Let's consider an example of motion, in which we want to use our energy concepts.

So suppose we have a ramp which is circular, radius r.

And we have an object here.

And now, here's a surface a certain distance, d, that has friction, where the coefficients of friction is non-uniform.

So we'll write it mu-0 And we'll write it as mu-1x.

We'll take x equals zero.

And then here, there's a spring and a wall.

And here there is no friction.

Now as we drop, assuming we have enough height here, h, so that this block will slide across the friction, make it all the way across and start compressing its spring.

What I'd like to find is how much the spring has been compressed.

OK, now how do we analyze this?

Well the key is to use our energy principle, where we have w-external equals the change in mechanical energy.

And the key is to, what we're going to do is the tool that we're going to use is what we call energy diagrams for the initial.

So what does this mean?

We want to choose, we want to first identify the initial and final states that we're referring to.

So, in our picture, here is our initial state.

And I'll draw the final state in when the spring has been compressed a distance, x-final.

So I drew it on my diagram.

So we have initial and final states.

And now what we want to do is choose reference points, zero-points, zero-point for each potential function.

And show that on our diagram.

So for the potential energy of gravity, here, if we chose this to be y, then this y equals 0.

And u at y equals 0 is 0.

And we'll call that the zero point for gravity.

And the zero point for the spring is when this x, now I'm going to call here x equals 0.

So, let's call this a variable, I can call it anything I want.

I'll call it u-final.

And this is where u is zero.

So u-final just measures the stretch of the spring.

And so at y equals 0 is 0.

And u spring little u equal 0, 0.

So now I can identify my energies.

So let's talk about the initial energy is all gravitational potential.

That's mgy.

It's starting at rest.

And e-final well, here, this is the distance where it comes to rest, also.

So there's no final kinetic energy.

There's no gravitational potential energy, because we're on the surface at y equals 0.

But our spring has been compressed by 1/2 k little-u-final square.

Now, in terms of our external work equals the change in mechanical energy, we have now identified the right-hand side using these tools of the energy diagrams.

And I can write my description u-final squared, minus mgy.

Now what I have to do is think about the friction force, f kinetic friction, as the object moves.

This friction force is non-uniform.

If we were to draw n, and mg, then our friction force here is equal to the integral minus the integral from x equals 0, to x equals d.

Equals 0 to x equal d, of the friction force, which is equal to the coefficient of friction, ukndx.

Now that 0 to d, notice that our coefficient of friction is varying.

And I chose it intentionally to show you that friction is really an integral.

So we have mu-0 plus mu-1 xmgdx.

Let's put our integration variable in there.

And these are just two separate integrals.

The first one is easy.

It's minus mu-0 mgd.

And the second one is -mu-1 mg.

And we're just integrating x-prime, dx prime between 0 and d.

So that's simply d-squared over 2.

And that's equal to ku-final squared minus mgy.

Let's write that as y-initial.

And we're starting it at y-initial equals h.

And so, even though this is complicated, I can now solve for how much this spring has been compressed with a little bit of algebra.

And so I'm just going to bring a bunch of terms over to the other side.

And take the square root of 2, divided by k of mgy-initial.

Minus mu-0 mgd.

Minus mu-1 mgd-squared over 2.

And that's how much the spring is compressed.

Notice what I did not do was divide this up into a bunch of different motions.

I picked an initial state.

I picked a final state.

I drew my energy diagrams with my zero points.

I described the key parameters of initial and final states, y-i and u-final.

I defined the initial mechanical energy, the final mechanical energy.

And then applied the work energy, work mechanical energy principle.

I had to integrate the friction force because it was non-trivial and solve for how much the spring has been compressed.