So we're going to return to our one-dimensional elastic collision with no external forces.

So we have object 1 moving with velocity V1 initial and object 2 maybe it's moving this way with V2 initial i hat, again, on a frictionless surface.

And we'll call that our initial state.

And here you can imagine we're going to use a ground reference frame.

So both objects are moving.

And our final state has object 1-- well, we don't know, again, which way it's going.

We can just say it bounced back.

And object 2 also bounced back.

But the goal of our problem, of course, is to determine these vectors.

And by knowing the vectors, we know which way they go.

Now because energy and momentum are constant, let's write down our two equations.

And I'm going to write them down, again, in terms of components.

So we have 1/2 m1 V1x initial squared plus 1/2 m2 V2x initial squared equals 1/2 m1 V1x final squared plus 1/2 m2 V2x final squared.

Now we're going to do some algebraic manipulations here.

So the first thing I'm going to do is just eliminate these halves because it's not necessary.

And I don't want to rewrite this equation.

And this is our fact that our energy is constant.

And now our condition that momentum is constant, we'll write this-- now, I'm going to leave a little space here intentionally.

And our condition that momentum is constant is m1 Vx initial plus m2 V2x initial equals m1 V1x final plus m2 V2x final.

Now this energy equation can be factored in by bringing all the m1 terms to one side and the M2 terms to the other side.

So when I write that, I'll need a little room.

I have m1 V1x initial squared minus V1x final squared.

And that's equal to m2 V2x final squared minus V2x initial squared.

So I've just brought those terms over to the other side.

Now likewise I'll do the same thing down here.

I have m1 V1x initial minus V1x final.

And that's equal to m2 V2x final minus V2x initial.

Now here comes the algebraic trick in which I'm going to linearize these systems.

This is a squared minus b squared, which factors into a plus b times a minus b.

So let's give ourselves a little room.

m1 V1x initial plus-- let's put the minus sign first.

minus V1x final times V1x initial plus V1x final.

Factored that term.

We have the same factoring on the other side.

So it's just identical, V2x final minus V2x initial times V2x final plus V2x initial.

Now let's call this equation 1a and our momentum factored as 2a.

Now if you notice, the momentum piece is appearing exactly there and exactly here.

So when I divide 1a by 2a-- and I'll just symbolically represent that-- then these two pieces cancel.

And that leads to just this term equal to that term.

And the significance, as you'll see when I write it out, 1 of x 1x final equals V2x final plus V2x initial.

I've solved the quad-- I've eliminated the squared terms, linearized the system.

Now I still want to write this equation in another way.

Another important point to notice is that this equation is independent of mass.

Now what I want to do is write this in terms of those concepts of relative velocity we had.

Remember just to motivate this, V relative by definition was V1 minus V2.

So let's write this in terms of the initial and the final.

So in order to do that, we have to bring this initial term over to here and this final term over to there.

And so this equation, which will give it a number 3, and now we'll modify that by calling it 3a.

We have V1x initial minus V2x initial.

Now notice the sign.

I'm going to want to keep the order of 1 and 2.

So I have to put a minus sign, V1x final minus V2x final.

And when written this way, this is the initial component of the relative velocity.

And in there is the final component of relative velocity.

So by combining these two equations, I have this remarkable result that V relative initial is minus V relative final.

And this condition is a very powerful tool for simply analyzing one-dimensional elastic and inelastic collisions.

I'd like to even give this a name.

I'd like to call it the energy momentum equation.

Now there's a lot of significant things about this.

So let's just think about it for a moment.

We have that V relative initial in magnitude is equal to V relative final.

And so right away, this gives us some insight into any collision.

We can see whether a collision, if we know what the relative initial velocity is, we know that the final relative velocity has the same magnitude but simply switches direction.

And that's a powerful tool in which to analyze collisions without doing a lot of algebra.