Let's consider a very famous problem the, Atwood machine.

We have a pulley, A, suspended from a ceiling.

And a rope is wrapped around the pulley.

And on each side of the rope, there's different masses.

So here is block 1, and block 2, and we can say here-- it doesn't matter-- but we'll say that M2 is bigger than M1.

And that gives us some intuition that we expect block 2 to go down and block 1 to go up.

Now in this problem there is friction between the rope and the pulley, so the rope is not sliding.

And what that means is that the pulley will rotate.

And also the mass of the pulley is not 0.

So these were all assumptions we made way back when were analyzing Newton's second law, but now we have to take into effect that there some rotational inertia to make the pulley start to have angular acceleration.

So what we'd like to do is to identify our three objects-- mass 1, the pulley, and mass 2.

And for mass 1 and mass 2, use Newton's second law, for the pulley, we'll use our torque relationships.

So let's begin by drawing our free body diagrams for object 1.

So we have tension in the rope pulling object 1 up, we have the gravitational force down.

And here it doesn't matter which way I'm going to choose my unit vectors, because I have this idea that M2 is bigger than M1.

I'm just going to choose j hat 1 up.

Now for block 2, I have M2g.

Now here's the place where lots of people get tripped up.

In the past we've been assuming that the tension in the rope is uniform everywhere.

In this problem, because the rope is not slipping and the pulley is not massless, the tension is not constant everywhere in the rope.

And we'll see more reasons why that can't be the case, so I'll have to identify a different tension on the other side T2 pulling the rope, pulling the block up.

And I'm going to choose j hat 2 down.

Notice my unit vectors are chosen in opposite directions.

The reason for that is that there's a constraint here that as block 2 goes down, block 1 goes up, if the acceleration of block 2 is positive, the acceleration of block 1 will also be positive if I choose unit vector pointing up.

So now I can write Newton's second law for both of these.

So for block 2, positive down, and 2g minus T2 equals M2a.

It's the same acceleration in the rope.

a is equal to a1, equal to a2, they're all positive.

And for 2, notice I have positive up minus M1g equals m1a.

So so far, these are my two equations.

I have three unknowns-- T1, T2, and the accelerations, and only two equations.

Now let's analyze the pulley.

So we have our pulley A. And what are the forces on the pulley?

Well forces and torques are a little bit different, but let's just draw our forces first.

There is a tension holding it up, we'll call that T3 from this rope pulling it up.

Now here's where we have to be careful.

Rope 1 is pulling the pulley down, that's what we called T1, so we draw a T1 on this side.

And the same rope on the other side is pulling the pulley down.

Notice that it is a different, T2.

Now this brings us to what we called our rotational coordinate system.

I do know expect that the angular acceleration of the pulley.

So as the pulley rotates in this problem, I expect that it's rotating in the direction like by the right hand rule, this way.

And so I expect to see the alpha pointing like that.

What I'll do is I'll choose a coordinate system for an angle theta.

And that will make positive direction like that for my rotational coordinate system.

Now when we write torque equals the moment of inertia of the pulley, times the angular acceleration, we have two torques.

The radius is R, radiuses of R. And if we're calculating the torque about the center of the pulley, we'll call that point 0, then each of these torques are in different directions.

So for instance, we would have to take for T1 going down and this vector from origin to where T1 is acting, we can extend that force.

And we see that that torque is pointing in the direction in-- direction like that.

And that's opposite or sign here.

So this torque, minus T1 R is negative.

What about the torque from the other force, T2?

Again, we would draw that vector.

Let's draw them over here.

We have T2 pointing down.

The point 0 is there.

The vector from 0 to 2 is pointing like that.

We extend that vector.

We put the arrow like that.

And we see that this one is pointing in our positive direction.

So we have a plus T2R equals IA alpha.

Now right away we can see why the tension in the strings is not the same.

Because if these tensions were the same, this quantity would be 0, but the tensions can't be the same because the pulley is rotating.

And that rotational inertia of the pulley is coming from the fact that the two torques are not the same on both sides.

So we now have our last equation here.

T2R minus T1R equals IA alpha.

But notice that I've introduced another variable here, so I guess again, I have four equations and only three unknowns.

We still have one last constraint.

Because the rope is sliding along the pulley and the radius is R, we know that a point on the rim of the pulley has acceleration A of the rope.

But the pulley has an alpha angular acceleration, so our constraint conditioned here is plus R alpha.

Now why did I put a plus sign?

Because when the pulley is rotating the direction shown, alpha will be positive.

This quantity, torque, will be bigger than this one, and so we'll have a positive alpha.

When I chose j hat 1 up and j hat 2 down, that made all my accelerations positive, and so I with the plus side.

If I had reversed my choice of unit vectors, then this would be a minus sign.

So you have to be extremely careful by making sure that the directions you chose for the linear force diagrams that give us A, and the direction we chose in our rotational coordinate system are consistent when we choose to relate the constraints between them.

So that's our last condition that a equals our R alpha.

So I now have four equations and four unknowns.

And if I want to solve for the acceleration a, then I have to need a strategy here.

And what I'll do to find a is I'll use this equation as my backbone.

Why did I do that?

Because I have the unknowns T2, T1, and alpha.

And I have separate equations that relate alpha to A, and T1 to a and T2 to a And so I can solve for T2.

And substitution here I'll need a little room.

So I get that T2 is M2g plus M2a.

I'm sorry, minus M2a times R. Now T1 is M1a plus M1g times R, and that's equal to times AI alpha, which is a over R.

So I'll now collect all of my a terms over here.

And so I get to M2gR minus M1gR, this term and this term, is equal to a times IA over R. Notice I have 2 plus signs here, so I get M1 plus M2 times R.

And I therefore conclude, we'll put it over here, that the acceleration a of is equal to M2 minus m1gR, divided by IA over R, plus M1 plus M2R.

Now, when I have this result, let's just check a number of things first.

Notice that if M2 is bigger than M1, a will be positive, which is what I expected.

Dimensionally, we have M2gR, down here, M2R, so this first term will have dimensions of acceleration g.

Now over here, we have IA over R, but remember moment of inertia is M times R square, so this also have the dimensions of mass and radius.

And so I have confidence dimensionally.

And the sine of A that this is the correct answer.

And that's how we solve the Atwood machine.