I would now like to calculate the moment of inertia of a uniform sphere.

And it has a mass m and radius r.

I'm going to look at three axes.

So I'll call this the x-axis, the y-axis, and the z-axis.

And first, let's calculate the moment about the z-axis.

So if I write down our definition, and I'm going to calculate it about the center of mass, so the moment about the z-axis.

How do we do that?

Well, we take a mass element, and have to be a little bit careful here.

Because if you think about what is our [? perp ?] for this mass element, it's actually x squared plus y squared, squared.

So the distance here, because it's going in a circle, the radius of that circle is x squared plus y squared.

So we have x squared plus y squared, and we're integrating over the sphere.

Now, this looks like a tough integral.

But let's now look at what would the moment of inertia be about the x-axis?

Well, the only difference here is I'm integrating now, first, if it's rotating about this axis, and I had my mass element, instead of x squared plus y squared about the z-axis, it's y squared plus z squared about the x-axis.

That's the perpendicular distance.

And if I calculate the moment of inertia about the y-axis, then same argument-- I have z squared plus x squared.

Now, the beauty of this problem-- and in physics, when we talk about beauty, we often talk about symmetry-- is that by the symmetry of the sphere, all of these moments are equal.

And let's call that I cm.

Now, if I add these three pieces together, what do I get?

So if I add I cm z plus I cm x plus I cm y, I get 3 times the moment about the center of the axes.

So what happens when I add these three integrals?

I get dm.

If you'll notice, x squared appears twice, y squared appears twice, and z squared appears twice.

So we get 2 times x squared plus y squared plus z squared.

But x squared plus y squared plus z squared is the radius of a small sphere of thickness dr.

And my mass element-- now I have to integrate over the sphere.

And so now our mass element, dm, is the volume density of this times the volume.

Now what is the volume density?

Well, that's the total mass over 4/3 pi r cubed.

And what is the volume of a sphere of radius r and thickness dr?

That's 4 pi r squared dr.

So if I put that into my expression, what I get-- let's just get rid of the 4 pi's, and I get 3m over r cubed times r squared dr.

And so our 3 I cm, I have a factor of 2, this integral becomes 2 times dm times r squared.

And the dm is 3m r cubed r squares dr times another r squared.

And what are we integrating over are r variable from?

These spherical shells that we're integrating outward go from r equal 0 to r equal capital R.

Notice that our 3 m's cancel.

And we'll just write this is I cm equals factor 2 times m over r cubed times the integral of r to the fourth dr from zero to R. And that's a simple integral to do.

r to the fourth is r to the fifth divided by 5.

And so we get 2 over 5 m R to the fifth over R cubed.

And we conclude that the moment of inertia about any of the axes of the sphere is 2/5 m R squared.

And in this calculation, it's a beautiful example of how we use the symmetry of the sphere to simplify very complicated integrals.