

Let's calculate the kinetic energy in the center of mass frame of two objects that are colliding.

I'm going to use prime for the center of mass frame velocity.

So we have V_1 prime.

And we have V_2 prime.

And recall two things that when we calculated V_1 prime in the center of mass frame, we found that this was equal to the reduced mass divided by m_1 times the relative velocities of the two objects, V_1 prime minus V_2 prime.

Now in the center of mass frame, this quantity is a reference frame independent.

And so we can just write that as $V_{1,2}$.

It's the relative velocity is reference frame independent.

So we also knew that V_2 prime was equal to minus μ over m_2 $V_{1,2}$.

So now let's calculate the kinetic energy in the center of mass frame.

So K_{cm} is $\frac{1}{2} m_1 V_1$ prime squared plus $\frac{1}{2} m_2 V_2$ prime squared.

Now let's use our results in terms of relative velocity.

So we have $\frac{1}{2} m_1 \mu$ over m_1 times $V_{1,2}$ squared-- because we're squaring that-- plus $\frac{1}{2} m_2 \mu$ -- the minus sign doesn't matter anymore because we're squaring it-- V_2 squared-- I could put that in there, it wouldn't matter.

And now what we have is we have $\frac{1}{2}$.

One of the m_1 s cancels so we have a μ squared over m_1 $V_{1,2}$ squared plus $\frac{1}{2}$ another μ square over m_2 times $V_{1,2}$ squared.

So I pull out the common terms, μ square, $V_{1,2}$ squared.

I have $\frac{1}{m_1} + \frac{1}{m_2}$.

But recall that the reduced mass was precisely $\frac{1}{m_1 + m_2}$.

And so I get $\frac{1}{2} \mu$ times $V_{1,2}$ squared is the kinetic energy in the center of mass frame.

So what that means is a way of thinking if this were just a simple reduced mass problem with a relative speed of $V_{1,2}$ squared, I can write down the kinetic energy.

But that's the kinetic energy in the center of mass frame.

And therefore, the change in kinetic energy in a collision is just $\frac{1}{2} \mu V_{1,2}^2$ final squared minus-- and it's the same μ , so we'll just put a parentheses-- minus $V_{1,2}$ initial squared.

And if the collision is elastic-- remember, our elastic collision show that $V_{1,2}$ initial was minus $V_{1,2}$ final.

So an elastic collision satisfies that condition.

And you can see directly that ΔK_{cm} is 0 in that case.

And a completely inelastic collision has the two objects moving together at the end.

So $V_{1,2}$ final is 0.

So completely inelastic is the condition that $V_{1,2}$ final is 0 because the objects stick together.

And then have a very simple result for the change in kinetic energy for completely inelastic collision.

It's only negative $\frac{1}{2} \mu V_{1,2}$ initial quantity squared.