}+\frac{1}{2} I_{C M} \omega^{2} \quad$ (kinetic energy for object moving and rolling)
$K E=\frac{1}{2} I_{P i v o t} \omega^{2} \quad$ (kinetic energy for object rotating around a fixed pivot)
Physical Constants:
$\mathrm{g}=9.8 \mathrm{~m} / \mathrm{s}^{2}$ Use the approximate value $\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}$ where told to do so.
$\mathrm{G}=6.67 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}$
$\mathrm{k}=1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K} \quad \mathrm{R}=8.31 \mathrm{~J} /(\mathrm{mol} . \mathrm{K})$
$0^{\circ} \mathrm{C}=273^{\circ} \mathrm{K}$
Density of water $=1,000 \mathrm{~kg} / \mathrm{m}^{3}$
Atmospheric pressure $=1.0 \times 10^{5} \mathrm{~Pa}$
Conversion reminder:
$\pi$ radians $=180^{\circ}$
Lazy Physicist's Favorite Angle: (to be used when calculators are not allowed):
$36.9^{\circ}$ and $53.1^{0}$ are the angles of a 3-4-5 right triangle so:
$\sin \left(36.9^{\circ}\right)=\cos \left(53.1^{\circ}\right)=0.60 \quad \cos \left(36.9^{\circ}\right)=\sin \left(53.1^{\circ}\right)=0.80$
$\tan \left(36.9^{\circ}\right)=0.75 \quad \tan \left(53.1^{\circ}\right)=1.33$
Other (possibly) Useful Trig Functions:

$$
\begin{gathered}
\cos \left(30^{\circ}\right)=\sin \left(60^{\circ}\right)=\sqrt{3} / 2 \quad \sin \left(30^{\circ}\right)=\cos \left(60^{\circ}\right)=1 / 2 \\
\cos \left(45^{\circ}\right)=\sin \left(45^{\circ}\right)=1 / \sqrt{2}
\end{gathered}
$$

Solution to a Quadratic Equation: If $a x^{2}+b x+c=0$ then $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

