

Let's now consider the generalization of the result that the torque on a single particle about a point causes the angular momentum of that particle about that point to change to a collection of particles.

So let's begin by indicating some  $i$ th particle with momentum  $P_i$  and some  $j$ th particle over here with momentum  $P_j$ .

And this is a big system of  $n$  particles.

And we'd like to calculate the angular momentum about some point,  $s$ .

So that angular momentum will consist of the direct product of the vector from  $s$  to the  $i$ th particle and the direct product of the vector  $r_{sj}$  to the  $j$ th particle.

So the angular momentum, total, will be the sum over all the particles, 1 to  $n$ , of this sum,  $r_{sj}$  cross  $P_j$ .

Now, what we'd like to do is, again, as before, take the time derivative of this angular momentum.

We have the sum--  $j$  goes from 1 to  $n$ .

We have two derivatives here, by the product rule, cross  $P_j$ , plus the sum,  $j$  goes from 1 to  $n$  of  $r_{sj}$  cross  $F_j$ , where we use the fact, as we did before, that the force on the  $j$ th particle is equal to the change in momentum of the  $j$ th particle.

Now, this first piece, again, the derivative of the vector from  $s$  to the  $j$ th part is the velocity.

So we have  $j$  equals 1 to  $n$  of  $v_j$ -- the velocity of the  $j$ th particle-- cross  $M_j$ ,  $P_j$ .

And that's 0, as we know that a vector cross product with itself is 0.

And this piece in here-- now, we have to be a little bit careful-- but  $r_{sj}$  cross  $F_j$  is the torque on the  $j$ th particle.

So that is the torque on the  $j$ th particle.

Now, remember that we showed that internal-- the forces on the  $j$ th particle could be both due to internal or external forces.

And as long as the internal forces pointed between two particles-- pointed along the line connecting those particles, the internal torques cancel in pairs.

And this will only be the external torque.

So we've assumed that the internal torques cancel in pair.

And that has to do with an assumption about the direction of the internal forces.

And so we can conclude that this is just the total external torque about the point  $s$ ,  $s$  total.

And we'll drop that total, and we'll conclude that the torque for our system of particles is just-- causes the angular momentum of the system of particles to change.

The calculation is exactly like the single particle, with a few subtleties that have to do with internal torques canceling in pairs.