## FINAL EXAM SAMPLE PROBLEMS

The Final Exam will be given on Monday, January 30 from 9 am - Noon
There will be no makeup exam.
You will be provided with a copy of the formula sheet during the exam. You may wish to memorize other formulas but a good understanding of the concepts and applicability of these formulas should allow you to do all of the problems on the exam.
The problems on the following pages are mostly problems from exams in previous 8.01L classes. Note that all topics for which you are responsible are NOT included in this sample. We have only included sample problems for the material since the last exam. You should also review the previous three exams (and the sample problems for them) as well as the material included on all of the homework assignments. The final exam will contain about 7-9 problems similar in difficulty to these, with roughly half the exam covering material since the last exam.
Some of these problems were from exams where calculators were allowed. You will not be allowed to use a calculator in this exam.

## Problem 1

a) A closed weather balloon with an initial volume $\mathbf{V}$ rises up in the atmosphere to a point where the outside air pressure is $1 / 10$ of the pressure at the surface and the temperature of the air has dropped by a factor of $\mathbf{3}$. Assume that the balloon is filled with an ideal gas and that the material of the balloon exerts no pressure and does not insulate so that the temperature and pressure are the same inside and outside the balloon. Does the volume of the gas increase, decrease, or stay the same as the balloon rises? Justify your answer.
b) A solid object of a material with a density of $4000 \mathrm{~kg} / \mathrm{m}^{3}$ has a volume of $0.002 \mathrm{~m}^{3}$ and thus a mass of 8 kg . How much force do you need to exert to support this object
i) In air.
ii) When it is completely submerged in water. The density of water is $1000 \mathrm{~kg} / \mathrm{m}^{3}$. Use $g=10 \mathrm{~m} / \mathrm{s}^{2}$ in this problem.
c) Assume that water is flowing through a rubber tube. Due to a weakness in the material, a section of the tube expands so it has a larger radius than the other parts. Describe what would happen to the water flow velocity and the water pressure in the enlarged section of the tube.

## Problem 2

a) Your physics professor is standing inside a cylinder of radius $r \square$ which is floating in equilibrium in water of density $\rho$. The cylinder is filled with air of negligible mass. The physicist and cylinder have a combined total weight of $W$. Find the depth $d \square$ to which the bottom of the cylinder sinks into the water.
b) Now assume that the cylinder starts at a higher position and
 falls down into the water. Find the acceleration when the bottom of the cylinder is at a depth of $d / 2$. Assume that you can ignore the viscous force of the water (the liquid equivalent of air resistance).

## Problem 3

a) Archeologists have recently uncovered evidence that people living in South America many centuries ago were able to climb to the summit of a mountain at a height of 6500 m above sea level. The air pressure at sea level is $1.013 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}$. Assuming that the atmosphere has an average density of $0.95 \mathrm{~kg} / \mathrm{m}^{3}$, find the air pressure at the top of this mountain.
b) Assume that a pressurized helium bottle contains gas at a pressure of $1.50 \times 10^{7} \mathrm{~N} / \mathrm{m}^{2}$ and is inside a building at a temperature of $20^{\circ} \mathrm{C}$. If you take the bottle outside to a New Year's Day parade where the temperature is $-20^{\circ} \mathrm{C}$, what will the pressure inside the bottle be once it has fully cooled?

## Problem 4

A standard track used in almost all running events consists of two straight parts and two semicircles, each of the four sections having roughly equal length of 100 m .
i) On the following drawing of a runner rounding a curve, draw labeled arrows to show all the forces acting on the runner. Your drawing should indicate clearly where the forces act and their directions. The arrow on the drawing points towards the center of the circle. Assume that the
 runner is moving with constant speed.
ii) Using your picture from part ii) and concepts of equilibrium, explain why it is impossible to run quickly around a curve standing straight up as shown (i.e. without leaning over).
iii) Draw a force diagram of a leaning runner and explain how leaning over makes rounding the curve possible i.e. what changes in the equilibrium equations. Show clearly the direction of the lean assuming that the center of the circle is to the left as in part ii) above.

## Problem 5

The physics of pushups: The drawing shows a simplified model of the force needed to do a pushup. Assume that a person's body is a bar of mass $\mathbf{M}$ with a moment of inertia about the pivot (the toes) of $\mathbf{I}_{\mathbf{T}}$. Assume that the center of mass is a distance $\mathbf{L}$ from the pivot and that the force shown (due to the arms) is purely vertical and applied at a distance
 of $4 \mathrm{~L} / 3$ from the pivot.
a) Draw a free-body diagram showing all the forces acting on the bar.
b) Find the value of the applied force $\mathbf{F}$ needed for the object to be in equilibrium and show that it is independent of the angle $\theta$ and thus independent of the height $h$.
c) What is the value of the force (magnitude and direction) exerted on the person by the ground? This means the force applied at the toes, not the force applied on the arms.

## Problem 6

A plank of mass M and length $L$ is held up a cable at each end. The left cable has a maximum tension of 3 Mg (any more force and the cable will break) while the right cable can exert a maximum force of only Mg . A load of bricks is placed on the plank with its center of mass located a distance D from the
 left edge (see drawing)
a) Considering only forces, how massive can the load of bricks be before the cables will fail?
b) Assume that the bricks have the maximum mass as calculated on part (a). Where must the bricks be placed (i.e. what is D ) so that neither cable exceeds its maximum tension?

## Problem 7

A plank of mass M and length L leans against a wall at one end and rests on a circular support at the other end. Assume that there is friction between the plank and the wall also between the ladder and the cylinder. Assume the plank is uniform so its center of mass is at the center. Note that the plank must hit the cylinder tangentially so the normal force from the cylinder is perpendicular to the plank and the friction force due to the cylinder is along the plank. Assume that the plank does not move.
a) Draw a clear diagram showing all the forces acting on the plank.
b) Choose and X and Y axis and write a set of force equations for those two directions.
c) Write a torque equation using the left end of the plank as the axis.
d) Repeat part (c) but using the right edge of the plank as the axis.

## Problem 8

A uniform bar of mass $\mathbf{M}$ and length $\mathbf{L}$ is attached to a wall by a frictionless hinge (i.e. there are no torques exerted by the hinge). The bar is released from rest in a horizontal position as shown. The bar has moments of inertia about its center and about its end

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\text { of } I_{c m}=M L^{2} / 12 \text { and } I_{e n d}=M L^{2} / 3, \text { respectively. }
$$

a) What is the angular acceleration of the bar at the instant that it is released?
b) At the instant after the bar is released, find the magnitude and direction of the force exerted on the bar by the hinge.
c) Use Work/Energy concepts to find the angular velocity of the bar when it has swung down to a vertical position and is just about the hit the wall.

## Problem 9

A small child runs and jumps onto a merry-go-round which has a moment of inertia $\mathbf{I}_{\mathbf{0}}$ about its axis of rotation. The child has mass $\mathbf{m}$ and is initially moving with velocity $\mathbf{v}$; she grabs onto a bar attached to the plate which is a distance $\mathbf{R}$ from the center of the merry-go-round. The merry go round was initially not rotating. Assume that the child is running tangential to the merry-go-round before jumping on as shown.
a) What is the magnitude of the final angular velocity of the merry-go-round, $\omega_{\text {final }}$ ? Give your answer in terms of $\mathbf{m}, \mathbf{v}$,
 $\mathbf{R}$ and $\mathbf{I}_{\mathbf{0}}$.
b) What fraction of the initial kinetic energy of the running child remains in the final system (i.e. what is $\mathrm{KE}_{\mathrm{F}} / \mathrm{KE}_{\mathrm{I}}$ )? Give your answer in terms of $\mathbf{I}_{\mathbf{0}}, \mathbf{m}, \mathbf{v}$, and $\mathbf{R}$.

## Problem 10

A gyroscope supported at one end is rotating as shown (the edge towards you is moving up, the top edge is moving away from you, the edge away from you is moving down, and the bottom edge is moving towards you). Assume that the support pivot can exert forces on the gyroscope but cannot exert any torques. For the multiple choice questions, you do not need to explain your answer.
i) In what direction does the angular velocity point?

ii) Consider torques around the pivot where the gyroscope is supported. Is there a torque around that point and if so, what force causes the torque?
iii) Consider torques around the pivot where the gyroscope is supported. If there is a torque, how would you describe the direction of that torque (there are 2 correct answers)?
Left Right Up Down Into the page Out of the page Other Clockwise Counter-clockwise
iv) In what direction do you predict the unsupported end of the gyroscope will move?

Left Right Up Down Into the page Out of the page Other
Problem 11
The physics of baseball.
TOP VIEW

BEFORE
Pivot


No torque


A baseball of mass 0.15 kg is initially traveling horizontally at $50 \mathrm{~m} / \mathrm{s}$. It is struck by a bat, after which the baseball is still traveling horizontally but in exactly the opposite direction from its initial motion at a speed of $40 \mathrm{~m} / \mathrm{s}$. Consider the collision of the bat and the ball. Assume that before the collision, the bat is moving in a horizontal circle at an angular velocity of $\omega$ $\mathrm{rad} / \mathrm{s}$. Assume that the player holding the bat exerts no torque. The bat has a moment of inertia of $0.30 \mathrm{~kg} \mathrm{~m} \mathrm{~m}^{2}$ about the pivot and the ball hits at a point that is 80 cm away from the pivot. After the collision, the bat is still swinging in the same direction around the same pivot but with a reduced angular velocity of $0.35 \omega$. Find the numerical value of $\omega$.

