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One of our classic problems to analyze using Newton's second law is the motion of two blocks with a rope that's wrapped around a pulley.

So imagine we have a pulley, P, and we hang from that block 1 and we hang from that block 2.

The pulley is suspended by another rope, and it's attached to a wall.

And let's now try to use Newton's second law to analyze this motion.

Now, going back to our methodology, the first thing we want to ask ourselves is to identify what are all the moving pieces?

So I'm going to make my first assumption here, that the rope/pulley a surface so that's right here is frictionless.

Now, what that assumption means is that the pulley will remain at rest.

And therefore, when we want to break this problem down, our first question is to identify all the moving objects.

And so we really have three.

We have mass, which we label by 1, 2, and the rope.

So our goal now is to identify all the moving objects and to draw free body force diagrams for each moving object.

Now, one of the things that I'm especially interested in talking about is to keep in mind to identify action-reaction pairs.

So as I go through and draw these free body diagrams, we want to ask ourselves, what forces form Newton's third law action-reaction pairs?

So what I'll do is I'll start with object 1.

Now, I'm object 1 has a gravitational force M 1 g.

And the rope is pulling object 1 up with the tension at the end of the rope T rope 1.

So the action- reaction pair to M 1 g is the Earth, is the force of this mass on the Earth, which we're not considering.

What about the rope?

So now, let's draw a picture of our rope.

And mass, this object here is pulling the rope down, and so we have, on the rope, object 1 pulling it down.

And this is our action-reaction pair.

What are the other forces on the rope?

Well over here, object 2, t 2, is pulling the rope down with the tension at the end of that rope.

Now, also, the pulley is exerting a force on the rope upwards, because we have that force.

And so these are the force diagrams on the rope.

Now, what about the action-reaction pair to T 2 rope?

Well, let's continue and draw 2.

We have, again, gravitational force on 2, the Earth is the action-reaction pair.

The force of 2 on the Earth upwards is equal to m 2g of the Earth downwards.

We're not drawing the Earth in this picture, so we don't show it.

Now, here is the force of the rope on 2.

And there is our other action-reaction pair.

So these are Newton's third law pairs in this object.

Now, now that we've identified all the forces, we want to apply Newton's second law to each of these objects that are moving.

So let's begin by remembering that if our rope-- our next assumption here, it's assumed that the mass of the rope is very light.

And then the tension in the rope is constant.

And the implication of that is T 2 r equals t 1 r.

And that's why we can now identify this as T and that as T.

Little bit later on, when we talk about pulleys that have mass with friction between them, the tension on the two sides of the pulley will not be the same.

But for the moment, we've made these assumptions, and that holds.

And now, we just really have objects 1 and object 2 analyze.

And so one of the things that helps a lot is, we need to choose unit vectors in order to write down the vector equations for Newton's second law.

So let's just here assume for ourselves that M 1 is greater than M 2.

This gives us some feeling for how the system moves.

I like to do this because it gives me a way to choose my unit vectors to make all the accelerations positive.

So when M 1 is bigger than M 2, I expect M 1 to go down and 2 go up.

And that's how I'm going to choose unit vectors j hat 1.

Now, here is the very interesting thing.

We choose a separate coordinate system for each object.

So I'm going to write that down.

We want to choose separate unit vectors for each object.

And our accelerations will be with respect to those unit vectors.

This is where a lot of people get tripped up.

So here, I'm choosing j hat 2.

And when I choose that, I expect a 2 to be positive because 2 is going up in the direction of j hat.

And here, I expect a 1 to be positive also, because 1 is going down.

Now I can draw Newton's second law on 1.

So I have F 1 equals M 1 a 1.

And now I analyze my forces, M 1 g is in the positive j hat direction, t is in the negative j hat direction, so I have M 1 g minus t equals M 1 a 1, which I'm expecting to be positive.

And in the same way, on 2, F 2 equals M 2 a 2.

I look at my force diagram.

Now, notice, j hat is up, so t is positive and 2 g is minus my direction, and 2 g equals M 2 a 2.

So I now have two equations, but when I look at these equations, I see that I have three unknowns.

I have t, a 1, and a 2.

But I have a constraint, because these objects are moving together and the way I've chosen the coordinate systems, a 1 is equal to plus a 2, both are positive with respect to my choice of coordinate systems.

And so these two as are the same.

And now I can solve my equations for t and a.

And let's quickly do that.

M 1 g minus t equals M 1 a.

And over here, let's solve for the tension, for the accelerations here.

T is equal to m 2 a plus m 2 g.

And when I substitute the t into that equation, we'll find some space for that.

we'll write that as, if I put in the M 2 minus M 2 a minus M 2 g and bring the M 2 a to the other side, then it's a little bit of algebra, but I think you'll trust me that this is M 1 minus M 2 g over M 1 plus M 2.

Now, we're almost done.

We need to check our answers.

First off, does it have the dimensions of acceleration?

Answer, yes, mass divided by mass, g has the dimensions of acceleration.

So that's my first check always with my algebra.

Second check, what if M 1 is equal to M 2?

Then the acceleration is 0, I expect them to be balanced.

I've said that M 1 is greater than M 2.

Positive sign, my A is positive.

That's how I set up my coordinate system, so I expect that too.

By the way, our symbol a was equal to those.

And so I'm pretty confident that my result is correct.

And that's how we apply Newton's second law to the pulley with several assumptions, frictionless surface, massless rope.

And think about how we chose our unit vectors.

Each object gets its separate coordinate system.

I'm not using the same coordinate system for each object.

I could've, but then there would be a subtlety to this condition, the accelerations would be opposite in sign.