We'd like to now calculate the angular momentum about an axis rotation of a symmetric object like a ring.

So here is our axis of rotation.

We're going to have omega pointing up.

That means omega's z will be positive, because that's going to be our k hat axis.

And this is a ring of mass m and radius r.

So here's our radius.

Now, and we're going to choose a point that lies somewhere on the symmetry axis of the ring.

And the object is rotating about that symmetry axis.

So it's passing through the center of mass.

Now, the way we'll do this is we'll divide our ring up into pairs of symmetric objects.

So we have an m here and a little delta m there.

And we're going to calculate the angular momentum first of that pair.

Well, we've already made that calculation that the angular momentum about s of the pair is twice the mass of each object.

That's the mass of the pair times r squared and it points in the omega direction.

And so to get the angular momentum of the ring, we'll just sum up a set of symmetric pairs until we expand out over the entire ring.

And so L of the ring about s is just the sum over the pairs of the mass of the pair times r squared omega.

And r squared and omega are all constants.

The total mass of the ring is just the sum over the pairs of the mass of the pair.

And that's the total mass m of the ring.

And so what we get is m r squared omega L of the ring s.

This is the moment of inertia about the axis for the ring, because all the mass is distributed a distance r away.

And so we see for a continuous symmetric body like a ring, again, because it's symmetric, the angular momentum only has a component along the axis of rotation.

Now, what if we had an extended symmetric object?

And let's see if I can possibly draw something like that.

So we'll draw a pear shaped object.

What is the moment of inertia of this pear shaped object, which is intended to be symmetric, about the z-axis?

Well, I can think of that pear shaped object as consisting of a ring and it's a solid object.

This is a solid symmetric object.

And then inside the ring is just more rings.

So the whole disk here is just a superposition of rings.

And so the moment of inertia-- so the angular momentum of just this disk is the moment of the disk about the axis.

And now I'll just add more disks that are symmetric.

And so by exactly the same calculation as I have here, the moment of this symmetric object is just-- the moment of inertia is more complicated for this object.

And it points in the omega direction.

So again, in conclusion, a symmetric object about the axis of rotation only has a component of L pointing in the direction of omega.

The constant of proportionality between the angular momentum and the angle the velocity is the moment of inertia about that axis.

This we need to calculate for each symmetric object.